CRB ANALYSIS OF NEAR-FIELD SOURCE LOCALIZATION USING UNIFORM CIRCULAR ARRAYS

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ABSTRACT

This paper is devoted to the Cramer Rao bound (CRB) on the azimuth, elevation and range of a narrow-band near-field source localized by means of a uniform circular array (UCA), using the exact expression of the time delay parameter. After proving that the conditional and unconditional CRB are generally proportional for constant modulus steering vectors, we specify conditions of isotropy w.r.t. the distance and the number of sensors. Then we derive very simple, yet very accurate non-matrix closed-form expressions of different approximations of the CRBs.

Index Terms— Cramer Rao bound, near field source localization, uniform circular array, isotropy.

1. INTRODUCTION

A considerable literature has been dedicated to the CRB on the direction of arrival (DOA) of narrow-band sources. However, most of it is limited the far-field case, meaning that a planar wavefront is impinging on the sensors array. The near-field assumption implies that the curvature of the waves has to be taken into account, resulting into more complicated models parameterized by the source DOAs and range. Hence, we can identify only very few studies of this kind. Inspired by the subspace-based DOA algorithms, early ones were based on an approximate propagation model based on second-order Taylor expansion of the time delay parameter [1, 2]. Only lately has the exact time delay formula been used [3] for the only uniform linear arrays (ULA).

The UCA has been popular with many applications [4, 5, 6] and dedicated estimation algorithms have been proposed for far-field source localization [12, 13]. Isotropy is a compelling feature of UCA. Not only does it mean that the CRB on the azimuth and elevation of a single source is uniform for all azimuths [7], but also that the two estimates are uncorrelated and that the CRB is not affected by the indeterminacy about the wave speed of propagation [8]. We are interested in analyzing the behavior of UCA for near-field sources, and in particular to what extend is isotropy maintained. Furthermore, very simple non-matrix closed-form expressions of approximations of the CRB on the azimuth, elevation and range are given, where the sixth-order approximation of the CRB on the azimuth turns out to be very accurate.

The paper is organized as follows. Section 2 formulates the problem and specifies the data model. Section 3 is devoted to the CRB derivations. After proving that the so-called conditional and unconditional CRB are generally proportional for constant modulus steering vectors, we specify conditions of isotropy w.r.t. the range and the number of sensors. We derive very simple non-matrix closed-form expressions of different approximations of the CRB. Finally some numerical illustrations are given in Section 4.

2. DATA MODEL

A UCA is made of P identical and omni-directional sensors $C_1, \cdots, C_P$ located in the (x, y) plane, at a distance r_0 from the origin O and an angle, respectively, $\theta_1, \cdots, \theta_P$ from $[O, x)$ such that $\theta_p \stackrel{\text{def}}{=} \theta - \frac{2\pi(p-1)}{P}$. It is radiated by a narrow-band source located in the antenna near-field, whose position is characterized by azimuth θ, elevation ϕ and range r as shown in Fig.1

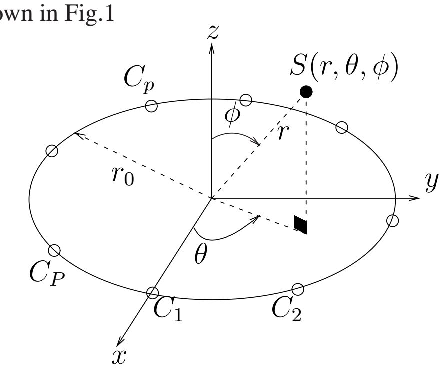

Fig.1 Uniform circular array and source DOAs.

Corrupted by an additive noise with complex envelope $\mathbf{n}(k)$, the complex envelope of the signals collected by the array of sensors is modelled as

$$\mathbf{x}_k = s_k\mathbf{a}(\boldsymbol{\alpha}) + \mathbf{n}_k, k = 1, ..., K,$$

where $\mathbf{a}(\boldsymbol{\alpha})$ is the so-called steering vector parameterized by $\boldsymbol{\alpha} = [\theta, \phi, r]^T$. Referenced to the origin, its is given by

$$\mathbf{a}(\boldsymbol{\alpha}) = [e^{i\tau_1(\boldsymbol{\alpha})}, ..., e^{i\tau_p(\boldsymbol{\alpha})}, ..., e^{i\tau_P(\boldsymbol{\alpha})}]^T$$

where $\tau_p(\boldsymbol{\alpha}) = 2\pi \frac{d_p}{\lambda_0}$ with $d_p = SO - SC_p$ and λ_0 is the propagation wavelength. In fact, τ_p is given by

$$\tau_p(\boldsymbol{\alpha}) = 2\pi \frac{r}{\lambda_0} \left(1 - \sqrt{1 - 2\frac{r_0}{r} \cos\theta_p \sin\phi + \frac{r_0^2}{r^2}} \right).$$

The usual statistical properties about $\mathbf{n}_k$ and s_k are the following: (i) s_k and $\mathbf{n}_k$ are independent, (ii) $(\mathbf{n}_k)_{k=1,...,K}$ are independent, zero-mean circular Gaussian distributed with covariance $\sigma_n^2 \mathbf{I}_P$, (iii) $(s_k)_{k=1,...,K}$ are assumed to be either deterministic unknown parameters (the so-called conditional or deterministic model), or independent zero-mean circular Gaussian distributed with variance σ_s^2 (the so-called unconditional or stochastic model).

3. CRB DERIVATION

3.1. General expression of the CRB

General compact expressions of the CRB, concentrated on the DOA parameter alone have been derived for these two models of sources (see e.g. [9] for one parameter per source and [10, Appendix D] for several parameters per source). Specialized to a single source with one or several parameters per source, these expressions $\text{CRB}_{\text{det}}(\boldsymbol{\alpha})$ and $\text{CRB}_{\text{sto}}(\boldsymbol{\alpha})$ have all been studied independently in the literature (see e.g. the recent papers [11] and [3] which even concludes by "extension of this work for stochastic sources is under consideration").

In fact, it can be proved (proof is not given here by lack of space) that the expressions $\text{CRB}_{\text{det}}(\boldsymbol{\alpha})$ and $\text{CRB}_{\text{sto}}(\boldsymbol{\alpha})$ for arbitrary parametrization $\boldsymbol{\alpha} = [\alpha_1, ..., \alpha_L]^T$ of the steering vector $\mathbf{a}(\boldsymbol{\alpha})$ related to array geometry or polarization such that $\|\mathbf{a}(\boldsymbol{\alpha})\|^2 = P$, are proportional[1], an issue previously overlooked. More precisely:

$$\text{CRB}_{\text{sto}}(\boldsymbol{\alpha}) = \left(1 + \frac{\sigma_n^2}{P\sigma_s^2}\right) \text{CRB}_{\text{det}}(\boldsymbol{\alpha}) = [\text{FIM}(\boldsymbol{\alpha})]^{-1} \quad (1)$$

where $\sigma_s^2 \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K |s_k^2|$ for the deterministic model of sources, with

$$\text{FIM}(\boldsymbol{\alpha}) = c_\sigma \left(P\mathbf{D}^H(\boldsymbol{\alpha})\mathbf{D}(\boldsymbol{\alpha}) - \mathbf{D}^H(\boldsymbol{\alpha})\mathbf{a}(\boldsymbol{\alpha})\mathbf{a}^H(\boldsymbol{\alpha})\mathbf{D}(\boldsymbol{\alpha}) \right) \quad (2)$$

where $c_\sigma \stackrel{\text{def}}{=} \frac{2K\sigma_s^4}{\sigma_n^2(\sigma_n^2 + P\sigma_s^2)}$ and $\mathbf{D}(\boldsymbol{\alpha}) \stackrel{\text{def}}{=} \left[\frac{\partial \mathbf{a}(\boldsymbol{\alpha})}{\partial \alpha_1}, ..., \frac{\partial \mathbf{a}(\boldsymbol{\alpha})}{\partial \alpha_L}\right]$.

In the addressed problem, $\mathbf{D}(\boldsymbol{\alpha}) = \left[\frac{\partial \mathbf{a}(\boldsymbol{\alpha})}{\partial \theta}, \frac{\partial \mathbf{a}(\boldsymbol{\alpha})}{\partial \phi}, \frac{\partial \mathbf{a}(\boldsymbol{\alpha})}{\partial r}\right] = [\mathbf{a}_\theta^{'}, \mathbf{a}_\phi^{'}, \mathbf{a}_r^{'}]$. Thanks to (1), we only consider the stochastic source model.

[1] Note that (1) is no longer valid for parametrizations for which $\|\mathbf{a}(\boldsymbol{\alpha})\|$ depends on $\boldsymbol{\alpha}$.

3.2. Exact Fisher information matrix

The elements of the FIM (2) are, for $i, j = 1, 2, 3$, given by

$$\text{FIM}_{i,j}(\boldsymbol{\alpha}) = c_\sigma[P(\mathbf{a}_{\alpha_i}^{'H}\mathbf{a}_{\alpha_j}^{'}) - (\mathbf{a}_{\alpha_i}^{'H}\mathbf{a})(\mathbf{a}_{\alpha_j}^{'H}\mathbf{a})^*],$$

where $\mathbf{a}_{\alpha_i}^{'H}\mathbf{a}_{\alpha_j}^{'} = \sum_{p=1}^P \tau_{p,\alpha_i}^{'}\tau_{p,\alpha_j}^{'}$, $\mathbf{a}_{\alpha_i}^{'H}\mathbf{a} = \sum_{p=1}^P \tau_{p,\alpha_i}^{'}$, and $\tau_{p,\alpha_i}^{'} \stackrel{\text{def}}{=} \frac{\partial \tau_p(\boldsymbol{\alpha})}{\partial \alpha_i}$. Consequently, we get

$$\begin{aligned}
\text{FIM}_{1,1}(\boldsymbol{\alpha}) &= c_\sigma^{'} \sin^2\phi \left[P\sum_{p=1}^P u_p^2 - \left(\sum_{p=1}^P u_p\right)^2 \right] \\
\text{FIM}_{2,2}(\boldsymbol{\alpha}) &= c_\sigma^{'} \cos^2\phi \left[P\sum_{p=1}^P v_p^2 - \left(\sum_{p=1}^P v_p\right)^2 \right] \\
\text{FIM}_{1,2}(\boldsymbol{\alpha}) &= c_\sigma^{'} \frac{\sin(2\phi)}{2} \left[\sum_{p=1}^P u_p \sum_{p=1}^P v_p - P\sum_{p=1}^P v_p u_p \right] \\
\text{FIM}_{3,3}(\boldsymbol{\alpha}) &= c_\sigma^{''} \left[P\sum_{p=1}^P (1+w_p)^2 - \left(\sum_{p=1}^P [1+w_p]\right)^2 \right] \\
\text{FIM}_{1,3}(\boldsymbol{\alpha}) &= c_\sigma^{'''} \sin\phi \left[\sum_{p=1}^P u_p \sum_{p=1}^P w_p - P\sum_{p=1}^P u_p w_p \right] \\
\text{FIM}_{2,3}(\boldsymbol{\alpha}) &= c_\sigma^{'''} \cos\phi \left[\sum_{p=1}^P v_p \sum_{p=1}^P w_p - P\sum_{p=1}^P v_p w_p \right]
\end{aligned}$$

where $u_p \stackrel{\text{def}}{=} \sin\theta_p/\sqrt{\beta_p}$, $v_p \stackrel{\text{def}}{=} \cos\theta_p/\sqrt{\beta_p}$, $w_p \stackrel{\text{def}}{=} \epsilon v_p \sin\phi - \beta_p^{-1/2}$, $\beta_p \stackrel{\text{def}}{=} 1 - 2\epsilon\cos\theta_p\sin\phi + \epsilon^2$ and $\epsilon \stackrel{\text{def}}{=} \frac{r_0}{r}$. There, we also define constants $c_\sigma^{'} \stackrel{\text{def}}{=} c_\sigma \left(\frac{2\pi r_0}{\lambda_0}\right)^2$, $c_\sigma^{''} \stackrel{\text{def}}{=} c_\sigma \left(\frac{2\pi}{\lambda_0}\right)^2$ and $c_\sigma^{'''} \stackrel{\text{def}}{=} c_\sigma \left(\frac{2\pi}{\lambda_0}\right)^2 r_0$.

Taylor series expansions of $1/\beta_p$ and $1/\sqrt{\beta_p}$ w.r.t. $\epsilon = r_0/r$, followed by elementary trigonometric relations, show that all elements of the FIM depend on the azimuth θ only through the sums $\sum_{p=1}^P \cos k\theta_p$ and $\sum_{p=1}^P \sin k\theta_p$ for $k = 1, 2, ...$, which can be easily simplified thanks to

$$\sum_{p=1}^P e^{ik\theta_p} = \begin{cases} Pe^{ik\theta} & \text{if } k/P \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

It turns to be that while the FIM is periodic in θ of period $2\pi/P$, as one may expect, it is not constant anymore, i.e. the UCA is no longer strictly isotropic in the near-field.

3.3. Isotropy of the UCA

We analyze the way isotropy, guaranteed at the antenna far-field [7], is deteriorated as a result of a decreasing source range r or a decreasing number of sensors P. Careful examination of the Taylor expansion of the elements of the FIM w.r.t. ϵ, where only the θ dependence is retained, yields to

$$\mathrm{FIM}_{i,j}(\boldsymbol{\alpha}) = \sum_{k=0}^{\infty}\left(\sum_{p=1}^{P} g_k^{(i,j)}(\cos\theta_p, \sin\theta_p)\right)\epsilon^k, \quad (4)$$

where $g_k^{(i,j)}$ is a polynomial expression of $\cos\theta_p$ and $\sin\theta_p$ of degree $k+2$, $k+1$ or k for $(i,j=1,2)$, $(i=1,2, j=3)$ or $(i=j=3)$ respectively. By linearizing this polynomial, we have for example for $i,j=1,2$:

$$g_k^{(i,j)}(\cos\theta_p, \sin\theta_p) = \sum_{\ell=0}^{k+2} c_{\ell,k}^{(i,j)}\cos(\ell\theta_p) + \sum_{\ell=1}^{k+2} s_{\ell,k}^{(i,j)}\sin(\ell\theta_p)$$

where $c_{0,k}^{(i,j)} = 0$ for odd degrees of $g_k^{(i,j)}$. Then, using (3), focusing on θ and carefully studying the first terms of the Taylor expansion in ϵ (4), we obtain

$$\mathrm{FIM}_{i,j}(\boldsymbol{\alpha}) = \sum_{k=0}^{\lfloor (P-3)/2\rfloor} b_{2k}^{i,j}\epsilon^{2k} + \sum_{k=P-2}^{\infty} b_k^{i,j}(\theta)\epsilon^k \quad (5)$$

$$\mathrm{FIM}_{i,j}(\boldsymbol{\alpha}) = \sum_{k=P-2}^{\infty} b_k^{i,j}(\theta)\epsilon^k \quad (6)$$

$$\mathrm{FIM}_{2,3}(\boldsymbol{\alpha}) = \sum_{k=3}^{P-1} b_k^{i,j}\epsilon^k + \sum_{k=P}^{\infty} b_k^{i,j}(\theta)\epsilon^k \quad (7)$$

$$\mathrm{FIM}_{3,3}(\boldsymbol{\alpha}) = \sum_{k=2}^{\lfloor (P-1)/2\rfloor} b_{2k}^{i,j}\epsilon^{2k} + \sum_{k=P}^{\infty} b_k^{i,j}(\theta)\epsilon^k, \quad (8)$$

for $P \geq 4$ and $i=j=1$ or $i=j=2$ (5), $i=1, j=2,3$ (6), where the terms $b_k^{i,j}$ and $b_{2k}^{i,j}$ do not depend on θ.

Note that zero-order terms of the block $[\mathrm{FIM}(\boldsymbol{\alpha})]_{(1:2,1:2)}$ derived from (5) and (6) give the $\mathrm{FIM}(\theta,\phi)$ of the far-field case. Using the decoupling of θ and ϕ of the far-field case that we uncover by (6), we have $\mathrm{CRB}_{\mathrm{FF}}(\theta) = 1/b_0^{1,1}$ and $\mathrm{CRB}_{\mathrm{FF}}(\phi) = 1/b_0^{2,2}$.

To illustrate the behavior of the FIM w.r.t. P, $\frac{\mathrm{FIM}_{1,1}(\boldsymbol{\alpha})}{c_\sigma c_{r_0}\sin^2\phi}$ is given by the following expressions for $P = 3,4,5,6$:

P	
3	$1-\epsilon^2\sin\phi\cos 3\theta + o(\epsilon^2)$
4	$1-\epsilon^2(\cos^2\phi - \sin^2\phi\sin 4\theta) + o(\epsilon^2)$
5	$1-\epsilon^2\cos^2\phi - \epsilon^3\sin^3\phi\cos 5\theta + o(\epsilon^3)$
6	$1-\epsilon^2\cos^2\phi - \epsilon^4(1-3\sin^2\phi+2\sin^4\phi$ $-\sin^4\phi\cos 6\theta) + o(\epsilon^4)$

with $c_{r_0} \stackrel{\mathrm{def}}{=} \left(2\pi\frac{r_0}{\lambda_0}\right)^2\frac{P^2}{2}$ and where $\lim_{\epsilon\to 0} o(\epsilon)/\epsilon = 0$. Consequently, from (5)-(8), the following can be concluded: For a number P of sensors, the FIM of the UCA does not depend on the azimuth up to the order $P-3$ in r_0/r. More precisely, we see from (6), that the parameters θ and (ϕ, r) are decoupled up to the order $P-1$ in r_0/r. Consequently, the azimuth CRB does not depend on the azimuth up to the order $P-1$ in r_0/r, in contrast to the elevation and range CRBs that require an order smaller or equal to $P-3$. *So, for r or P fixed, isotropy increases when P or r increases, respectively, and the azimuth CRB is much less sensitive to the azimuth than those of the elevation and range.*

3.4. Closed-form expression of the CRB

To further improve our understanding of the near-field CRBs, we investigate different closed-form expressions of the approximations of the CRB on $\boldsymbol{\alpha}$. Note that to obtain these approximations, we have to elaborate a little bit the necessary order of the Taylor expansions of each term of $\mathrm{FIM}(\boldsymbol{\alpha})$ because the order in ϵ of these terms may be different.

For $P > 8$, we have proved that[2]

$$\begin{aligned}
\frac{\mathrm{FIM}_{1,1}(\boldsymbol{\alpha})}{c_\sigma c_{r_0}} &= \sin^2\phi\left[1-\epsilon^2\cos^2\phi + \epsilon^4 g_1(\sin^2\phi)\right. \\
&\qquad \left. + \epsilon^6 g_2(\sin^2\phi)\right] + o(\epsilon^7) \\
\mathrm{FIM}_{1,2}(\boldsymbol{\alpha}) &= o(\epsilon^7) \\
\mathrm{FIM}_{1,3}(\boldsymbol{\alpha}) &= o(\epsilon^7) \\
\frac{\mathrm{FIM}_{2,2}(\boldsymbol{\alpha})}{c_\sigma c_{r_0}} &= \cos^2\phi\left[1-\epsilon^2\left(1-\frac{5}{2}\sin^2\phi\right)\right] + o(\epsilon^3) \\
\frac{\mathrm{FIM}_{3,3}(\boldsymbol{\alpha})}{c_\sigma c_{r_0}} &= \frac{\sin^2\phi}{16 r_0^2}\left[\sin^2\phi + \epsilon^2 g_3(\sin^2\phi)\right]\epsilon^4 + o(\epsilon^7) \\
\frac{\mathrm{FIM}_{2,3}(\boldsymbol{\alpha})}{c_\sigma c_{r_0}} &= -\frac{\sin\phi\cos\phi}{r_0}\left(1-\frac{7}{8}\sin^2\phi\right)\epsilon^3 + o(\epsilon^3),
\end{aligned}$$

with

$$\begin{aligned}
g_1(\sin^2\phi) &\stackrel{\mathrm{def}}{=} 1-3\sin^2\phi+2\sin^4\phi \\
g_2(\sin^2\phi) &\stackrel{\mathrm{def}}{=} -1+6\sin^2\phi-10\sin^4\phi+5\sin^6\phi \\
g_3(\sin^2\phi) &\stackrel{\mathrm{def}}{=} 16-33\sin^2\phi+\frac{35}{2}\sin^4\phi.
\end{aligned}$$

This expression of $\mathrm{FIM}(\boldsymbol{\alpha})$ allows us to derive after some manipulations of $\mathrm{FIM}^{-1}(\boldsymbol{\alpha})$ the following closed-form expressions of the CRBs:

$$\begin{aligned}
\mathrm{CRB}(\theta) = \mathrm{CRB}_{\mathrm{FF}}(\theta)\Big[1+\cos^2\phi\frac{r_0^2}{r^2} \\
+ \sin^2\phi\cos^2\phi\frac{r_0^4}{r^4} + h_1(\sin^2\phi)\frac{r_0^6}{r^6} + o\left(\frac{r_0^7}{r^7}\right)\Big] \quad (9)
\end{aligned}$$

$$\mathrm{CRB}(\phi) = \mathrm{CRB}_{\mathrm{FF}}(\phi)\left[1+h_2(\sin^2\phi)\frac{r_0^2}{r^2} + o\left(\frac{r_0^2}{r^2}\right)\right] \quad (10)$$

$$\mathrm{CRB}(r) = \frac{16}{c_\sigma c_{r_0}\sin^4\phi}\frac{r^4}{r_0^2}\left[1+h_3(\sin^2\phi)\frac{r_0^2}{r^2} + o\left(\frac{r_0^2}{r^2}\right)\right], \quad (11)$$

where

$$\begin{aligned}
\mathrm{CRB}_{\mathrm{FF}}(\theta) &= 1/(c_\sigma c_{r_0}\sin^2\phi) \\
\mathrm{CRB}_{\mathrm{FF}}(\phi) &= 1/(c_\sigma c_{r_0}\cos^2\phi)
\end{aligned}$$

[2]For $P > 6$, $\mathrm{FIM}_{1,1}(\boldsymbol{\alpha}) = c_\sigma c_{r_0}\sin^2\phi[1-\epsilon^2\cos^2\phi+\epsilon^4 g_1(\sin^2\phi) + o(\epsilon^5)$ and $\mathrm{FIM}_{1,2}(\boldsymbol{\alpha}) = \mathrm{FIM}_{1,3}(\boldsymbol{\alpha}) = o(\epsilon^5)$ and the other FIM expressions remain valid.

denote the CRB on the azimuth and elevation in the far field case [7], and where we have introduced

$$\begin{aligned} h_1(\sin^2\phi) &\stackrel{\text{def}}{=} -\sin^2\phi + 3\sin^4\phi - 2\sin^6\phi \\ h_2(\sin^2\phi) &\stackrel{\text{def}}{=} \frac{16}{\sin^2\phi} + \frac{39}{4}\sin^2\phi - 27 \\ h_3(\sin^2\phi) &\stackrel{\text{def}}{=} 5 - \frac{21}{4}\sin^2\phi. \end{aligned}$$

Using the decoupling between θ and (ϕ, r), note that the second-order (resp. fourth-order) expansion in r_0/r of $\mathrm{CRB}(\theta)$ in (9) is still valid for only $P > 4$ (resp. $P > 6$).

Interestingly, when the source is known to be in the (x, y) plane, we deduce from the $\mathrm{FIM}(\boldsymbol{\alpha})$ for $\phi = \pi/2$ that

$$\begin{aligned} \mathrm{CRB}(\theta) &= \mathrm{CRB}_{\mathrm{FF}}(\theta)\left[1 + o\left(\frac{r_0^7}{r^7}\right)\right] \\ \mathrm{CRB}(r) &= \frac{16}{c_\sigma c_{r_0}}\frac{r^4}{r_0^2}\left[1 - \frac{r_0^2}{2r^2} + o\left(\frac{r_0^2}{r^2}\right)\right], \end{aligned}$$

where here $\mathrm{CRB}_{\mathrm{FF}}(\theta) = 1/c_\sigma c_{r_0}$. Note that the first order term r_0/r vanishes in all the expressions of the CRB for $P > 6$. Furthermore, for a source located in the plane (x, y), the CRB on the azimuth is very insensitive to the range. This contrasts with the near field expression of the CRB for the ULA [3] for which, $\mathrm{CRB}(\theta)$ includes a first order term in r_0/r and thus is much more sensitive to the range than for the UCA. Finally, note the simplicity of our closed-form expressions w.r.t. the complicated non interpretable closed-form expressions obtained for the ULA [2, 3].

4. ILLUSTRATIONS

In order to characterize the *level* of isotropy, we introduce a non-isotropy criterion

$$\rho = \sup_\theta \frac{|\mathrm{CRB}(\theta) - \overline{\mathrm{CRB}(\theta)}|}{\overline{\mathrm{CRB}(\theta)}}$$

(where $\overline{\mathrm{CRB}(\theta)}$ denotes the mean of $\mathrm{CRB}(\theta)$ w.r.t. θ) that we illustrate in Fig.2 for a UCA with half-wavelength inter-sensors spacing, and a source emitting at 1 MHz[3]. This figure shows that the isotropy is much more sensitive to P than to r.

The validity of some approximate closed-form expressions of the CRB is illustrated for a source located with an azimuth $\theta = 70°$ and elevation $\phi = 70°$. Fig.3 and 4 compare the approximate ratios $\mathrm{CRB}(\theta)/\mathrm{CRB}_{\mathrm{FF}}(\theta)$ and $\mathrm{CRB}(\phi)/\mathrm{CRB}_{\mathrm{FF}}(\phi)$ given by (9) and (10) to the exact ones and Fig.5 compares the approximate $\mathrm{CRB}(r)$ (11) to the exact one. We see from these three figures that the proposed approximations are very accurate from the ratio $r/r_0 = 2$ for $P = 10$ and from $r/r_0 = 4$ for $P = 7$.

[3]Note that these characteristics impact only r_0, but not the different ratios of CRB.

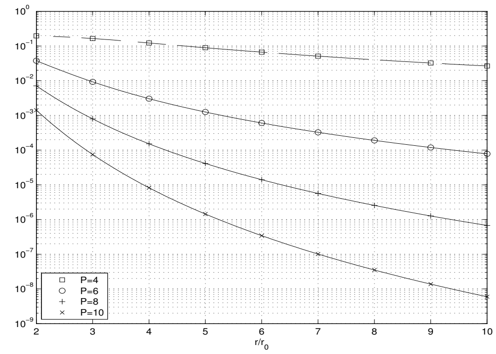

Fig.2 Non-isotropy criterion ρ w.r.t. r and P.

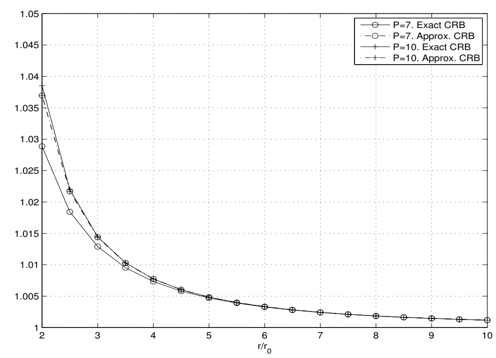

Fig.3 Approximate and exact ratios $\mathrm{CRB}(\theta)/\mathrm{CRB}_{\mathrm{FF}}(\theta)$.

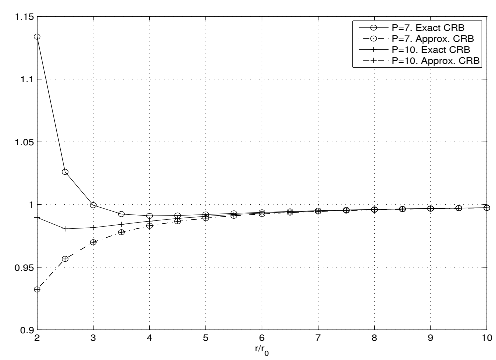

Fig.4 Approximate and exact ratios $\mathrm{CRB}(\phi)/\mathrm{CRB}_{\mathrm{FF}}(\phi)$.

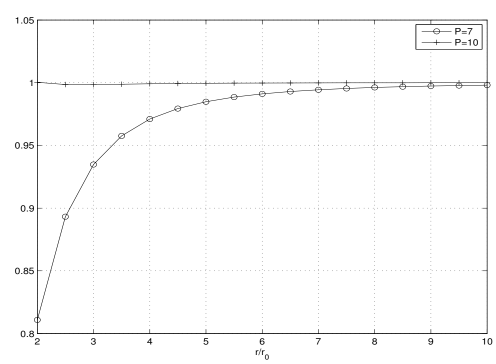

Fig.5 Ratio of the approximate $\mathrm{CRB}(r)$ to the exact one.

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