DOA ESTIMATION OF COHERENT TARGETS IN MIMO RADAR

Wei Zhang[⋆†] *Wei Liu*[†] *Ju Wang*[⋆] *Siliang Wu*[⋆]

[⋆] School of Information and Electronics
Beijing Institute of Technology, Beijing, 100081, China

[†] Communications Research Group
Department of Electronic and Electrical Engineering
University of Sheffield, Sheffield, S1 3JD, United Kingdom

ABSTRACT

A simple scheme for direction of arrival (DOA) estimation of coherent targets in a multiple-input multiple-output (MIMO) radar is proposed. It is based on the idea of joint transmission and reception diversity smoothing. Compared to the existing transmission diversity smoothing (TDS) method, the major advantage of the new scheme is that more covariance matrices are available for averaging to decorrelate the coherent signals, leading to a better estimation result. Moreover, it is able to identify much more coherent targets than the TDS method when sparse arrays are used.

Index Terms— MIMO radar, DOA estimation, coherent targets, transmission-reception diversity smoothing.

1. INTRODUCTION

Unlike the standard phased-array radar, MIMO radar employs multiple transmit antennas for emitting orthogonal waveforms and multiple receive antennas for receiving the echoes reflected by the targets [1, 2, 3]. Two types of MIMO radar have been investigated: MIMO radar with widely separated antennas [4], and those with colocated antennas [5]. In this paper, we focus on the type with colocated antennas.

Many techniques have been proposed (see [6, 7, 8, 9, 10, 11, 12] for details) for DOA estimation in MIMO radar, by assuming that all targets are uncorrelated with each other, so that the traditional eigenspace-based algorithms, such as MUSIC [13] and ESPRIT [14], can be employed for multiple target localization. However, in many radar applications, the received echo signals from different targets are considered as coherent, which implies that the eigenspace-based methods cannot be directly used for DOA estimation due to the ill conditioning problem of the covariance matrix. Spatial smoothing technique is a classic method to decorrelate the signals in the data covariance matrix [15, 16, 17]. The drawback with this approach is the decrease of the array aperture and the degrees of freedom (DOFs), resulting in lower resolution and accuracy. To overcome the coherent-source localization problem in MIMO radar, a preprocessing technique referred to as TDS is used to spatially smooth the signal covariance matrix [18]. The basic idea is to form a new covariance matrix with decorrelated signal subspace by summing the covariance matrices corresponding to the receive antennas together. Unlike the traditional spatial smoothing technique, the TDS method does not decrease the physical array aperture.

Following the TDS method, we here propose a further improved method for dealing with multiple coherent targets. Since linearly independent waveforms are transmitted simultaneously via multiple antennas in a MIMO radar, we can obtain a data matrix based on a set of virtual antennas. A $K_t \times K_r$ transmitting-receiving window is then employed to slide over this data matrix, where K_t and K_r represent the transmitting and receiving dimensions of the sliding window. Due to existence of the phase-shift factor between the subblock data, the corresponding covariance matrices can be employed to perform spatial smoothing for reconstructing the full-rank signal covariance matrix. Since both transmission and reception diversity smoothing is utilized, the proposed method has more covariance matrices for the smoothing operation, and therefore, can achieve a better estimation result than the TDS method. Moreover, the proposed method is able to identify much more coherent targets than the TDS method when sparse arrays are used.

This paper is organized as follows. In Section 2, the signal model for MIMO radar is given. The proposed method is introduced in Section 3. Simulation results are presented in Section 4 and conclusions drawn in Section 5.

2. SIGNAL MODEL FOR MIMO RADAR

Consider a MIMO radar system with a uniform linear array (ULA) of M antennas for transmitting and a ULA of N antennas for receiving. Both the transmit and receive arrays are assumed to be closely located in space so that any target located in the far-field can be seen at the same direction by both arrays. The M transmit antennas are used to transmit M orthogonal waveforms. It is assumed that K coherent targets are present in the same range cell. Consequently, the received data of the lth snapshot at the output of the matched filters at the receiver can be expressed as [1, 3]

$$\begin{aligned}\mathbf{x}[l] &= \big[x_{1,1}[l], x_{2,1}[l], \cdots, x_{M,1}[l], x_{1,2}[l], x_{2,2}[l], \cdots, \\ &\quad x_{M,2}[l], \cdots, x_{1,N}[l], x_{2,N}[l], \cdots, x_{M,N}[l]\big]^T \\ &= \sum_{k=1}^{K} \mathbf{a}_r(\theta_k) \otimes \mathbf{a}_t(\theta_k) b_k[l] + \mathbf{z}[l] \\ &= [\mathbf{a}_r(\theta_1) \otimes \mathbf{a}_t(\theta_1), \mathbf{a}_r(\theta_2) \otimes \mathbf{a}_t(\theta_2), \cdots, \\ &\quad \mathbf{a}_r(\theta_K) \otimes \mathbf{a}_t(\theta_K)]\mathbf{b}[l] + \mathbf{z}[l] \qquad (1)\end{aligned}$$

where $x_{m,n}[l]$ is the received data at the nth receive antenna associated with the mth transmit antenna, $[\cdot]^T$ denotes the transpose operation, θ_k is the DOA of the kth target, $\otimes$ stands for the Kronecker product operator, $b_k[l]$ is the complex-valued reflection coefficient of the kth target,

$$\mathbf{b}[l] = \left[b_1[l], b_2[l], \cdots, b_K[l]\right]^T, \quad (2)$$

$$\mathbf{a}_t(\theta_k) = [1, e^{-j2\pi d_t \sin(\theta_k)/\lambda}, \cdots, e^{-j2\pi(M-1)d_t \sin(\theta_k)/\lambda}]^T \quad (3)$$

$$\mathbf{a}_r(\theta_k) = [1, e^{-j2\pi d_r \sin(\theta_k)/\lambda}, \cdots, e^{-j2\pi(N-1)d_r \sin(\theta_k)/\lambda}]^T \quad (4)$$

are the transmit and receive steering vectors in direction θ_k, with d_t and d_r, respectively, being the adjacent antenna spacing for the transmit and receive arrays and λ denoting the wavelength, and $\mathbf{z}[l]$ is the vector of noise, which is assumed to be zero-mean independent and identically distributed complex Gaussian [19].

3. PROPOSED METHOD

3.1. Construction of Full-rank Signal Covariance Matrix

First, we form an $M \times N$ matrix $\mathbf{Y}[l]$ directly from $\mathbf{x}[l]$. The nth column of $\mathbf{Y}[l]$ is the received data at the nth receive antenna associated with the M transmit antennas, and $\mathbf{Y}[l]$ is then given by

$$\mathbf{Y}[l] = \begin{bmatrix} x_{1,1}[l] & x_{1,2}[l] & \cdots & x_{1,N}[l] \\ x_{2,1}[l] & x_{2,2}[l] & \cdots & x_{2,N}[l] \\ \vdots & \vdots & \ldots & \vdots \\ x_{M,1}[l] & x_{M,2}[l] & \cdots & x_{M,N}[l] \end{bmatrix} = \mathbf{A}_t \Xi \mathbf{A}_r^T + \mathbf{Z}[l] \quad (5)$$

where

$$\mathbf{A}_t = [\mathbf{a}_t(\theta_1), \mathbf{a}_t(\theta_2), \cdots, \mathbf{a}_t(\theta_K)], \quad (6)$$

$$\mathbf{A}_r = [\mathbf{a}_r(\theta_1), \mathbf{a}_r(\theta_2), \cdots, \mathbf{a}_r(\theta_K)], \quad (7)$$

$$\Xi = \mathrm{diag}\left[b_1[l], b_2[l], \cdots, b_K[l]\right], \quad (8)$$

and $\mathbf{Z}[l]$ denotes the $M \times N$ noise matrix.

Define a $K_t \times K_r$ matrix $\mathbf{Y}_{i,j}[l]\big(1 \le i \le M - K_t + 1, 1 \le j \le N - K_r + 1\big)$, which is the received data from the jth to the $(j+K_r-1)$th columns of $\mathbf{Y}[l]$ and from the ith to the $(i+K_t-1)$th rows of $\mathbf{Y}[l]$. Now we introduce the notation $\mathrm{Vec}(\cdot)$ for a matrix operation that stacks the columns of a matrix to form a new column vector, and define complex scalars $\{\alpha_k = e^{-j2\pi d_t \sin(\theta_k)/\lambda}, \beta_k = e^{-j2\pi d_r \sin(\theta_k)/\lambda}\}_{k=1}^K$. We can then form the following data vectors:

$$\begin{aligned} \mathbf{y}_{i,j}[l] &= \mathrm{Vec}\big(\mathbf{Y}_{i,j}[l]\big) \\ &= \sum_{k=1}^{K} \big(\mathbf{a}_r^{(K_r)}(\theta_k) \otimes \mathbf{a}_t^{(K_t)}(\theta_k)\big)\alpha_k^{i-1}\beta_k^{j-1} b_k[l] \\ &\quad + \mathbf{z}_{i,j}[l], \\ i &= 1, \cdots, M - K_t + 1, \\ j &= 1, \cdots, N - K_r + 1, \end{aligned} \quad (9)$$

where $\mathbf{a}_r^{(K_r)}(\theta_k)$ and $\mathbf{a}_t^{(K_t)}(\theta_k)$ are the $K_r \times 1$ and $K_t \times 1$ truncated versions of the steering vectors $\mathbf{a}_r(\theta_k)$ and $\mathbf{a}_t(\theta_k)$, respectively. The covariance matrix corresponding to $\mathbf{y}_{i,j}[l]$ is given by

$$\mathbf{R}_{i,j} = E\big[\mathbf{y}_{i,j}[l]\mathbf{y}_{i,j}^H[l]\big] \quad (10)$$

where $E[\cdot]$ denotes the expectation operation and $[\cdot]^H$ represents the Hermitian transpose. Like the classical spatial smoothing technique [16], we can sum all the $\hat{\mathbf{R}}_{i,j}$ together to spatially smooth the signal covariance matrix:

$$\mathbf{R} = \frac{\sum_{i=1}^{(M-K_t+1)} \sum_{j=1}^{(N-K_r+1)} \mathbf{R}_{i,j}}{(M - K_t + 1)(N - K_r + 1)}. \quad (11)$$

In practice, the sample covariance matrix of (10)

$$\hat{\mathbf{R}}_{i,j} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{y}_{i,j}[l]\mathbf{y}_{i,j}^H[l] \quad (12)$$

is used, where L is the number of snapshots.

3.2. Selection of K_t and K_r

As shown in (9) and (11), the effective aperture and the number of covariance matrices defined in (10) are related to K_t and K_r. In this section, the selection of K_t and K_r is investigated for two cases of a MIMO radar system.

3.2.1. Filled ULA for both the transmit and receive arrays

First, we consider the case with both the transmit and receive arrays being filled ULAs [5], i.e., half-wavelength interelement spacing with $d_t = d_r = \lambda/2$. In this case, the $K_t K_r \times 1$ vector $\mathbf{a}_r^{(K_r)}(\theta_k) \otimes \mathbf{a}_t^{(K_t)}(\theta_k)$ has only $(K_t + K_r - 1)$ distinct elements; in fact, this appears to be the smallest possible number of distinct elements, and there are $(M - K_t + 1)(N - K_r + 1)$ number of $\mathbf{R}_{i,j}$ defined in (10); nevertheless, only $(M - K_t + 1 + N - K_r)$ distinct $\mathbf{R}_{i,j}$ are actually used for spatial smoothing. Therefore, in order to identify K coherent targets, K_t and K_r should satisfy the following conditions:

$$\begin{aligned} K_t + K_r - 1 &> K \\ M - K_t + 1 + N - K_r &> K. \end{aligned} \quad (13)$$

It can be seen that an enhanced spatial resolution will be obtained by increasing the value of K_t or K_r. However, the number of covariance matrices $\mathbf{R}_{i,j}$ will decrease in such a case, leading to a decrease of the maximum number of coherent targets that can be identified by the proposed method. Consequently, these is a trade-off between the array aperture and the number of coherent targets identified by the proposed method. In particular, when the following condition

$$K_t + K_r - 1 = M - K_t + 1 + N - K_r \quad (14)$$

is met, i.e., $K_t + K_r = \frac{M+N+2}{2}$, the maximum number of coherent targets that can be identified will be obtained. On the other hand, the proposed method will be equivalent to the TDS method when K_t and K_r are set to M and 1, respectively. So the TDS method can be considered as a special case of the proposed one. By setting K_t and K_r properly, the equivalent aperture of the proposed method can be larger than that of the TDS method.

3.2.2. Filled ULA for transmit array but sparse ULA for receive array

When the transmit array is a filled ULA and the receive array is a sparse ULA with $M/2$-wavelength interelement spacing, the virtual aperture of the MIMO radar system is a filled-element ULA with MN distinct elements [5]. The vector $\mathbf{a}_r^{(K_r)}(\theta_k) \otimes \mathbf{a}_t^{(K_t)}(\theta_k)$ for this case has $K_t K_r$ distinct elements, and there are $(M - K_t + 1)(N - K_r + 1)$ distinct $\mathbf{R}_{i,j}$ defined in (10) actually used for spatial smoothing. Similarly, the following conditions

$$\begin{aligned} K_t K_r &> K \\ (M - K_t + 1)(N - K_r + 1) &> K \end{aligned} \qquad (15)$$

should be satisfied to identify K coherent targets. In this case the maximum number of coherent targets which can be identified by the proposed method is obtained when $(K_t K_r) = (M - K_t + 1)(N - K_r + 1)$. For simplicity, we set $K_t = \frac{M+1}{2}$ and $K_r = \frac{N+1}{2}$ in our proposed method, and then, the maximum number of coherent targets that can be identified by the proposed method is $\frac{(M+1)(N+1)}{4} - 1$. Note that if $N > 3$, the number of coherent targets identifiable by the proposed method is larger than $M - 1$, which is the maximum number of identifiable targets by the TDS method.

4. SIMULATIONS

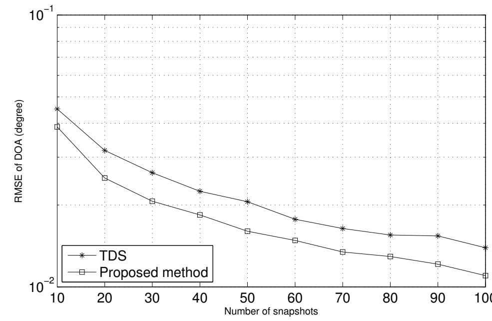
In this section, simulations are carried out to investigate the performance of the proposed method compared with the TDS method. We consider a MIMO array configuration where a ULA of $M = 10$ antennas is used for transmitting and a ULA of $N = 10$ antennas for receiving. Two scenarios are considered. In the first scenario, three coherent targets with the same signal-to-noise ratio (SNR) are located at the angles $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$ and $\theta_3 = 30^\circ$, respectively. In the second scenario, the three angles are changed to $\theta_1 = 50^\circ$, $\theta_2 = 60^\circ$ and $\theta_3 = 70^\circ$. All simulations are averaged over 500 independent runs. Define the root mean squared error (RMSE) as

$$\frac{1}{K} \sum_{k=1}^{K} \sqrt{\frac{1}{500} \sum_{n=1}^{500} (\theta_k - \hat{\theta}_{n,k})^2} \qquad (16)$$

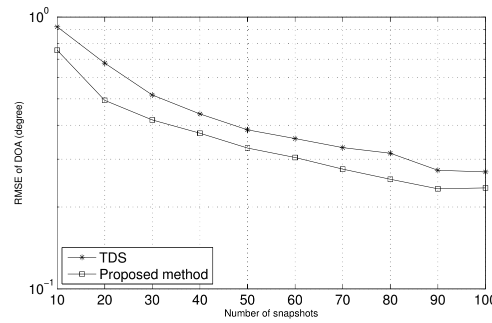
where $\hat{\theta}_{n,k}$ is the estimate of θ_k at the nth run.

4.1. Example 1: both the transmit and receive arrays are filled ULAs

In the first example, both the transmit and receive arrays are arranged with half-wavelength spacing between adjacent antennas. To form the same aperture as the TDS method, the proposed one chooses $K_t = 5$ and $K_r = 6$. The performance of the two methods is studied using the ESPRIT-based algorithm [7]. Fig. 1 shows the RMSEs of DOA estimation versus the number of snapshots for SNR $= 20$ dB. Fig. 2 shows the RMSEs of DOA estimation as a function of input SNR for $L = 50$. As shown, the proposed method has achieved higher estimation accuracy than the TDS method. This is because, although there are only 10 distinct covariance matrices defined in (10), the proposed method has actually used $(M - K_t + 1)(N - K_r + 1) = 30$ covariance matrices for spatial smoothing, and thus obtained a better conditioned estimate of the covariance matrix.

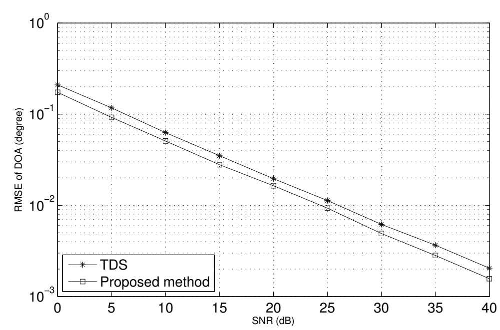


(a) First signal scenario

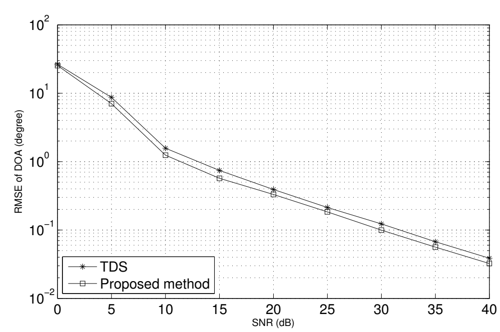


(b) Second signal scenario

Fig. 1. RMSEs of DOA estimation versus the number of snapshots.



(a) First signal scenario



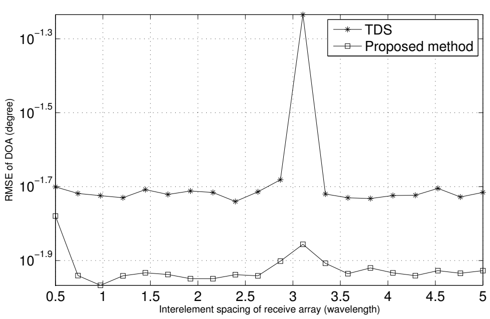
(b) Second signal scenario

Fig. 2. RMSEs of DOA estimation versus input SNR.

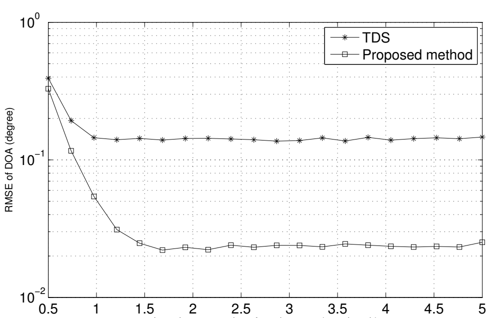
4.2. Example 2: the receive array is a sparse ULA

In the second example, the transmit array is a filled ULA, while the receive array is a sparse ULA. SNR $= 20$ dB and $L = 50$. Fig. 3 shows the effect of interelement spacing of the receive array on the estimation performance for the two different signal scenarios considered in Example 1. From the two figures, we see that the proposed method outperforms the TDS method significantly, especially for the second scenario, as the interelement spacing of the receive array increases.

Now assume that 11 coherent targets are located at the angle region $[-80°, 70°]$, with equal angle interval of $15°$. Both K_t and K_r are set to 6 for the proposed method. Two cases of receive arrays with $d_r = 3\lambda$ and $d_r = 5\lambda$, respectively, are considered. In this scenario, the TDS method fails because the number of coherent targets is larger than the maximum number allowed by it. On the other hand, the proposed one has $K_tK_r = 36$ distinct elements in the vector $\mathbf{a}_r^{(K_r)}(\theta_k) \otimes \mathbf{a}_t^{(K_t)}(\theta_k)$ and has $(M - K_t + 1)(N - K_r + 1) = 25$ distinct covariance matrices defined in (10) for spatial smoothing. Therefore, the proposed method can localize all the coherent targets. With SNR $= 20$ dB and $L = 50$, the spatial spectrum of the proposed method by applying the classic MUSIC algorithm is shown in Fig. 4. We can clearly see from the two figures that there are 11 largest peaks located at the real DOAs of the targets, and they have been identified successfully.

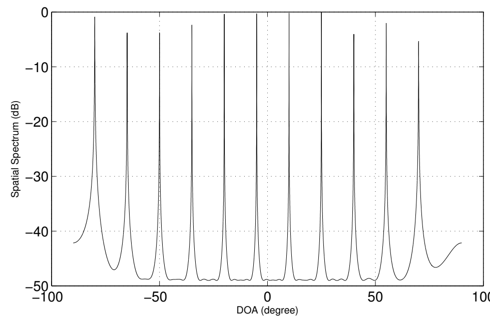


(a) First signal scenario

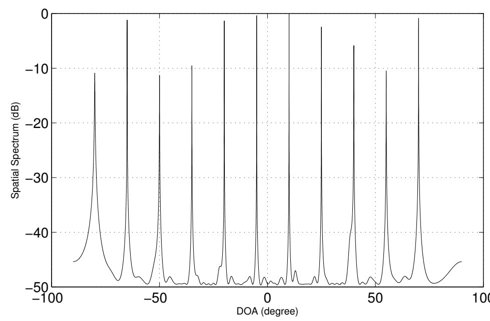


(b) Second signal scenario

Fig. 3. RMSEs of DOA estimation versus d_r.



(a) $d_r = 3\lambda$



(b) $d_r = 5\lambda$

Fig. 4. Spatial spectrum of the proposed method.

5. CONCLUSIONS

A novel improved DOA estimation method for coherent targets has been introduced for MIMO radar systems. Different from the existing TDS method, the proposed one exploits both transmission and reception diversity smoothing to tackle the ill-conditioning problem of the covariance matrix. It can achieve better estimation accuracy than the TDS method since there are more covariance matrices available for spatial smoothing. On the other hand, the number of coherent targets that can be identified by the proposed method is much larger than that of the TDS method when the receive array is a sparse one. The effectiveness of the proposed method has been demonstrated by extensive simulation results.

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