MULTI-FREQUENCY TUNING TECHNIQUE FOR DISTRIBUTED SENSOR LOCALIZATION

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ABSTRACT

In this paper, we propose a novel angle of departure (AOD) aided sensor localization algorithm. Through the multiple frequency tuning technique at a fixed-spaced two-antenna anchor node, a non-uniform linear array can be rebuilt at each ground sensor node. We convert the angle estimation into a real number reconstruction by Chinese remainder theorem (CRT) and then provide a new algorithm to solve the congruence problem. Compared with the conventional spectral searching algorithms, the proposed one is computationally efficient and also performs much better than other CRT reconstruction algorithms. The validity and the advantages of the proposed algorithm is proved by numerical simulations.

Index Terms— Localization, multiple frequencies tuning, angle of departure, Chinese remainder theorem (CRT).

1. INTRODUCTION

Information fusion between the sensor spatial attribute and the artificial perception of external environment is now playing an important role in many wireless distributed systems, e.g., wireless sensor networks (WSNs). In that sense, sensor localization has attracted much attention in recent years [1]. A distributed sensor localization scheme usually relies on one or several auxiliary parameters such as received signal strength, time/time difference of arrival or the connectivity such as average one-hop distance. Particularly, angle information is also utilized in [2–4], where the single antenna sensor node conducts angle of departure (AOD) estimation for further self-localization. Besides the above narrow-band systems, a interference field [5] created by two linear chirp waves with slight frequency difference is used to convert the AOD estimation into a frequency estimation problem. Given that the system cost and the total energy consumption, however, it is impractical for each anchor node to equip antenna array or to adjust the system to be a wide-band one. In addition, it is also improper for each sensor node to perform complicate angle estimation algorithm if one wants to prolong the sensor's life as long as possible.

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To cope with the aforementioned problems, we try to devise a novel distributed AOD aided sensor localization scheme. With the help of multiple frequencies tuning at a fixed-spaced two-antenna anchor node, the unattended sensor node can rebuild a non-uniform linear array to estimate the angle information with respect to its own position. To relieve the computational burden of sensor node, we first utilize a subspace iteration algorithm to abstract the spatial character of line of sight signal component, and further convert the angle estimation into a real number construction from the perspective of Chinese remainder theorem (CRT). We also propose a new algorithm to solve this congruence problem. Note that an apparent discrepancy with the conventional uniform linear array with antenna spacing being half the wavelength, our multiple frequency synthetic array (MFSA) is actually of spatial sub-Nyquist sampling.

2. AOD AIDED LOCALIZATION SYSTEM MODEL

2.1. System Depiction and Signal Model

Consider a wireless networks includes an anchor node equipped with two omnidirectional antennas spacing d and a number of unattended single antenna sensor nodes. The anchor node is assumed to possess sufficient energy and is arranged at a much higher position, so that it can connect all sensor nodes in a line of sight (LOS) way. As a transmitter, each antenna of anchor node is assigned by a unique pilot sequence, e.g., let $c_m(t)$ be the baseband signal of the mth antenna, which satisfies $\int_0^{T_s} c_m(t)c_k^*(t)dt = \delta(m-k)$, where T_s denotes a basic duration. The localization data frame format is shown in Fig.1, where the preamble is used for symbol synchronization and the following N repeated pilot sequences are used for angle estimation. Let θ denotes the AOD of LOS signal between transmitter and sensor node. Although the localization system can work at different frequency points, it is still a narrow-band system. For that, one can refers to the IEEE 802.15.4 protocol.

When the system works at frequency f_i with wavelength λ_i, the baseband signal received at sensor node is given by,

$$r_i(t) = \sum_{m=1}^{2} h_i e^{-j\phi_m(\theta)} c_m(t - nT_S - \tau) + w(t) \quad (1)$$

where $\phi_m(\theta) = 2\pi \frac{d}{\lambda_i}(m-1)\cos\theta$ denotes the relative phase between the mth antenna and the reference one. For convenience, we herein let the first antenna be the reference, i.e., $\phi_1(\theta) = 0$. The path loss $h_i = \lambda_i \sqrt{G} e^{-j2\pi l/\lambda_i}/(4\pi l)$, where G denotes the equivalent antenna field radiation patterns in the LOS direction. For omnidirectional antenna in our system, it has $G = 1$. Delay τ is caused by the transmitting distance l from anchor node to sensor node. $w(t)$ is Gaussian white noise with zero-mean and variance σ_n^2, which is independent of angle parameter. In addition, the network is usually static, therefore h_i and θ are assumed to be time invariable in the whole localization stage. Note that we disregard the carrier frequency offset of the transceiver and assume it has been compensated perfectly beforehand.

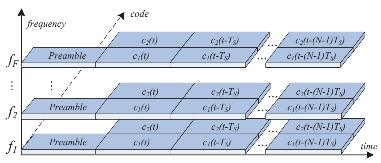


Fig. 1. The frame format of pilot signals.

After performing symbol synchronization and the match filtering, i.e., $r_i(m,n) = \int_{(n-1)T_s}^{nT_s} r_i(t) c_m^*(t) dt$, we can easily obtain

$$\mathbf{r}_i(n) = h_i \mathbf{a}_i + \bar{\mathbf{w}}_i(n). \tag{2}$$

where $\mathbf{r}_i(n) = [r_i(1,n),\, r_i(2,n)]^T$, $\mathbf{a}_i = [1,\, e^{-j2\pi \frac{d}{\lambda_i}\cos\theta}]^T$, and $\bar{\mathbf{w}}_i(n)$ is noise vector after the linear transformation.

When stacking all N measurements, we have

$$\mathbf{X}_i = \mathbf{a}_i \mathbf{S}_i + \mathbf{N}_i \tag{3}$$

where $\mathbf{X}_i = [\mathbf{r}_i(1), \cdots, \mathbf{r}_i(N)]$, $\mathbf{S}_i = h_i \mathbf{1}^T$ with $\mathbf{1} = [1, \cdots, 1]^T$ and $\mathbf{N}_i = [\bar{\mathbf{w}}_i(1), \cdots, \bar{\mathbf{w}}_i(N)]$. We name $\mathbf{r}_i(n)$ in (2) as a pseudo-snapshot for the discrepancy with the conventional array snapshot.

2.2. Multi-Frequency Synthetic Array

Intuitively, if we just consider the second element of $\mathbf{a}_i$, i.e., $a_i = e^{-j2\pi \frac{d}{\lambda_i}\cos\theta}$, $i = 1, \cdots, F$, they manifest a virtual non-uniform array structure with array manifold vector $[1, a_1, a_2, \cdots, a_F]^T$. More important, this virtual array also implies the AOD information. Note that $a_1, \cdots, a_F$ are not derived from the physical antennas but are synthesized by way of multiple frequencies turning, so we name it as multi-frequency synthetic array (MFSA) for short, the philosophy of which is shown in Fig.2.

According to [6], the ambiguity, i.e., $\exists\, \psi \in [0, \pi)$ such that $\mathbf{a}(\theta) = \mathbf{a}(\psi)$, is a vital problem for non-uniform array; therefore we give the following theorem to guarantee the unambiguity of MFSA, the similar discussion is also appeared in [7].

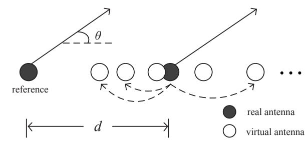


Fig. 2. The description of MFSA.

Theorem 1 *For the MFSA with selected wavelength $\lambda_i \triangleq Q\eta_i, i = 1, \cdots, F$, where Q denotes a quantification factor and η_i's are integers, it is unambiguity providing that η_i's are pair-wisely co-prime and $\min\{\eta_1, \cdots, \eta_F\} > \sqrt{2d/Q}$.*

Proof 1 *The requirement of angle ambiguity can be simplified as $e^{-j2\pi \frac{d}{\lambda_i}\cos\theta} = e^{-j2\pi \frac{d}{\lambda_i}\cos\psi}$, so we further have*

$$2\pi d(\cos\theta - \cos\psi)/\lambda_i = 2k_i\pi. \tag{4}$$

For all F ambiguity equations in (4), define $\rho_i = (\cos\theta - \cos\psi)$ for the ith one and $d_i = 2d/\lambda_i = 2d/(Q\eta_i)$. if considering the condition $\eta_i > \sqrt{2d/Q}$, we know that $|\ \rho_i\ | \leq 2$ and $k_i \leq \lfloor d_i \rfloor < d_i < \eta_i$; moreover, ρ_i falls into some discrete values, i.e., $\rho_i = \{0, \pm 2k_i/d_i\}$. We now consider ρ_i and ρ_j with respect to the co-prime integers η_i and η_j. Assume that there exists non-zero $\bar{k}_i < \eta_i$ and non-zero $\bar{k}_j < \eta_j$ to make $\rho_i = \rho_j$, then $\bar{k}_i\eta_i = \bar{k}_j\eta_j$ if neglecting the common coefficient $2d/Q$. It means that $\eta_i/\eta_j = \bar{k}_j/\bar{k}_i$, which is a contradiction to the fact that η_i and η_j are pair-wisely co-prime integers. Hence, we have

$$\{\rho_1 \cap \rho_2 \cap \cdots \cap \rho_F\} = \{0\}. \tag{5}$$

The above result manifests that $\cos\theta = \cos\psi$, in other words, the MFSA is unambiguity.

3. THE PROPOSED ESTIMATION ALGORITHM

To achieve the angle estimation, we have to acquire the spatial character of LOS signal. One well-known measure is based on the eigenvalue decomposition. We herein exploit a alternative algorithm which can decrease the computational burden.

Beforehand, we stack the observation data $\mathbf{X}_i, i = 1, \cdots, F$, defining $\mathbf{X} = \left[\mathbf{X}_1^T, \mathbf{X}_2^T, \cdots, \mathbf{X}_F^T\right]^T = \mathbf{AS}$, where $\mathbf{A} = \text{diag}\{\mathbf{a}\}$ with $\mathbf{a} = [\mathbf{a}_1; \cdots; \mathbf{a}_F]$ and $\mathbf{S} = [\mathbf{S}_1; \cdots; \mathbf{S}_F]$, for calculating the correlation matrix. We show the iterative subspace algorithm [8] for spatial character extract in Algorithm 1.

According to the above algorithm, we can get the estimation of a_i, i.e., $\hat{a}_i = \hat{\mathbf{a}}(2i)/\hat{\mathbf{a}}(2i-1)$, where $\hat{\mathbf{a}}(i)$ denotes the ith element of vector $\hat{\mathbf{a}}$.

3.1. CRT-based Algorithm

Chinese remainder theorem [9] tells that a positive integer K can be uniquely reconstructed from its remainders modulo L

Algorithm 1 Spatial Character Extract

Input: The correlation matrix, $\mathbf{R} = \mathbf{X}\mathbf{X}^H \in \mathbb{C}^{2F\times 2F}$;
Output: The estimation of $\mathbf{a}$

Initialization: $l = 1$;
  $\mathbf{q}^{(0)} = [1, 0, \cdots, 0]^T$, $\mathbf{q}^{(1)} = [1, 1, \cdots, 1]^T$;
while $\parallel \mathbf{q}^{(l)} - \mathbf{q}^{(l-1)} \parallel \geq \epsilon$ **do**
  $\mathbf{q}^{(l)} = \mathbf{R}\mathbf{q}^{(l-1)}$;
end while
return $\hat{\mathbf{a}} = \mathbf{q}^{(l)}/\mathbf{q}^{(l)}(1)$.

positive integer $M_1, \cdots, M_L$, if $K < lcm\{M_1, \cdots, M_L\}$, where $lcm\{\cdot\}$ denotes the least common multiple operation, and furthermore it provides a simple reconstruction formula if all moduli M_i are co-prime [10]. We now convert the angle estimation problem into a real number reconstruction.

Let $\hat{\phi}_i = \text{angle}\{\hat{a}_i\} = \phi_i + \Delta\phi_i$, where ϕ_i is true value, $\Delta\phi_i$ is error, and angle$\{\cdot\}$ is to obtain the phase. If the phase is defined in $[-\pi, \pi]$, then it should be adjusted as below,

$$\hat{\phi}_i = \begin{cases} \hat{\phi}_i & \text{if } \hat{\phi}_i \geq 0 \\ \hat{\phi}_i + 2\pi & \text{if } \hat{\phi}_i < 0 \end{cases}. \tag{6}$$

Regardless of error, we know that

$$d\cos\theta = m_i\lambda_i + \frac{\phi_i}{2\pi}\lambda_i = m_iQ\eta_i + b_i, \tag{7}$$

where m_i is the unknown wrapping integer. For convenience, we define $D = d\cos\theta$ and $b_i = \frac{\phi_i}{2\pi}Q\eta_i$. Obviously, $D \leq d$. In practice, the observed phase usually has error, e.g., $\hat{b}_i = b_i + \Delta b_i = \frac{\phi_i}{2\pi}Q\eta_i + \frac{\Delta\phi_i}{2\pi}Q\eta_i$. Now the question is how to reconstruct a real number D from its F erroneous remainders.

As we know that when no errors occur, all remainders b_i have a common remainder b^c divided by Q, i.e., $b_i = q_iQ + b^c$, $q_i = \lfloor b_i/Q \rfloor$, $\lfloor\cdot\rfloor$ stands for the flooring operation. In conventional CRT [10], if we define $D_0 \triangleq \lfloor D/Q \rfloor$, then D can be uniquely reconstructed by,

$$D = QD_0 + b^c \tag{8}$$

where $D_0 = \sum_{i=1}^{F} \bar{\gamma}_i\gamma q_i \mod \Gamma$, $\Gamma \triangleq \eta_1\eta_2\cdots\eta_F$ and $\gamma_i \triangleq \Gamma/\eta_i$. $\bar{\gamma}_i$ is the modular multiplicative inverse of γ_i modulo η_i, i.e., $\bar{\gamma}_i\gamma_i \equiv 1 \mod \eta_i$. Note that the above formula requires all η_i's are co-prime integers, which means that the system bandwidth is usually very wide. In addition, the analysis in [10] manifests that reconstruction formula is error sensitive. We propose a novel divide-and-rule algorithm as shown in Algorithm 2, which actually divide the whole MFSA into multiple sub-MFSA only consisting two virtual antennas.

Given that the remainder error in our algorithm, we have $\hat{b}^c = \frac{1}{F}\sum_{i=1}^{F}\hat{b}_i^c$ with $\hat{b}_i^c = \hat{b}_i - Q\lfloor\hat{b}_i/Q\rfloor$, then $\hat{q}_i = \lfloor(\hat{b}_i - \hat{b}^c)/Q\rfloor$. In addition, we also define a special function $\mathcal{F}(\bar{D}) = \sum_{i=1}^{F} d^2(\hat{r}_i, \bar{D} \mid \lambda_i)$, where the circular distance $d(x, y \mid C) \triangleq x - y - k_0C$ with $k_0 = [(x-y)/C]$, operator $[x] = \lfloor x + 0.5 \rfloor$.

Algorithm 2 CRT-based AOD Estimation

Input: $\hat{\phi}_i, i = 1, \cdots, F$;
Output: The estimation of θ

Form the first congruence problem via $\{\hat{\phi}_i\}_{i=1}^F$ and the second one via $\{-\hat{\phi}_i\}_{i=1}^F$ utilizing (6);
Select all $J = \binom{F}{2}$ frequency pairs to estimate D_j's by (8) for each problem;
Define data set $\mathbb{D} = \{\hat{D}_j \mid \hat{D}_j \leq d, j = 1, \cdots, J\}$ for the first problem and $\bar{\mathbb{D}}$ for second one based on the same condition;
Minimize function $\mathcal{F}(\bar{D})$ in set $\mathbb{D} \cup \bar{\mathbb{D}}$, then $\hat{\theta} = \cos^{-1}(\hat{D}/d)$ if $\hat{D} \in \mathbb{D}$ or $\hat{\theta} = \pi - \cos^{-1}(\hat{D}/d)$ if $\hat{D} \in \bar{\mathbb{D}}$.
return $\hat{\theta}$.

Remark 1: The reason we adopt two versions of phase is that D appears to be negative number when $\theta \in (\pi/2, \pi]$. To do so, if one version of phase can give a successful estimation; conversely, its counterpart functions like a erroneous one, the estimation by which will be larger than d. This feasibility is due to the error sensitivity of CRT.

Remark 2: For each sub-MFSA formed by two frequencies, in practice, $\{\eta_i, \eta_j\}$ usually have a greatest common divisor (GCD) M_{ij}, therefore the angle unambiguity condition should be revised as $\min\{\eta_i, \eta_j\} > \sqrt{2dM_{ij}/Q}$. In addition, $\min lcm\{\eta_i, \eta_j\}$ should be larger than or equal to d/Q for the basic requirement of CRT.

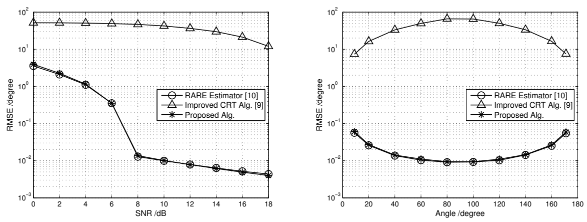
3.2. Discussion

Note that the spatial character extraction has computational complexity $\mathcal{O}(4F^2)$, which is in some extent computational efficient comparing with the $\mathcal{O}(8F^3)$ of eigenvalue decomposition. In addition, different from the searching algorithm [11], the proposed CRT-based one is a kind of closed-form solution, consequently, it is very appropriate for sensor node to achieve self-localization.

On the other hand, the proposed divide-and-rule algorithm has a improved performance comparing with other CTR algorithms. The reason is rooted in the fact that the more erroneous remainders used in reconstruction formula the larger deviation to real value we can get no matter how to choose the reference remainder in [10]. In our algorithm, the rejection of improper candidates guarantees a improved estimation.

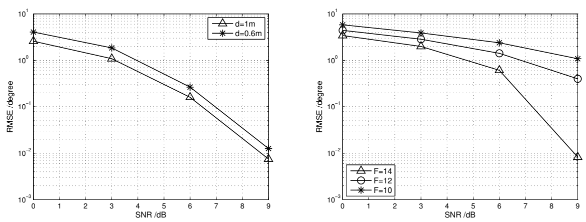
4. SIMULATIONS

To prove the effectiveness of the proposed localization scheme, we refer to the IEEE 802.15.4 protocol, in which it specifies 16 channels with carrier frequencies span from 2.405GHz to 2.480GHz in 5MHz steps with wavelength $\lambda \in \{0.1247, 0.1245, 0.1242, \cdots, 0.1215, 0.1212, 0.1210\}$m, so $J = \binom{16}{2} = 120$. The quantification factor $Q = 0.0001$ and $N = 200$. The signal power is normalized as one, so that the



(a) (b)

Fig. 3. The RMSE performance comparison.



(a) (b)

Fig. 4. The RMSE performance with system parameters.

signal-noise-ratio (SNR) is defined as $10\log_{10}(1/\sigma_n^2)$. For comparison, the RARE estimator [11] searches the angle with step 0.0001rad in $[0, \pi]$, and the improved CRT algorithm [10] use $\{0.1240, 0.1230, 0.1210\}$m to perform calculation. The spatial character extract in our proposed localization scheme has the iteration suspending condition $\epsilon = 10^{-3}$. For assessment, the root mean square error (RMSE) is defined as $\sqrt{E(\hat{\theta}-\theta)^2}$. Each simulation result is the statistics of 2000 independent trials.

Fig.3 (a) illustrates that the proposed algorithm has a obviously superiority over the one in [10] and nearly the same performance as the RARE estimator, in which the AOD of LOS signal $\theta = 3\pi/10$, $d = 0.8$m. Fig.3 (b) gives a sampled glance at how the estimation performance varies with the different AOD, where we fix SNR=9dB and other system parameters are the same as previous example.

In addition, we also consider the effect of parameters d and F on the system performance, the RMSE performance of which are shown in Fig.4. In this example, we test parameter d by utilizing all 16 frequency points, and then fix $d = 1$m to test F (i.e., selecting the first F frequency points in set λ for angle estimation). The AOD of LOS signal is set as $\theta = 4\pi/7$. From the statistical results, we can see that the accuracy of angle estimation improves with the increase of frequency points and antenna spacing. Therefore, for a fixed antenna spacing localization system, one should select as more frequency points as possible to guarantee a better estimation.

5. CONCLUSION

Through the virtual non-uniform array generated by the multiple frequencies tuning, we have introduced a novel AOD based sensor self-localization scheme. To decrease the computational complexity, we first utilized iterative subspace algorithm to extract the spatial character of LOS signal, and then converted the angle estimation into a real number reconstruction from the perspective of CRT. An effective algorithm was also proposed to cope with this congruence problem.

6. REFERENCES

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