MULTIPLE-CHANNEL DETECTION IN ACTIVE SENSING

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ABSTRACT

The problem of detecting the presence of a known signal in multiple channels of additive noise, such as occurs in active radar with a single transmitter and multiple geographically distributed receivers, is addressed via coherent multiple-channel techniques. The transmitted signal replica is treated as one channel in a suitable M-channel detector with the remaining $M-1$ channels comprised of data from the receivers. To accommodate this approach, an invariance result for the distribution of the eigenvalues of a Gram matrix is derived. The result implies that false alarm probabilities for any detector based on a function of the eigenvalues are not changed if one data channel contains a signal replica, provided that the other $M-1$ channels only Gaussian noise and all channels are independent.

Index Terms— Multiple-channel detection, Matched filter, Active radar, Passive coherent radar, Eigenvalue distribution invariance

1. INTRODUCTION

Tests for detecting the presence of a common but unknown signal in $M \geq 2$ noisy channels have been studied extensively in connection with passive localization of emitters for such applications as passive sonar, non-invasive monitoring of mechanical systems, and electronic support. In particular, the magnitude-squared coherence (MSC) estimate has seen wide application in situations involving $M=2$ channels for over five decades (see, e.g., [1, 2] and the papers included in [3]). Estimates of multiple coherence were investigated and applied as detection statistics for passive detection with $M>2$ channels in the 1970s and 1980s [4, 5]. The generalized coherence (GC) estimate was introduced in 1988 [6], and its performance as a statistic for passive multiple-channel detection were studied extensively during the 1990s [7, 8, 9, 10]. The rise of MIMO systems in sensing and communications has recently led to renewed interest in multiple-channel detection. A generalized likelihood ratio test (GLRT) for spatial correlation among a collection of complex circular Gaussian signals with unknown arbitrary covariance matrices was derived in [11]. GLRTs and locally most powerful invariant tests for vector-valued random processes with covariance matrices of known rank were developed in [12, 13] and a Bayesian test for diagonal covariance matrix versus arbitrary non-diagonal covariance matrix for a zero-mean complex Gaussian $M \times N$ matrix was derived in [14].

A common feature of the MSC and GC statistics as well as statistics derived in [11, 12, 13] is that they are functions of the eigenvalues of some non-negative definite $M \times M$ matrix obtained from samples of the M data channels as a Gram matrix. In [15], the distribution of the MSC estimate was shown not to depend on the distribution of the data on one of the two channels provided the two channels are independent and the second channel contains only white Gaussian noise. This invariance property was given a geometric interpretation in [16]. At the time, the principal reason for interest in this invariance was that MSC methods were being used in practice with reference channels on which a signal had already been detected by single-channel means. Obtaining a two-channel detection is valuable in localization of the signal source, and the invariance result lent confidence that detection thresholds on the MSC estimate chosen to provide desired false alarm probabilities would remain approximately valid as long as the second channel contained only noise. A similar result was given for the GC estimate for $M \geq 2$ channels in [9]. This same paper remarked that the result implies one of the M channels can be replaced by an exact signal replica without changing the distribution of the detection statistic under the signal-absent hypotheses on the other $M-1$ channels, thus suggesting the potential utility of the GC estimate in active sensing. Another application in which this formulation is of potential value is passive coherent radar where one channel contains the direct-path signal from the transmitter of opportunity and the goal is to detect time-shifted and Doppler-shifted replicates of this signal in other receiver channels [17].

Invariance of the distribution of the GC estimate to the distribution of data on one channel is a consequence of the distribution of the determinant of a certain Gram matrix not depending on one of the data vectors. This paper extends this invariance result to the distributions of the individual eigenvalues of that matrix, endowing the distribution of any statistic formed as a function of these eigenvalues with the desired invariance. Section 2 establishes the mathematical framework for the remaining parts of the paper. The main theoretical result on eigenvalue invariance is presented in Section 3, followed by a brief analysis of the optimality of MSC-based detection with one receiver in Section 4. Performance results for multiple-channel detection using a signal replica, based on Monte Carlo simulations, are shown and discussed in Section 5.

2. MATHEMATICAL FORMULATION

Consider M complex N-vectors $\mathbf{x}_1, \ldots, \mathbf{x}_M$ with $M < N$ corresponding to segments of time-series data from a collection of M receivers. Denoted by $\mathbf{X}$ the $N \times M$ matrix whose m^{th} column is $\mathbf{x}_m / ||\mathbf{x}_m||$. The GC estimate for $\mathbf{x}_1, \ldots, \mathbf{x}_M$ is [16]

$$\gamma^2(\mathbf{x}_1, \ldots, \mathbf{x}_M) = 1 - \det \mathbf{X}^\dagger \mathbf{X}$$

where $\mathbf{X}^\dagger$ denotes the hermitian transpose of $\mathbf{X}$. The $(i,j)^{\text{th}}$ element of the non-negative definite $M \times M$ matrix $\mathbf{X}^\dagger \mathbf{X}$ is the complex inner product

$$g_{ij} = \left\langle \frac{\mathbf{x}_i}{||\mathbf{x}_i||}, \frac{\mathbf{x}_j}{||\mathbf{x}_j||} \right\rangle$$

By construction, γ^2 is invariant to unknown complex gains on the channels; i.e., $\gamma^2(\alpha_1 \mathbf{x}_1, \ldots, \alpha_M \mathbf{x}_M) = \gamma^2(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ for any non-zero complex scalars $\alpha_1, \ldots, \alpha_M$. With $M = 2$, the GC estimate reduces to the MSC estimate [1]; i.e.,

$$\gamma^2(\mathbf{x}_1, \mathbf{x}_2) = \frac{|\langle \mathbf{x}_1, \mathbf{x}_2 \rangle|^2}{||\mathbf{x}_1||^2 ||\mathbf{x}_2||^2}, \tag{1}$$

the distribution of which was shown in [15] not to depend on the distribution of $\mathbf{x}_1$ provided it is independent of $\mathbf{x}_2$ and $\mathbf{x}_2 \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbb{I})$. In [16], these conditions for invariance are relaxed slightly to include other spherically symmetric distributions on $\mathbf{x}_2$. A similar result was shown to hold for the GC estimate on more than two channels in [9].

As mentioned in Section 1, interest in invariance results of this kind was motivated by a desire to apply detector based estimates of coherence, whether MSC or GC, in situations in which one channel is known not to contain only noise. Recent results have shown that GLRT and optimal Bayesian multiple-channel detectors for signals of known rank under various assumptions about noise levels on the channels are based on functions of the eigenvalues of $\mathbf{X}^\dagger \mathbf{X}$ [11, 12, 14, 13, 18]. For example, in [18], the GLRT is derived for the problem

$$\begin{aligned} H_0 &: \mathbf{X} \sim \mathcal{CN}(0, \sigma^2 \mathbb{I}) \\ H_1 &: \mathbf{X} \sim \mathcal{CN}(SA, \sigma^2 \mathbb{I}) \end{aligned} \tag{2}$$

where a K-dimensional signal subspace is defined by an unknown $N \times K$ complex matrix $\mathbf{S}$ whose columns are orthonormal vectors in $\mathbb{C}^N$, and the element a_{km} of the unknown $K \times M$ complex matrix $\mathbf{A}$ is the complex amplitude of the component of the signal received at sensor m and in the subspace corresponding to the k^{th} column of $\mathbf{S}$. Specifically, the generalized likelihood ratio (GLR) for this rank-K detection problem is shown to be

$$e^{\frac{N}{\sigma^2} \sum_{i=1}^{K} \lambda_i}$$

where $\lambda_i, i = 1, ..., M$ are the eigenvalues of $\mathbf{X}^\dagger \mathbf{X}$ arranged in non-increasing order. In particular, for $K = 1$, the largest eigenvalue is a sufficient statistic for the GLR.

The rank-one situation is typical in active radar involving a single transmitter and in passive coherent radar exploiting a single transmitter of opportunity. Thus invariance of the distribution of the individual eigenvalues of $\mathbf{X}^\dagger \mathbf{X}$, and of the largest eigenvalue in particular, is of interest for these applications to enable one channel to contain a noiseless signal replica (for active radar) or a high-SNR direct-path signal (for passive coherent radar) without altering the detection thresholds corresponding to desired false alarm probabilities. In certain cases, these thresholds can be derived analytically using the known (if rather intractable) distributions of eigenvalues of complex Wishart-distributed matrices [19, 20, 21].

3. INVARIANCE OF EIGENVALUE DISTRIBUTION

Consider the $M \times M$ Gram matrix $\mathbf{X}^\dagger \mathbf{X} = [\mathbf{g}_{ij}]$ of the normalized signal vectors, as defined in Section 2. The objective of this section is to show that the distribution of the eigenvalues of this matrix does not depend on the distribution of one of the data vectors, $\mathbf{x}_m$, provided that $\mathbf{x}_j \sim \mathcal{CN}[\mathbf{0}, \sigma^2 \mathbb{I}]$ for $j \neq m$ and all of the vectors are independent. There is no loss in generality in assuming $m = 1$; i.e., that the first channel is the one with arbitrary distribution. This is because $\mathbf{x}_m$ and $\mathbf{x}_1$ can be exchanged in $\mathbf{X}$ via right multiplication by a unitary permutation matrix $\mathbf{P}$. The new Gram matrix $\mathbf{P}^\dagger \mathbf{X}^\dagger \mathbf{X} \mathbf{P}$ is unitarily similar to $\mathbf{X}^\dagger \mathbf{X}$, and thus has the same eigenvalues. Properties of Gram matrices are discussed in [22, Ch. 8].

Since the data vectors are normalized, the Gram matrix has the form

$$G = X^\dagger X = \begin{bmatrix} 1 & g_{12} & \cdots & g_{1M} \\ g_{12}^* & 1 & \cdots & g_{2M} \\ \vdots & & \ddots & \vdots \\ g_{1M}^* & g_{2M}^* & \cdots & 1 \end{bmatrix}$$

where g_{ij}^* denotes the complex conjugate of g_{ij}. Let U be a unitary matrix of dimension $(M-1) \times (M-1)$ such that

$$U^\dagger \begin{bmatrix} 1 & g_{23} & \cdots & g_{2M} \\ g_{23}^* & 1 & \cdots & g_{3M} \\ \vdots & & \ddots & \vdots \\ g_{2M}^* & g_{3M}^* & \cdots & 1 \end{bmatrix} U = \begin{bmatrix} \eta_2 & & 0 \\ & \ddots & \\ 0 & & \eta_M \end{bmatrix}$$

Then denoting

$$V = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & U & \\ 0 & & & \end{bmatrix},$$

yields

$$V^\dagger G V = \begin{bmatrix} 1 & g_{12} & g_{13} & \cdots & g_{1M} \\ g_{12}^* & \eta_2 & 0 & \cdots & 0 \\ g_{13}^* & 0 & \eta_3 & \cdots & 0 \\ \vdots & 0 & & \ddots & 0 \\ g_{1M}^* & 0 & \cdots & 0 & \eta_M \end{bmatrix}$$

Since V is unitary, $V^\dagger G V$ has the same eigenvalues and eigenvectors as G. The characteristic polynomial of $V^\dagger G V$ is

$$P(\lambda) = \det \begin{bmatrix} 1 - \lambda & g_{12} & \cdots & g_{1M} \\ g_{12}^* & \eta_2 - \lambda & & 0 \\ \vdots & & \ddots & \\ g_{1M}^* & 0 & & \eta_M - \lambda \end{bmatrix}$$

Expanding the determinant upon the first row

$$\begin{aligned} C(\lambda) = (1 - \lambda) \prod_{j=2}^{M} (\eta_j - \lambda) - |g_{12}|^2 \prod_{j=3}^{M} (\eta_j - \lambda) \\ -|g_{13}|^2 \prod_{j \neq 3} (\eta_j - \lambda) - \cdots - |g_{1M}|^2 \prod_{j \neq M} (\eta_j - \lambda) \end{aligned} \tag{3}$$

Note that

$$|g_{1j}|^2 = \left| \left\langle \frac{\mathbf{x}_1}{||\mathbf{x}_1||}, \frac{\mathbf{x}_j}{||\mathbf{x}_j||} \right\rangle \right| = \frac{|\langle \mathbf{x}_1, \mathbf{x}_j \rangle|^2}{||\mathbf{x}_1||^2 \, ||\mathbf{x}_j||^2}$$

which is the MSC estimate for the data vectors $\mathbf{x}_1$ and $\mathbf{x}_j$.

If $\mathbf{x}_2, \ldots, \mathbf{x}_M$ are $\mathcal{CN}[\mathbf{0}, \sigma^2 \mathbb{I}]$ random vectors that are each independent of $\mathbf{x}_1$, the invariance result for the distribution of the MSC estimate proven in [15, 16] implies the distributions of the scalar random variables $|g_{1j}|^2$ for $j = 2, \ldots, M$ do not depend on the distribution of $\mathbf{x}_1$ provided it is independent of each $\mathbf{x}_j$ for $j > 1$. From (3), the only potential dependence of the terms of $C(\lambda)$ on the distribution of $\mathbf{x}_1$ is through $|g_{1j}|^2$ for $j = 2, \ldots, M$. Thus the distributions of all the coefficients of $C(\lambda)$, and hence the distributions of the eigenvalues of $\mathbf{G}$, do not depend on the distribution of $\mathbf{x}_1$ if $\mathbf{x}_1$ is independent of the $\mathcal{CN}[\mathbf{0}, \sigma^2 \mathbb{I}]$ random vectors $\mathbf{x}_j$ for $j = 2, \ldots, M$.

The invariance of the distribution of the largest eigenvalue of G with $M = 3$ was tested empirically as follows. In a 1000-trial and

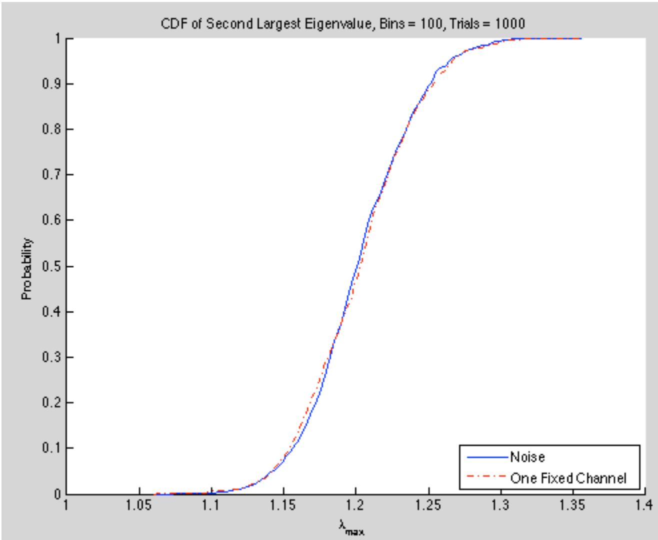

Fig. 1. Empirical distributions for the largest eigenvalues of 1000 3×3 Gram matrices used in a Kolmogorov-Smirnov goodness of fit test. The solid line represents the CDF of the largest eigenvalue statistic where there is independent white Gaussian noise on all three channels, while the dashed line represents the CDF of the largest eigenvalue when $\mathbf{x}_1$ is fixed and the other two channels are independent white Gaussian noise.

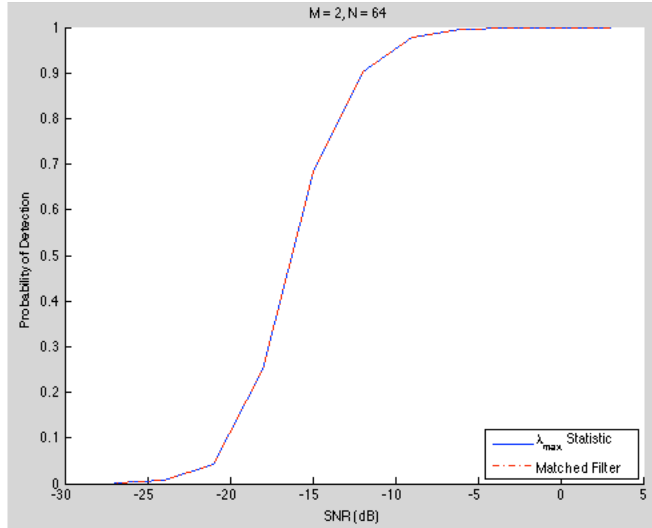

Fig. 2. Probability of detection (P_d) as a function of signal-to-noise ratio for a fixed false alarm probability of $P_f = 0.1$ using two detectors: (a) the matched filter for a signal with unknown phase and (b) the detector based on the largest eigenvalue of the 2×2 Gram matrix $\mathbf{X}^\dagger \mathbf{X}$ with a noiseless signal replica on one channel (from simulations). In each case, the data vector lengths are $N = 64$. Close alignment of the matched filter curve (dashed) and the curve for the detector based on the largest eigenvalue (solid) is consistent with theoretically predicted equivalence of the detectors.

a 100,000-trial Monte Carlo simulation, the largest eigenvalue of a Gram matrix formed from a fixed deterministic complex vector $\mathbf{x}_1$ of length $N = 256$ and realizations of two iid $\mathcal{CN}[\mathbf{0}, \mathbb{I}]$ random vectors $\mathbf{x}_2$ and $\mathbf{x}_3$ were collected (Case 1). The largest eigenvalue of a Gram matrix formed from realizations of three iid $\mathcal{CN}[\mathbf{0}, \mathbb{I}]$ random vectors were also collected (Case 2). The sample distributions of these two sets of eigenvalues, shown in Fig. 1, were compared using a Kolmogorov-Smirnov goodness of fit test with a significance threshold of 5%. With 1000 data, the test indicated the Case 1 data and the Case 2 data were drawn from the the same distribution with a confidence 89.72%. With 100,000 data, the confidence rose to 99.95%.

4. OPTIMALITY WITH ONE RECEIVER

Among the most studied active sensing situations entails a monostatic radar that transmits a rank-one signal and detects echoes of that signal in noise. The equivalence of three approaches to this problem is discussed in this brief section.

In detecting a complex signal vector s that is known up to a complex scalar gain in additive complex white Gaussian noise, the most powerful invariant detector (normalized matched filter) has the form

$$\to H_1 = \left\{ \left| \left\langle \frac{s}{||s||}, \frac{\mathbf{x}}{||\mathbf{x}||} \right\rangle \right| > \tau \right\}$$

where $\mathbf{x}$ is the received data and τ is a threshold [23]. Denoting $\mathbf{x}_1 = s$ and $\mathbf{x}_2 = \mathbf{x}$, this becomes $\to H_1 = \{|g_{12}| > \tau\}$. In the MSC detector (1), the statistic is $\gamma^2(\mathbf{x}_1, \mathbf{x}_2) = |g_{12}|^2$. The GLRT detectors for rank-one signals in [12] and [18] are based on the largest eigenvalue of $\mathbf{X}^\dagger \mathbf{X}$, which in this case is $\lambda_{\max} = 1 + |g_{12}|$.

Since all three of these detection statistics are monotonically related, the corresponding detectors are equivalent and all share the optimality properties known for the matched filter. This is supported by the simulation results shown in Fig. 2.

5. APPLICATION TO ACTIVE SENSING

This section presents results from simulation of an active sensing scenario with two receivers. A noiseless replica of the transmitted signal s was used as $\mathbf{x}_1$, while $\mathbf{x}_2$ and $\mathbf{x}_3$ are independent, zero-mean, unit-variance complex Gaussian noise vectors under H_0. Under H_1, a signal component $\alpha_m s$ with $\alpha_m > 0$ for $m = 2, 3$ and s realized from $\mathcal{CN}[\mathbf{0}, \mathbb{I}]$ was added to the noise on channels 2 and 3. The largest eigenvalue of $\mathbf{X}^\dagger \mathbf{X}$ is used as the detection statistic. This is illustrative of the active sensing scheme proposed for $M - 1$ receivers, where a noise-free signal replica is used as one channel in and M-channel detector.

Fig. 3 shows results obtained with $N = 64$ and $\alpha_2 = \alpha_3 = \alpha$ (i.e., equal SNRs on the two receiver channels). Data obtained under H_0 were used to determine thresholds to achieve a false alarm probability $P_f = 0.1$. A normalized matched filter detector was applied to the transmitted signal replica $\mathbf{x}_1$ and the data from the first sensor $\mathbf{x}_2$. From this, an SNR value of -12dB was determined to be required for a probability of detection $P_d = 0.9$ with only one receiver channel. This is illustrated by the dashed vertical and horizontal lines in the figure. Finally, the largest eigenvalue detector was applied to the noiseless replica and the two receiver channels simultaneously. The required SNR value, with equal SNRs on both receiver channels, required to obtain $P_d = 0.9$ and $P_f = 0.1$ was -15dB. This is shown by the the dot in Fig. 3. Single receiver regions of detectability lie to the right of the vertical dashed line and above the horizontal dashed line. Processing both receiver channels simultaneously the largest eigenvalue coherence detector allows a signal

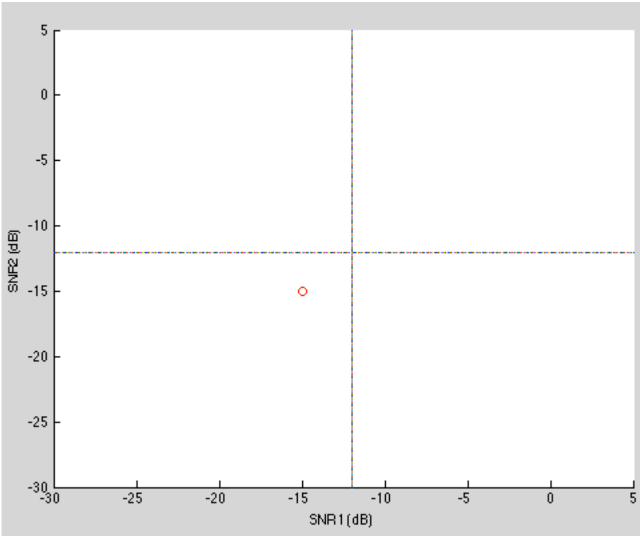

Fig. 3. SNRs on two receiver channels needed to achieve $P_f = 0.1$ and $P_d = 0.9$ for infinite SNR on one channel. The dashed lines show the SNR values (in dB) on one receiver channel required to achieve this performance level with the matched filter. The marked point shows the equal-channel SNR required to achieve this performance with a three-channel coherence detector with a noiseless signal replica on one channel and using $\lambda_{\max}$ as the detection statistic.

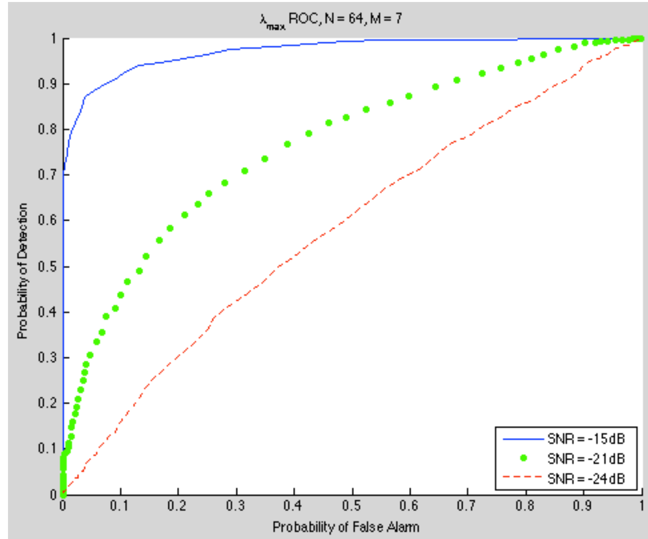

Fig. 4. Detection probability (P_d) versus false alarm probability (P_f) for a seven-channel largest eigenvalue detector with sequence length $N = 64$. Channel one contains a noiseless signal replica, while the remaining six channels have equal SNR values. The solid line represents an SNR of -18dB, the dotted line represents an SNR of -21dB, and the dashed line represents an SNR of -24dB.

to be detected on both receiver channels when it is too weak to be detected on either receiver channel by a matched filter.

Fig. 4 depicts receiver operating characteristic (ROC) curves obtained using a seven-channel largest eigenvalue statistic in the active sensing case with $N = 64$ and with equal SNRs on the six receiver channels. The simulation used 1000 trials, each involving a complex white Gaussian signal and additive complex white Gaussian noise on channels 2–7.

6. CONCLUSION

Motivated by the roles played by the individual eigenvalues of the Gram matrix of normalized data vectors in recently derived multiple-channel detectors for signals of known rank, the distribution of the eigenvalues of such a matrix have been shown to be invariant with respect to the statistical behavior of one data vector provided that the remaining data vectors contain only white Gaussian noise and all the data vectors are independent. This result subsumes a 1997 invariance result for the determinant of the Gram matrix in [9]. The invariance enables detection thresholds for detectors based on functions of the eigenvalues to be used in situations where one channel is known not to contain only noise, such as active sensing or passive coherent radar, without altering the false alarm probabilities. An example using a three-channel coherence detector with a noiseless signal replica and two channels of receiver data was delineated. This example showed that there exists cases where such a configuration enables a multiple channel detection even when single-channel matched filters fail to detect on either receiver channel.

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