COORDINATED BLIND CALIBRATION FOR TIME INTERLEAVED ADCS

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ABSTRACT

This paper presents a novel digital blind calibration method for time interleaved analog to digital converters (TIADCs). A simple cost function based on the cross-correlation of channel statistics is used to derive a steepest descent algorithm for the compensation of timing mismatch errors. Instead of calibrating the timing mismatches independently for each channel, only one adaptation channel needs to be calibrated within a closed loop. The calibration of the rest of the channels can be coordinated according to a scaling relationship established during an initialization stage. As a result, both the computational complexity and convergence speed of the proposed algorithm can be improved significantly with little loss in calibration performance.

***Index Terms*—** Farrow structure, fractional delay filter, steepest descent

1. INTRODUCTION

Analog-to-Digital Converters (ADCs) serve as the gate connecting the real world with today's digital world. There has always been an increasing need for faster and more accurate ADCs in modern communication systems. Similar to what has happened with multi-core CPUs in the PC industry, parallelism is the most natural way to extend the existing technology for ADCs. The idea of the time interleaved ADC system (TIADC) is very easy to state. A group of slower ADCs takes samples alternately and the data from all channels are multiplexed into a single output stream with a faster equivalent sampling speed.

Figure 1 shows the basic structure of an ideal TIADC system. The input analog signal is sampled at frequency F_s/M in each channel, where M is the number of channels. The phase of each channel ADC is separated by $2\pi/M$. The sampled data from the M channels are multiplexed to yield a digital output $y(n)$ at an equivalent sampling frequency of F_s.

The foregoing comments assume ideal models, but there are always mismatch errors between channels due to the ADC manufacturing process variations, environment variables and clock jitter. Three kinds of mismatch errors in a TIADC system are significant:

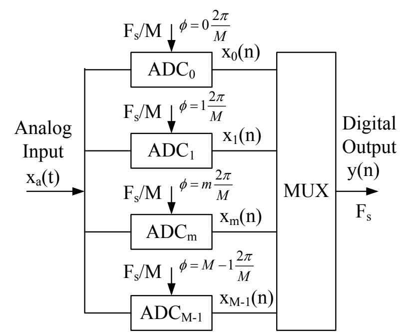

Fig. 1. An ideal TIADC system

- Offset error o_m: The ground voltage level of each ADC is not exactly identical.
- Gain error g_m: The analog gain differs among ADCs.
- Timing error $\Delta_{t,m}$: The relative clock delays between pairs of ADCs are not all identical.

Earlier work [1, 2] gives a detailed analysis of the channel mismatch effects in TIADC systems. For a sinusoidal input signal at frequency f_0, gain and timing mismatch errors will cause spectral distortions at $f_0 \pm \frac{m}{M}F_s,\ m = 1, ..., M-1$ while offset mismatch errors introduce distortions at $\frac{m}{M}F_s$. As a result, the overall system suffers a reduced Signal to Noise plus Distortion Ratio (SINAD) and Spurious Free Dynamic Range (SFDR).

The correction of mismatches can be accomplished in either the analog [3] or digital domain [4, 5]. Because of their flexibility, digital compensation methods are more attractive and have been heavily studied. Digital calibration methods can be classified into two categories. Active calibration methods [6] send a known pilot signal to the TIADC system and estimate the mismatches according to the digital output. As accurate and fast as these methods are, they unavoidably interrupt the normal ADC operation which is unacceptable in most real-time applications.

Blind calibration methods [7, 8, 9], on the other hand, perform calibration while the TIADC is in normal operation. The blind assumption doesn't mean that we are completely agnostic about the input signal. Different algorithms make certain

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assumptions about properties of the input signal. As a result, various limitations exist, e.g., oversampling the input signal [4], or restricting the number of channel numbers to two [8], or four [9]. Computation can also be an issue, e.g., [7] proposes an adaptive algorithm based on minimizing the mean square difference between the outputs of all adjacent channels, but it is computationally too expensive to be practical.

This paper proposes a coordinated blind adaptive calibration algorithm to compensate the timing mismatches in the TIADC system. Figure 2 shows the block diagram of the proposed digital calibration technique. Channel 0 is set as the reference channel. Farrow-structured fractional delay (FD) filters [10, 11] are applied to correct the timing mismatches in a continuous manner without an online filter design process. Offset and gain mismatches are relatively easy to estimate and compensate—these are corrected sequentially by the offset/gain compensation module. The timing compensation module requires a closed loop adaptation scheme, but we show that only one channel must be fully adapted. This simplification relies on a normalized relationship among the timing mismatches of each channels with regard to an adaptation channel that can be established after an initialization step. As a result, the overall computational complexity is reduced significantly and the convergence speed is accelerated as well.

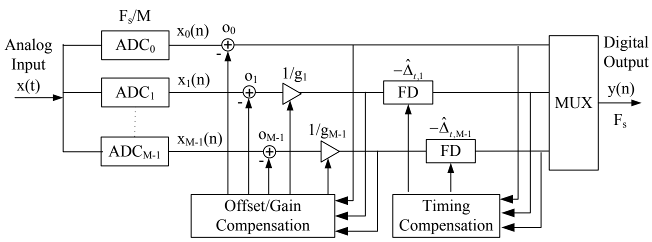

Fig. 2. The proposed calibration scheme

2. ALGORITHM DERIVATION

2.1. Blindness assumption of the input signal

Suppose the input signal $x(t)$ is a zero mean, ergodic, wide sense stationary process, the sampled signal at channel m is

$$x_m[n] = o_m + g_m x((nM+m)T_s + \Delta_{t,m}), \quad n = 0, ..., N-1,\ m = 0, ..., M-1, \tag{1}$$

where $o_m, g_m, \Delta_{t,m}$ are the offset, gain, and timing mismatches at channel m, N is the number of samples in each channel and M is the number of channels.

2.2. Offset/gain mismatch estimation

The offset mismatches can be estimated according to

$$E[x_m(n)] = o_m \qquad m = 0, ..., M-1. \tag{2}$$

After compensating the offset mismatches, the gain mismatches can be estimated according to

$$g_m = g_0 \sqrt{\frac{E[x_m^2[n]]}{E[x_0^2[n]]}}, \quad m = 1, ..., M-1, \tag{3}$$

where the gain g_0 is assumed to be the gain in channel 0, which is set as the reference channel. In other words, we can only estimate the relative gains for channel 1 to $M-1$.

2.3. Timing mismatch estimation

Once the gains are estimated and corrected to g_0, the most difficult task remains—timing mismatch estimation and compensation. In this section, a cost function that equals the sum of squared cross-correlation differences between pairs of adjacent channels is defined. When the timing mismatches are small compared to the sampling interval, an approximate scaling among the timing mismatches at each channel can be established by observing the cross-correlation of consecutive channels. In the adaptation process, a steepest descent algorithm is executed to adjust the timing mismatch at a single adaptation channel, leaving the timing mismatches in the rest of the channels to be adjusted in a coordinated manner according to the scaling relationship established beforehand.

According to assumption on the input signal, we have

$$\begin{aligned} E[x_m[n]x_{m-1}[n]] =& g_0^2 E[x((nM+m)T_s + \Delta_{t,m}) \\ & x((nM+m-1)T_s + \Delta_{t,m-1})] \\ =& g_0^2 R_x(T_s + \Delta_{t,m} - \Delta_{t,m-1}), \end{aligned} \tag{4}$$

where $R_x(\tau)$ is the unknown auto-correlation function of $x(t)$. When the timing mismatch differences are very small, i.e., $|\Delta_{t,i} - \Delta_{t,j}| \ll T_s, \ \forall i \neq j$, we can approximate $R_x(T_s + \Delta_{t,m} - \Delta_{t,m-1})$ according to a first-order Taylor expansion:

$$\begin{aligned} E[x_m[n]x_{m-1}[n]] \approx& g_0^2 R_x(T_s) \\ &+ g_0^2 R_x'(T_s)(\Delta_{t,m} - \Delta_{t,m-1}). \end{aligned} \tag{5}$$

Next, we can define a sub-cost function as

$$\begin{aligned} J_{m-1} =& E[x_m[n]x_{m-1}[n]] - E[x_{m-1}[n]x_{m-2}[n]] \\ \approx& g_0^2 R_x'(T_s)(\Delta_{t,m} - 2\Delta_{t,m-1} + \Delta_{t,m-2}), \\ & m = 2, ..., M-1, \end{aligned} \tag{6}$$

and we include the last channel by invoking wraparound

$$\begin{aligned} J_{M-1} =& E[x_0[n+1]x_{M-1}[n]] - E[x_{M-1}[n]x_{M-2}[n]] \\ \approx& g_0^2 R_x'(T_s)(\Delta_{t,0} - 2\Delta_{t,M-1} + \Delta_{t,M-2}). \end{aligned} \tag{7}$$

To unify the definition of (6) and (7), we can redefine the sub-cost function as

$$\begin{aligned} J_{m-1} = & E[x_{m \bmod M}[n + \lfloor \frac{m}{M} \rfloor] x_{m-1}[n]] \\ & - E[x_{m-1}[n]x_{m-2}[n]] \quad m = 2, ..., M. \end{aligned} \tag{8}$$

Then we define the overall cost function as

$$J = \sum_{m=1}^{M-1} J_m^2, \tag{9}$$

which is minimized to zero when all the timing mismatches are equalized. An adaptive algorithm to update all channels can be derived from the gradient of the cost function. Suppose we apply a delay $\hat{\Delta}_{t,m}$ to the signal $x_m[n]$ by means of digital fractional delay filters so that the new delay becomes $\Delta_{t,m} - \hat{\Delta}_{t,m}$. By denoting the compensated signal sequence as $\hat{x}_m[n]$ and the new sub and overall cost functions after compensation as $\hat{J}_m$ and $\hat{J}$, we can evaluate the gradient of $\hat{J}$ according to (6, 7, 9):

$$\frac{\partial \hat{J}}{\partial \hat{\Delta}_{t,m}} = -g_0^2 R_x'(T_s)(\hat{J}_{m+1} - 2\hat{J}_m + \hat{J}_{m-1}) \tag{10}$$
$$m = 1, ..., M-1, \quad \hat{J}_0 = 0, \quad \hat{J}_M = 0.$$

The exact values of $R_x'(T_s)$ and g_0 are unknown. However, as long as we can determine the sign of $R_x'(T_s)$, we can still determine the direction of the gradient which is sufficient for the steepest descent algorithm to work.

Suppose $x(t)$ is a random process bandlimited to $[-\frac{F_s}{2}, \frac{F_s}{2}]$, we can show that $R_x'(T_s) \leq 0$. According to the steepest descent algorithm, we can update the estimate of $\hat{\Delta}_{t,m}$ by

$$\hat{\Delta}_{t,m}[k+1] = \hat{\Delta}_{t,m}[k] - \mu_m(\hat{J}_{m+1}[k] - 2\hat{J}_m[k] + \hat{J}_{m-1}[k]) \qquad m = 1, ..., M-1, \tag{11}$$

where k is the update step and μ_m is a positive step size. Therefore, for each update step k, we need to collect N samples of $x_m(n)$ for all M channels, evaluate $\hat{J}_m$, and update the delay estimates according to (11).

By assuming that the timing delays are constant during the adaptation process, we can significantly simplify the update scheme by the following coordinated adaptive algorithm. According to (6) and (7), we have

$$\gamma_1 = \frac{J_1}{J_2} \approx \frac{\Delta_{t,2} - 2\Delta_{t,1}}{\Delta_{t,3} - 2\Delta_{t,2} + \Delta_{t,1}} \tag{12}$$
$$\gamma_{m-1} = \frac{J_{m-1}}{J_m} \approx \frac{\Delta_{t,m} - 2\Delta_{t,m-1} + \Delta_{t,m-2}}{\Delta_{t,m+1} - 2\Delta_{t,m} + \Delta_{t,m-1}} \qquad m = 3, .., M-2$$
$$\gamma_{M-2} = \frac{J_{M-2}}{J_{M-1}} \approx \frac{\Delta_{t,M-1} - 2\Delta_{t,M-2} + \Delta_{t,M-3}}{-2\Delta_{t,M-1} + \Delta_{t,M-2}}$$

Therefore, we have $M-2$ equations and $M-1$ variables $\Delta_{t,1}$ to $\Delta_{t,M-1}$. Without loss of generality, we can set channel #1 as the reference channel. Denote α_m as the normalization coefficient at channel m with respect to the reference channel,

$$\Delta_{t,m} = \alpha_m \Delta_{t,1} \quad m = 2, ..., M-1. \tag{13}$$

We can rewrite (12) as

$$\mathbf{A}\boldsymbol{\alpha} = \mathbf{b}, \tag{14}$$

where,

$$\boldsymbol{\alpha} = [\alpha_2, \alpha_3, ..., \alpha_{M-1}]^T, \tag{15}$$
$$\mathbf{b} = [-\gamma_1 - 2, 1, 0, ..., 0]^T, \tag{16}$$

$\mathbf{A}$ is a band (2,1) matrix whose element a_{ij} is zero when $j < i-2$ and $j > i+1$. The diagonal row vectors of $\mathbf{A}$ are

$$\mathbf{a}_1 = [-2\gamma_1 - 1, \gamma_1], \tag{17}$$
$$\mathbf{a}_2 = [2 + \gamma_2, -2\gamma_2 - 1, \gamma_2], \tag{18}$$
$$\mathbf{a}_m = [-1, 2 + \gamma_m, -2\gamma_m - 1, \gamma_m], \tag{19}$$
$$m = 3, ..., M-3, \tag{20}$$
$$\mathbf{a}_{M-2} = [-1, 2 + \gamma_{M-2}, -2\gamma_{M-2} - 1]. \tag{21}$$

After resolving $\boldsymbol{\alpha}$ during the initialization stage, we can coordinate the delay compensations as

$$\hat{\Delta}_{t,m} = \alpha_m \hat{\Delta}_{t,1} \quad m = 2, ..., M-1, \tag{22}$$

As a result, we just need to focus on adapting the time delay $\hat{\Delta}_{t,1}$ on channel 1, the delays for the other $M-2$ channels can be calculated directly from (22) instead of adaptively updated using (11). In other words, we only need to keep track of the data sequences $x_0[n]$, $x_1[n]$ and $x_2[n]$. Not only does the proposed algorithm reduce computational complexity, but it also accelerates the convergence speed. Imagining the overall cost function $\hat{J}$ as a function of $\hat{\Delta}_{t,m}$ in a multi-dimensional space, the proposed algorithm will follow a straight line from the initial choice of $\hat{\Delta}_{t,m}$ to the optimality location of $\hat{J}$, which is the shortest path available.

3. SIMULATION EXAMPLE

Two examples (for 3 channels and 8 channels) of TIADC timing mismatch calibration with the proposed coordinated adaptation algorithm are shown in Fig. 3. The timing compensations $\hat{\Delta}_{t,m}$ are updated once every $N = 5000$ samples for each channel. The equivalent overall sampling frequency is F_s = 1 GHz. The input multi-sine signal has 3 components whose frequencies, amplitudes and phases are unknown. The timing mismatches are assumed to be uniformly distributed over $[-0.1\frac{T_s}{M}, 0.1\frac{T_s}{M}]$, where M is the number of channels.

Figure 3(a-c) shows a three channel $(M = 3)$ TIADC example. The step size is set as $\mu_m = 0.5T_s$. The convergence path on the overall cost function surface in Fig. 3(a) follows a straight line from the initial choice of $\hat{\Delta}_{t,1}$ and $\hat{\Delta}_{t,2}$ towards their true values when the coordinated algorithm is used. In contrast, the convergence path of the independent adaptation scheme follows a path that is normal to the contour of the overall cost function. Because of the shortened convergence path, the convergence speed of the cost function and estimated

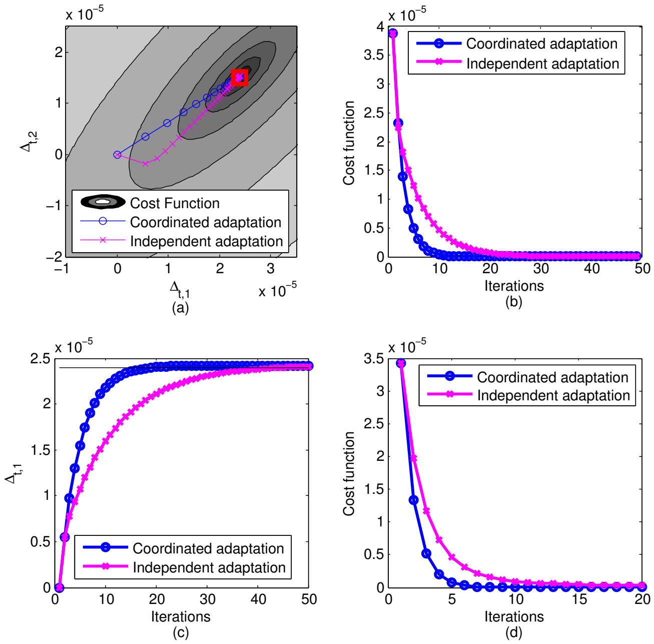

Fig. 3. TIADC timing mismatch compensation performance: (a) convergence paths on the cost function surface for $M = 3$. (b) cost function convergence speed for $M = 3$. (c) convergence of the timing mismatch estimation in one channel, for $M = 3$. (b) cost function convergence for $M = 8$.

timing mismatches versus the number of iterations can be accelerated by a factor of 2.5 as seen in Fig. 3(b-c).

Figure 3(d) compares the convergence speeds for an eight-channel TIADC example where the input signal is still the same multi-sine signal. The step size is set as $\mu_m = 0.2T_s$. In this case, the speed up of the convergence is about a factor of 2 for the proposed coordinated adaptation algorithm.

Figure 4 shows a Monte Carlo experiment on the performance of the proposed algorithm. The input multisine signal has 3 sinusoidal components whose frequencies, amplitude and phases are unknown. The timing mismatch differences $\Delta_{t,i} - \Delta_{t,j}, \forall i \neq j$ are assumed to be uniformly distributed in $[-\beta\frac{T_s}{M}, \beta\frac{T_s}{M}]$, where M = 8 is the number of channels. The step size of the proposed algorithms is set as $\mu = \beta\frac{T_s}{M}$. The number of iterations is set to 10.

The normalized timing mismatch compensation RMS error is defined as

$$\Delta_{\text{RMSE}} = \frac{M}{T_s}\sqrt{\frac{1}{M-1}\sum_{m=1}^{M-1}(\Delta_{t,m} - \hat{\Delta}_{t,m})^2}. \qquad (23)$$

The proposed algorithms are claimed convergent if the compensated overall cost function $\hat{J}$ is smaller than the uncompensated cost function J.

Figure 4(a) shows the relationship between Δ_{RMSE} and β. Each point on the figure is averaged over 1000 experiments. For both the independent and coordinated adaptation schemes, Δ_{RMSE} increases linearly with regard to β, which suggests that performance of the proposed algorithms degrades when the timing mismatch differences $|\Delta_{t,i} - \Delta_{t,j}|$ increase. The independent adaptation algorithm is more robust as it is guaranteed to converge over the entire range. By contrast, the coordinated adaptation algorithm may diverge when the small timing mismatch difference assumption is not valid. Figure 4(a) evaluates Δ_{RMSE} only when both algorithms converge. Figure 4(b) shows the probability of convergence for the coordinated adaptation algorithm. Combining 4(a) with (b), we can conclude that both algorithms exhibit similar timing compensation performance. The independent adaptation algorithm is more robust than the coordinated adaptation algorithm in terms of convergence. However, when the coordinated adaptation algorithm converges, it reduces the overall computational complexity by a factor of $M - 2$. As a result, we can apply the coordinated adaptation algorithm initially. Should it diverge, we can switch to the independent adaptation algorithm for a few iterations to narrow down the compensated timing mismatch differences, which increases the likelihood that the coordinated adaptation algorithm will converge thereafter.

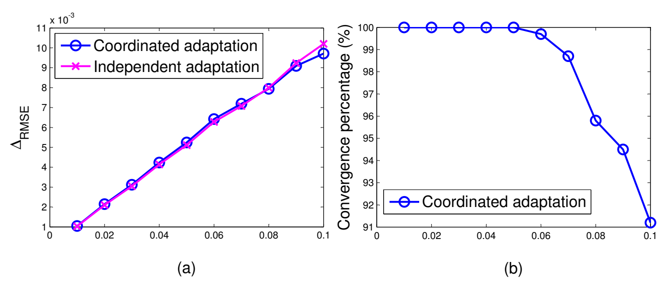

Fig. 4. Monte Carlo experiments on the performance of the proposed algorithms after 10 iterations. (a) normalized timing mismatch compensation RMS error as a function of the timing mismatch difference variation range, (b) probability of convergence of the coordinated adaptation algorithm.

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