

# BLIND CALIBRATION AND REMEDY FOR MIS-ORIENTED VELOCITY-SENSOR TRIADS

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## ABSTRACT

This paper introduces a new blind calibration algorithm as well as a new remedy procedure for velocity-sensor triads' mis-orientations. The word "blind" indicates the proposed method requires no priori knowledge of incident sources' direction-of-arrivals (DOAs), and allows the arrays of velocity-sensor triads to be arbitrarily separated at unknown locations in three-dimensional space. Moreover, the present work is applicable to the arrays of velocity-sensor triads with unknown inter-triad gain and phase deviations. Simulation results verify the efficacy and versatility of this proposed scheme.

**Index Terms**— velocity-sensor triad, direction of arrival estimation, partly calibrated arrays, mis-orientation

## 1. INTRODUCTION

### 1.1. The Velocity-Sensor Triad

An velocity-sensor triad (a.k.a. vector-hydrophone) consists of three identical, but orthogonally oriented, acoustic *velocity*-sensors – all spatially co-located in a point-like geometry. The entire velocity-sensor triad thus distinctly measures all three Cartesian components of the particle-velocity vector. The velocity-sensor triad thus treats the acoustic wavefield as a vector-field (i.e., the particle-velocity field), not merely as a scalar field (i.e., pressure-field), as by the customary microphone or hydrophone. More precisely, an acoustic *vector*-sensor (placed at the origin of the three-dimensional Cartesian coordinates) would have this  $3 \times 1$  array-manifold [1], [2], in response to a unit-power incident acoustic wave that has traveled through an homogeneous isotropic medium:

$$\mathbf{a} \stackrel{\text{def}}{=} \begin{bmatrix} u(\theta, \phi) \\ v(\theta, \phi) \\ w(\theta) \end{bmatrix} \stackrel{\text{def}}{=} \begin{bmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{bmatrix}, \quad (1)$$

where  $0 \leq \theta \leq \pi$  symbolizes the elevation-angle measured from the vertical  $z$ -axis,  $0 \leq \phi < 2\pi$  denotes the azimuth-angle measured from the positive  $x$ -axis,  $u(\theta, \phi)$  refers to the direction-cosine along the  $x$ -axis,  $v(\theta, \phi)$  represents the direction-cosine along the  $y$ -axis, and  $w(\theta)$  refers to the

direction-cosine along the  $z$ -axis. The first, second, and third components in  $\mathbf{a}(\theta, \phi)$  correspond to the acoustic *velocity*-sensors aligned along the  $x$ -axis, the  $y$ -axis, and the  $z$ -axis, respectively.

### 1.2. Literature Review of Relevant Works

This presently proposed algorithm could be related to a class of array calibration algorithms and direction finding algorithms [3, 4, 5, 6, 7], that handle an array of ideal subarrays with *non*-ideal inter-subarray relationships. Amongst these series works, only [5] is noticed that allows small inter-subarray mis-orientations, which is nonetheless limited to one dimensional rotation on the azimuthal plane, but not the full trivariate Euler angles for three-dimensional mis-orientation.

A few calibration algorithms are devised for acoustic vector-sensor's mis-orientation. [8, 9] can handle vector-sensor's mis-orientation, but they all (unlike the present work) perform *aided* calibration (as opposed to "blind" calibration), necessitating cooperative emitters to impinge from prior known directions-of-arrival. It's worthy noting that at least three reference signals impinging from different DOAs are required for [8] to determine the state of misorientation of each vector hydrophone in the array. [10] introduces a focusing technique based calibration algorithm, for a single vector-sensor with time-variant mis-orientation, but assumes ideal (or prior known) gain and phase for the single vector-sensor.

### 1.3. This Works Contributions

This present work's contributions may be concluded in several regards:

1. A new blind calibration and remedy scheme, which requires no priori knowledge of incident sources' DOAs, is proposed to estimate and rectify the full trivariate Euler angles for three-dimensional mis-orientation;
2. The new calibration method allows the velocity-sensor triads with unknown inter-triad gain and phase deviations to be arbitrarily separated at unknown locations in three-dimensional space.

3. The new calibration method has no constraints on the number of emitters.

## 2. MATHEMATICAL MODELING OF MIS-ORIENTATION IN AN VELOCITY-SENSOR TRIAD

The array manifold in (1) is ideal, in presuming perfect gain responses, as well as presuming perfect conformity to the nominal locations and orientations. If this hypothetical ideality is violated by the  $\ell$ th velocity-sensor triad, its array manifold would become

$$\mathbf{a}_k^{(\ell)} = \mathcal{G}^{(\ell)} \mathbf{R}(\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)}) \begin{bmatrix} u_k \\ v_k \\ w_k \end{bmatrix} \times e^{j \frac{2\pi f_k}{c} \{u_k x^{(\ell)} + v_k y^{(\ell)} + w_k z^{(\ell)}\}} \quad (2)$$

In the above,

1. the mis-orientation is represented by a  $3 \times 3$  matrix,  $\mathbf{R}(\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)})$  where

$$\begin{aligned} & \mathbf{R}(\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)}) \\ &= \begin{bmatrix} \cos \gamma^{(\ell)} & \sin \gamma^{(\ell)} & 0 \\ -\sin \gamma^{(\ell)} & \cos \gamma^{(\ell)} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta^{(\ell)} & \sin \beta^{(\ell)} \\ 0 & -\sin \beta^{(\ell)} & \cos \beta^{(\ell)} \end{bmatrix} \\ & \times \begin{bmatrix} \cos \alpha^{(\ell)} & 0 & \sin \alpha^{(\ell)} \\ 0 & 1 & 0 \\ -\sin \alpha^{(\ell)} & 0 & \cos \alpha^{(\ell)} \end{bmatrix}. \end{aligned} \quad (3)$$

Any mis-orientation in three-dimensional space may be represented by the model in (3), which involves three sequential rotations: 1) rotation by an angle of  $\alpha^{(\ell)}$  on the  $x$ - $z$  plane, 2) then a second rotation by an angle of  $\beta^{(\ell)}$  on the  $y$ - $z$  plane, and 3) lastly a third rotation by an angle of  $\gamma^{(\ell)}$  on the  $x$ - $y$  plane. Without loss of generality, let the  $\ell = 1$ st velocity-sensor triad serve as orientation reference. I.e.  $\alpha^{(1)} = 0, \beta^{(1)} = 0, \gamma^{(1)} = 0$ .

2. the unknown complex-valued gain deviation for the  $\ell$ th velocity-sensor triad is represented by  $\mathcal{G}^{(\ell)}$ ,
3.  $e^{j \frac{2\pi f_k}{c} (x^{(\ell)} u_k + y^{(\ell)} v_k + z^{(\ell)} w_k)}$  represents the  $k$ th source's *spatial* phase factor relating the  $\ell$ th velocity-sensor triad's *arbitrary unknown* location  $(x^{(\ell)}, y^{(\ell)}, z^{(\ell)})$  relative to the reference location of  $(0, 0, 0)$ , with  $f_k$  denoting the  $k$ th narrowband source's center frequency, and  $c$  symbolizing the propagation speed.

Stacking all  $L$  vector sensors' individual steering vectors, the entire array is characterized by the  $3L \times 1$  steering vector,

$$\mathbf{a}_k = \left[ \left( \mathbf{a}_k^{(1)} \right)^T, \dots, \left( \mathbf{a}_k^{(L)} \right)^T \right]^T.$$

## 3. MEASUREMENT DATA MODEL

Consider  $K$  number of incident signals, At the  $\ell$ th velocity-sensor triad, the following  $3 \times 1$  data vector is received at time  $t$ :

$$\mathbf{z}^{(\ell)}(t) = \sum_{k=1}^K \mathbf{a}_k^{(\ell)} s_k(t) + \mathbf{n}(t). \quad (4)$$

The  $k$ th signal is a pure tone  $s_k(t) = \sqrt{\mathcal{P}_k} e^{j(2\pi f_k t + \varphi_k)}$ , at frequency  $f_k$ , distinct from the other  $K - 1$  frequencies. Moreover,  $\mathcal{P}_k$  denotes the signal power (a priori unknown to the receiver);  $\varphi_k$  signifies a random phase (uncorrelated to all other random entities). It is further assumed that  $(\theta_k, \phi_k) \neq (\theta_j, \phi_j), \forall k \neq j \in \{1, \dots, K\}$ . The additive noise  $\mathbf{n}(t)$  is modeled as zero mean, spatio-temporally uncorrelated, with (a priori unknown) power  $\mathcal{P}_N$ .

The present problem aims to estimate the sources' arrival angles, given the  $M \times N$  collected data  $\mathbf{Z} = [\mathbf{z}(T_s), \dots, \mathbf{z}(NT_s)]$ , where  $M = 3L$ ,  $N > 2M + 1$ <sup>1</sup> and  $T_s$  symbolizes the a priori known time sampling period.

## 4. PROPOSED CALIBRATION AND REMEDY METHOD FOR VELOCITY-SENSOR TRIADS' MIS-ORIENTATION

### 4.1. Estimate Each Source's Steering Vector

Refer to "Uni-Vector-Hydrophone ESPRIT" algorithm [11], form the  $M \times N$  data-matrices  $\mathbf{Z}_1 = [\mathbf{z}(t_1), \mathbf{z}(t_2), \dots, \mathbf{z}(t_N)]$  and  $\mathbf{Z}_2 = [\mathbf{z}(t_1 + \Delta_T), \mathbf{z}(t_2 + \Delta_T), \dots, \mathbf{z}(t_N + \Delta_T)]$ . Form the  $2M \times N$  data-matrix  $\mathbf{Z} = [\mathbf{Z}_1^T, \mathbf{Z}_2^T]^T$ . Eigen-decompose  $\mathbf{Z}\mathbf{Z}^H$ , such that  $\mathbf{E}_s = [\mathbf{E}_1^T, \mathbf{E}_2^T]^T$  is a  $2M \times K$  matrix (the signal subspace eigenvector matrix), whose  $K$  columns are the  $K$  principal eigenvectors associated with the  $K$  largest magnitude eigenvalues. Define the  $K \times K$  matrix,

$$\mathbf{\Psi} \stackrel{\text{def}}{=} (\mathbf{E}_1^H \mathbf{E}_1)^{-1} (\mathbf{E}_1^H \mathbf{E}_2) = \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T},$$

where the  $k$ th eigenvalue of  $\mathbf{\Psi}$  equals  $[\mathbf{\Phi}]_{k,k} = e^{j2\pi f_k \Delta_T}$ ,  $\forall k = 1, \dots, K$ , and the corresponding right eigenvector constitutes the  $j$ th column of  $\mathbf{T}$ . The  $K$  impinging sources' steering vectors may be estimated, to within complex value multiplicative scalars, as

$$[\hat{\mathbf{a}}_1, \dots, \hat{\mathbf{a}}_K] = \frac{1}{2} \{ \mathbf{E}_1 \mathbf{T}^{-1} + \mathbf{E}_2 \mathbf{T}^{-1} \mathbf{\Phi}^{-1} \}.$$

These  $K$  *unknown* complex value multiplicative scalars arise from the eigen decomposition of  $\mathbf{\Psi}$ .

Now we exploit the velocity sensor's direct measurement

<sup>1</sup>For a justification of this inequality, please refer to [11].

of the Cartesian direction cosine:

$$\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1 \approx c_k \mathcal{G}^{(\ell)} u_k^{(\ell)} e^{j \frac{2\pi f_k}{c} (x^{(\ell)} u_k + y^{(\ell)} v_k + z^{(\ell)} w_k)}, \quad (5)$$

$$\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_2 \approx c_k \mathcal{G}^{(\ell)} v_k^{(\ell)} e^{j \frac{2\pi f_k}{c} (x^{(\ell)} u_k + y^{(\ell)} v_k + z^{(\ell)} w_k)}, \quad (6)$$

$$\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_3 \approx c_k \mathcal{G}^{(\ell)} w_k^{(\ell)} e^{j \frac{2\pi f_k}{c} (x^{(\ell)} u_k + y^{(\ell)} v_k + z^{(\ell)} w_k)}. \quad (7)$$

where the approximations are caused by noises in the data model,  $c_k$  is an unknown complex value scalar introduced from eigen decomposition.

Both azimuth and elevation angles are embedded wholly within any one velocity-sensor triad's own data. Thus, the  $k$ th source's azimuth and elevation direction of arrivals can be estimated with respect to the  $\ell$ th velocity-sensor triad via (5)-(7) as

$$\begin{aligned} \hat{\theta}_k^{(\ell)} &= -\frac{\pi}{2} \left( \operatorname{sgn} \left( \Re \left\{ \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1}{\cos(\hat{\phi}_k^{(\ell)}) \begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_3} \right\} \right) - 1 \right) \\ &\quad + \arctan \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1}{\cos(\hat{\phi}_k^{(\ell)}) \begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_3}, \quad (8) \\ \hat{\phi}_k^{(\ell)} &= \begin{cases} -\frac{\pi}{2} \left( \operatorname{sgn} \left( \Re \left\{ \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_2}{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1} \right\} \right) - 1 \right) + \arctan \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_2}{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1}, & \text{if } \phi \in [0, \pi), \\ -\frac{\pi}{2} \left( \operatorname{sgn} \left( \Re \left\{ \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_2}{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1} \right\} \right) - 3 \right) + \arctan \frac{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_2}{\begin{bmatrix} \hat{\mathbf{a}}_k^{(\ell)} \end{bmatrix}_1}, & \text{if } \phi \in [\pi, 2\pi). \end{cases} \quad (9) \end{aligned}$$

where  $\Re\{\cdot\}$  returns the real part of  $\{\cdot\}$  and  $\operatorname{sgn}(\cdot)$  signifies the sign of the real-value number inside the parentheses  $(\cdot)$ .

## 4.2. Calibration of Velocity-Sensor Triad Mis-Orientation

The mis-orientation matrix given in (3) relates the reference (i.e. the 1st) velocity-sensor triad's array manifold to the  $\ell$ th velocity-sensor triad's array manifold as  $\mathbf{a}_k^{(\ell)} = \mathbf{R}(\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)}) \mathbf{a}_k^{(1)}$ . Let  $\mathbf{a}_k^{(1,\ell)} \stackrel{\text{def}}{=} \left[ \left( \mathbf{a}_k^{(1)} \right)^T, \left( \mathbf{a}_k^{(\ell)} \right)^T \right]^T$ .

With azimuth and elevation angles estimated w.r.t. the 1st velocity-sensor triad  $\hat{\theta}_k^{(1)}, \hat{\phi}_k^{(1)}$ , and to-be-estimated mis-orientation parameters for the  $\ell$ th velocity-sensor triad  $\alpha, \beta, \gamma$ ,

$\mathbf{a}_k^{(1,\ell)}$  may be written in the following form:

$$\begin{aligned} \mathbf{a}_k^{(1,\ell)} &= \underbrace{\begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{R}(\alpha, \beta, \gamma) \end{bmatrix}}_{\stackrel{\text{def}}{=} \mathbf{\Upsilon}_k^{(\alpha, \beta, \gamma)}} \left\{ \mathbf{I}_2 \otimes \begin{bmatrix} \sin(\hat{\theta}_k^{(1)}) \cos(\hat{\phi}_k^{(1)}) \\ \sin(\hat{\theta}_k^{(1)}) \sin(\hat{\phi}_k^{(1)}) \\ \cos(\hat{\theta}_k^{(1)}) \end{bmatrix} \right\} \\ &\quad \times \underbrace{\begin{bmatrix} \mathcal{G}^{(1)} e^{j \frac{2\pi f_k}{c} \{u_k x^{(1)} + v_k y^{(1)} + w_k z^{(1)}\}} \\ \vdots \\ \mathcal{G}^{(L)} e^{j \frac{2\pi f_k}{c} \{u_k x^{(L)} + v_k y^{(L)} + w_k z^{(L)}\}} \end{bmatrix}}_{\stackrel{\text{def}}{=} \mathbf{h}} \end{aligned} \quad (10)$$

where  $\mathbf{I}_3$  signifies  $3 \times 3$  identity matrix,  $\mathbf{0}_{3 \times 3}$  denotes  $3 \times 3$  zero matrix and  $\otimes$  symbolizes Kronecker product.

Substitute (10) into the MUSIC equation, which may be expressed as  $\left( \mathbf{a}_k^{(1,\ell)} \right)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}_k^{(1,\ell)} = 0$ , where the columns of  $6 \times (6 - K)$  matrix  $\mathbf{E}_n$  contains  $6 - K$  eigenvectors spanning the noise subspace of  $\mathbf{Z}^{(1,\ell)} \mathbf{Z}^{(1,\ell)H}$ , where  $\mathbf{Z}^{(1,\ell)} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3 \times (M-3)} \\ \mathbf{0}_{3 \times (\ell-1)} & \mathbf{I}_3 & \mathbf{0}_{3 \times (M-\ell-2)} \end{bmatrix} \mathbf{Z}$ , which denotes  $6 \times N$  data matrix collected from the 1st velocity-sensor triad and the  $\ell$ th mis-oriented velocity-sensor triad. Then, we have

$$\mathbf{h}^H \underbrace{\mathbf{\Upsilon}_k^{(\alpha, \beta, \gamma)H} \mathbf{E}_n \mathbf{E}_n^H \mathbf{\Upsilon}_k^{(\alpha, \beta, \gamma)}}_{\stackrel{\text{def}}{=} \mathbf{C}_k^{(\alpha, \beta, \gamma)}} \mathbf{h} = 0 \quad (11)$$

If  $\theta \neq \{0, \pi\}$ , since  $\mathbf{h}^H \neq 0$ , (11) can be true only if  $\mathbf{C}_k^{(\alpha, \beta, \gamma)}$  drops rank (i.e.  $\det\{\mathbf{C}_k^{(\alpha, \beta, \gamma)}\} = 0$ ). [4] shows that the rank of  $\mathbf{C}_k^{(\alpha, \beta, \gamma)}$  drops (i.e.,  $\text{rank}\{\mathbf{C}_k^{(\alpha, \beta, \gamma)}\} < L$ ) if and only if  $\alpha = \alpha^{(\ell)}, \beta = \beta^{(\ell)}, \gamma = \gamma^{(\ell)}$ . Therefore,  $(\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)})$  can be estimated as

$$\hat{\alpha}^{(\ell)}, \hat{\beta}^{(\ell)}, \hat{\gamma}^{(\ell)} = \arg \min_{\alpha, \beta, \gamma} \left\{ \sum_{k=1}^K \det \mathbf{C}_k^{(\alpha, \beta, \gamma)} \right\} \quad (12)$$

where  $\det\{\cdot\}$  is the determinant of matrix.

## 4.3. Remedy for Velocity-Sensor Triad Mis-orientation

The "raw" remedy procedure presented in [8, 10] accommodates the vector-sensor's orientation errors directly on the received data (i.e. the raw data), thus noise would be cross-correlated among the three component-sensors at each vector-sensor triad. Hence, this phenomenon may be evitable if the orientation errors are rectified on the signal subspace.

Having identified the orientation of each velocity-sensor triad by the foregoing calibration algorithm, the direction-cosines for the  $k$ th source at the  $\ell$ th ( $\ell > 1$ ) velocity-sensor

triad need be modified as follow to accommodate the velocity-sensor triads' mis-orientation:

$$\underbrace{\begin{bmatrix} \hat{u}_k^{\ell, \text{remedy}} \\ \hat{v}_k^{\ell, \text{remedy}} \\ \hat{w}_k^{\ell, \text{remedy}} \end{bmatrix}}_{\stackrel{\text{def}}{=} \mathbf{p}_k^{(\ell)}} = \left( \mathbf{R} \left( \hat{\alpha}^{(\ell)}, \hat{\beta}^{(\ell)}, \hat{\gamma}^{(\ell)} \right) \right)^{-1} \begin{bmatrix} \hat{u}_k^{\ell} \\ \hat{v}_k^{\ell} \\ \hat{w}_k^{\ell} \end{bmatrix} \quad (13)$$

where

$[\hat{u}_k^{\ell}, \hat{v}_k^{\ell}, \hat{w}_k^{\ell}]^T = [\sin \hat{\theta}_k^{(\ell)} \cos \hat{\phi}_k^{(\ell)}, \sin \hat{\theta}_k^{(\ell)} \sin \hat{\phi}_k^{(\ell)}, \cos \hat{\theta}_k^{(\ell)}]^T$  are the direction-cosines which are estimated from the signal subspace as illustrated in Section 4.1. Moreover, as the signal-to-noise-ratio in the signal subspace would be higher than that in the received data, better direction-finding accuracy would be expected if the remedy procedure is performed on the signal subspace.

With  $[\hat{u}_k^{\ell}, \hat{v}_k^{\ell}, \hat{w}_k^{\ell}]^T$  thus being rectified, the  $k$ th source's direction-cosines  $\mathbf{p}_k \stackrel{\text{def}}{=} [u_k, v_k, w_k]^T$  may be estimated as

$$\hat{\mathbf{p}}_k = \arg \min_{\mathbf{p}_k} \sum_{\ell=1}^L \left| \mathbf{p}_k - \mathbf{p}_k^{(\ell)} \right|^2,$$

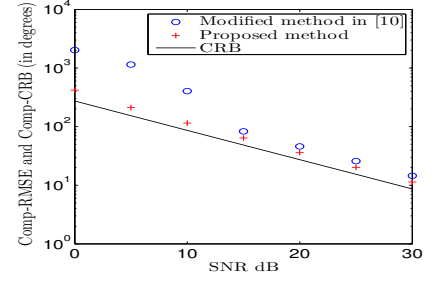
The well known solution for this optimization problem is  $\hat{\mathbf{p}}_k = \frac{1}{L} \sum_{\ell=1}^L \mathbf{p}_k^{(\ell)}$ . Then, the estimates of sources' direction-of-arrivals may be found as:

$$\hat{\theta}_k = \arccos[\hat{\mathbf{p}}_k]_3 \quad (14)$$

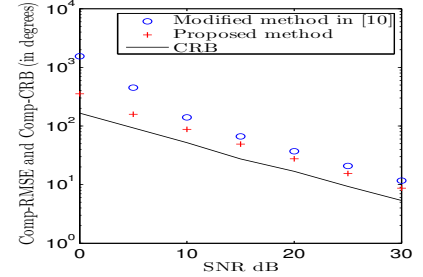
$$\hat{\phi}_k = \arctan \frac{[\hat{\mathbf{p}}_k]_2}{[\hat{\mathbf{p}}_k]_1} \quad (15)$$

## 5. NUMERICAL RESULTS

The simulation scenario is detailed below: There exist  $K = 2$  pure-tone incident signals, at digital frequencies  $f'_1 = 0.47$ , and  $f'_2 = 0.16$ , which are a priori unknown to the receiver. Each pure tone has a random phase, uniformly distributed between  $[0, 2\pi)$  radians, independently generated at each Monte Carlo trial, and independent across the signals. The emitters' respective azimuth and elevation directions of arrivals are  $(\theta_1, \phi_1) = (80^\circ, 50^\circ)$ , and  $(\theta_2, \phi_2) = (40^\circ, -155^\circ)$ , also a priori unknown to the receiver. There exist  $L = 5$  non-ideal velocity-sensor triads, with unknown locations, unknown complex-valued gain deviations, and mis-locations. These  $L = 5$  velocity-sensor triads' mis-orientations are  $(\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}) = (0^\circ, 0^\circ, 0^\circ)$ , and  $(\alpha^{(2)}, \beta^{(2)}, \gamma^{(2)}) = (2^\circ, 5^\circ, 4^\circ)$ , and  $(\alpha^{(3)}, \beta^{(3)}, \gamma^{(3)}) = (3^\circ, 1^\circ, 2^\circ)$ , and  $(\alpha^{(4)}, \beta^{(4)}, \gamma^{(4)}) = (5^\circ, 2^\circ, 1^\circ)$ , and  $(\alpha^{(5)}, \beta^{(5)}, \gamma^{(5)}) = (6^\circ, 3^\circ, 4^\circ)$ . At each Monte Carlo trial, the collected data consist of  $N = 80$  snapshots, corrupted by additive Gaussian noise, white over time, white also across all velocity sensors. Each icon in Figures 1a-1b consists of  $I = 500$  statistically independent Monte Carlo trials.



**Fig. 1a.** Composite mean square error (Comp-RMSE) in estimating the mis-orientation angles.



**Fig. 1b.** Composite mean square error (Comp-RMSE) in estimating the azimuth-elevation directions-of-arrival.

For comparison, the method proposed in [10] is modified to adapt for current situations: 1) time segments in [10] are replaced by spatial segments (i.e. different velocity-sensor triads), 2) MUSIC algorithm used in [10] for DOA estimation is replaced by RARE algorithm in [4] so as to accommodate unknown gain/phase deviation and unknown subarray locations.

## 6. CONCLUSION

The proposed calibration algorithm works well in presence of velocity-sensor triad's unknown location and unknown complex-valued gain deviation. The remedy procedure is performed on the signal subspace which is more robust to noise contamination.

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<sup>2</sup>The composite root-mean-square-error (R.M.S.E.) and composite CRB are respectively defined as  $\frac{1}{KT} \sum_{k=1}^K \sum_{i=1}^I \sqrt{\frac{\delta_{\theta,k,i}^2 + \delta_{\phi,k,i}^2}{2}}$  and  $\frac{1}{K} \sum_{k=1}^K \sqrt{\frac{\text{CRB}(\theta_k) + \text{CRB}(\phi_k)}{2}}$ . Herein,  $\delta_{\theta,k,i}$  ( $\delta_{\phi,k,i}$ ) symbolizes the  $i$ th Monte Carlo trial's estimation error for  $\theta_k$  ( $\phi_k$ ).

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