ON THE STRUCTURE OF THE MULTI-MODE FILTERS FOR PASSIVE WAVEFRONT CURVATURE RANGING IN A DISTRIBUTED ARRAY SYSTEM *

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ABSTRACT

This work presents some new results on the structures of, and our interpretation on, the multi-rank filters used for passive wavefront curvature (WFC) ranging. Such a WFC ranging systems uses a large-scale distributed arrays with many spatially separated modular arrays, operating under environments subject to a spatial coherence loss. Working on the modular array level beamformed data, the multi-rank filters along with the weighting coefficients provide further spatial filtering capability to rake in spatial coherence existing in the distorted wavefronts impinging on different modular arrays. Such multi-rank filters can improve ranging performance through different combining schemes, beyond what achieved by the bearing-only based triangulation. For a real-valued inter-module spatial coherence matrix, the derived multi-rank filters follow a nicely balanced structure comprised of in-phase and quadrature (I/Q) modes with varying spatial directions. The results provide a simple solution for us to discovering levels of coherence existing in different modes for multi-mode combining.

Index Terms— Asymptotic Results, Eigen-Modes, Large-Scale Array of Arrays, Passive Source Localization

1. INTRODUCTION

In a typical passive sensing system [1, 2] deployed for surveillance, a large number of distributed modular arrays or sub-arrays are used to form a composite array network. In our application, the aim is to passively detect, localize and track a distant source of interest using a towed array system comprised of multiple array modules. Under such a passive operation condition, the distant source can be viewed as a far-filed source on each individual morular array level. Hence, we can use modular level sub-array beamforming to improve data quality and at the same time to reduce data dimensionality. To achieve the ranging capability at long range, sub-arrays need to be distributed spatially so that wavefront curvature can be sensed. However, in an underwater acoustic environment as illustrated in Fig.1, the inhomogeneous random media, such as turbulence, internal waves and currents, can distort wavefront arriving on a distributed array system, causing spatial coherence loss [5, 6, 7, 8].

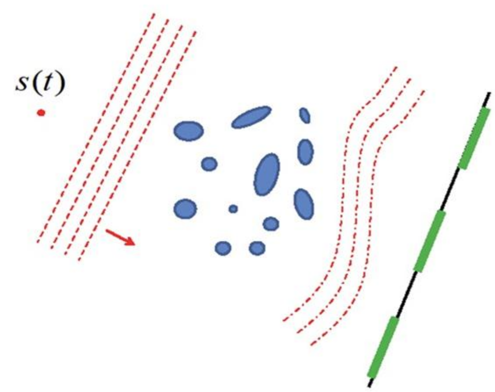


Fig. 1. An illustration of underwater acoustic channel's random media effect, which causes distortions on the wavefronts impinging on a distributed array system.

As can be imagined from the above illustration that the impact of wavefront distortion is connected to a smearing-like effect of a point source perceived by a distributed array system. This is also connected to the spatial coherence loss of the spherical wavefront caused by random phase jitters (phase discontinuities) existing among array modules [3, 4, 6].

Most of the previous work on the effects of spatial coherence loss was focused primarily on the beamforming and direction of arrival (DoA) estimation applications [7, 8, 11, 12]. While our primary goal of this research is on the passive ranging beyond the traditional triangulation method [2] (a non-coherence solution). To model and combat such spatial coherence loss, we developed in [9] a two-stage multi-rank maximum likelihood solution to a passive ranging system using wavefront curvature (WFC) sensed by an array of three modular arrays in environments subject to signal coherence loss.

In this paper we examine the structure of the optimum filter weights when the array becomes large. We show that these multi-rank modes tend to become Fourier-like vectors that occur in in-phase and quadrature pairs.

*THIS WORK HAS BEEN PARTIALLY SUPPORTED BY THE OFFICE OF NAVAL RESEARCH UNDER THE GRANT N00014-12-1-0215 AND THE NUWC.

2. NOTATION AND THE MULTI-RANK PROCESSOR

The key components in the multi-rank processor are a bank of multi-mode eigen-filters and a combiner. Built from the knowledge of spatial coherence model, such components further process the beamformed outputs from small-size modular arrays for passive ranging. To ease the presentation on the multi-mode filter, let us briefly summarize the multi-rank ranging solution in [9] as follows. The source's range and bearing information are revealed by the peak of the compressed likelihood function in the parameter space,

$$\begin{bmatrix} \hat{r} \\ \hat{\theta} \end{bmatrix} = \arg\max_{r,\theta} J_{ML}(r,\theta), \tag{1}$$

with

$$J_{ML}(r,\theta) = \sum_{i=1}^{L} \frac{\lambda_i\, \eta_{SNR}}{1+\lambda_i\, \eta_{SNR}} \left|\mathbf{v}_i^H \mathbf{y}(t)\right|^2. \tag{2}$$

Here, $\mathbf{v}_i$ and λ_i are the eigen-vector and eigen-value pair of the inter-module spatial coherence matrix $\mathbf{R}_\rho$ in eq. (6); $\eta_{SNR} = \sigma_s^2/\sigma_n^2$ is the sample SNR on the data from each array channel; the $L \times 1$ vector $\mathbf{y}(t) = \mathbf{y}(t; r, \theta, f)$ collects beamformed data from all L modular arrays, i.e.,

$$\mathbf{y}(t; r,\theta,f) = \begin{bmatrix} \mathbf{s}_1^H(r,\theta,f)\cdot \mathbf{x}_1(t) \\ \vdots \\ \mathbf{s}_L^H(r,\theta,f)\cdot \mathbf{x}_L(t) \end{bmatrix},$$

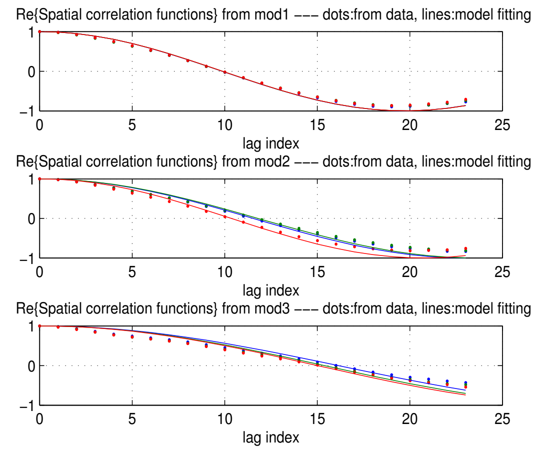
with the kth sub-arrays' steering vector,

$$\mathbf{s}_k(r,\theta,f) = \begin{bmatrix} \exp\left\{-j2\pi f \frac{\|\mathbf{p}_{k,n}-\mathbf{p}_s(r,\theta)\|}{c}\right\} \\ \downarrow\; n = 1,2,\ldots,N_t \end{bmatrix}_{N_t\times 1},$$

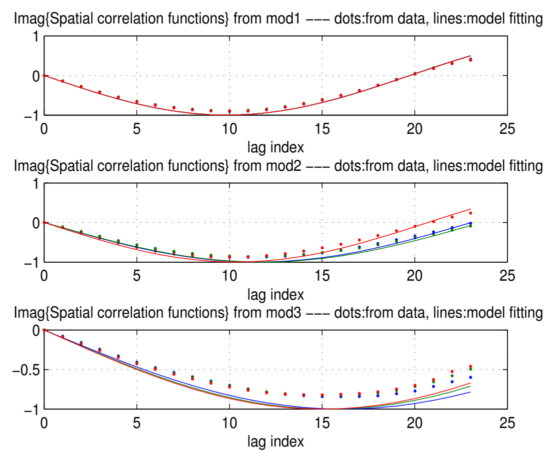
working on data $\mathbf{x}_k(t)$ collected by the kth modular array. Here, the $\mathbf{p}_{k,n}$ denotes the coordinate of the nth hydrophone in the kth modular array; the $\mathbf{p}_s$ denotes the source's coordinate. When applying the above procedure to our sea test data, the soundness of the first-stage beamforming operation on modular array level become evident in the bearing-rate results obtained on modular array level.

3. SPATIAL COHERENCE

The focus of this work is to study the structure of the multi-rank filters and the combiner contained in the multi-rank processor outlined in eq. (2). As can be seen from the notation and results shown in the Fig.2 that different levels of spatial coherence exist in the data collected by a distributed array system. Shown in Fig.2, are the spatial correlation functions (SCFs), either synthesized using an analytical model (lines) or extracted from the cross-spectral density matrix (CSDM) calculated from the data out of modular level sub-arrays, at a processing frequency.



(a) Real parts of the spatial correlation function (SCF).



(b) Imaginary parts of the spatial correlation function (SCF).

Fig. 2. The SCFs observed by a distributed arrays consists of three volumetric modular arrays. Results from data-driven CSDM are shown by dots, and SCF model based syntheses are shown by lines.

3.1. Intra-module Spatial Coherence

Under the far-field assumption and the presence of random scatter induced path loss [14, 15], the normalized spatial correlation matrix for the data on a modular array level, follows a Hermitian Toeplitz structure,

$$\mathbf{R}_{mod} = \text{Toeplitz}(\boldsymbol{\rho}, \boldsymbol{\rho}^H), \tag{3}$$

with complex-valued correlation vector $\boldsymbol{\rho}$ taking the form of,

$$\boldsymbol{\rho} = \exp\left\{-j\frac{2\pi}{\lambda}\mathbf{p}_{mod}(d)\cdot \boldsymbol{\kappa}(\boldsymbol{\theta}_s)\right\}. \tag{4}$$

Here the $N \times 3$ matrix $\mathbf{p}_{mod}(d) = \begin{bmatrix} \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \end{bmatrix}$ contains a collection of position vectors of a uniform linear array (ULA) within a modular array, and the vector

$$\boldsymbol{\kappa}(\boldsymbol{\theta}_s) = \begin{bmatrix} \sin(\theta_z) \cdot \cos(\theta) \\ \sin(\theta_z) \cdot \sin(\theta) \\ \cos(\theta_z) \end{bmatrix}$$

is simply a 3-D wave vector along the direction of the source of interest. Note that due to the different spatial location of each modular array, the source's far-field bearing (the wave vector) depends on the *apparent* bearing seen at each modular array. As can be seen from Fig.2, the complex-valued spatial correlation estimated out of data collected on modular array fit nicely into a far-field analytical model. In our application, due to the relative small-size of modular array in comparison to the source distance, intra-module coherence can be maintained in many processing bands. This facilitates the coherent sub-array beamforing operation on modular-array level.

Our special focus in this work is on how to characterize the special structures, hence, to understand the roles of the multi-mode filters $\mathbf{v}_i$ as well as the weighting coefficients λ_i in (2), for a large-scale distributed array.

3.2. Inter-module Spatial Coherence

The wiggling wavefronts perceived at spatially separated array modules motivates us to introduce an inter-module coherence matrix $\mathbf{R}_\rho$. In doing so, the data on the whole array

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \vdots \\ \mathbf{x}_L(t) \end{bmatrix} = \sigma_s \cdot \mathbf{S}_{st}(r,\theta,f) \cdot \mathbf{s}_\rho(t) + \mathbf{n}(t). \tag{5}$$

will have the following correlation matrix,

$$\mathbf{R}_\mathbf{x} = \sigma_s^2 \cdot \mathbf{S}_{st}(r,\theta,f)\,\mathbf{R}_\rho\,\mathbf{S}_{st}^H(r,\theta,f) + \sigma_n^2\mathbf{I}, \tag{6}$$

with

$$\mathbf{S}_{st}(r,\theta,f) = \begin{bmatrix} \mathbf{s}_1(r,\theta,f) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_2(r,\theta,f) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{s}_L(r,\theta,f) \end{bmatrix}_{N\times L}$$

being the the whole array's steering matrix. Note that when there is no coherence loss among distributed sub-array modules, $\mathbf{R}_\rho = \mathbf{1}_L \cdot \mathbf{1}_L^T$, hence, the signal term in $\mathbf{R}_\mathbf{x}$ becomes a rank-1 term, commonly assumed in a traditional array processing (a fully coherent system),

$$\sigma_s^2 \cdot \mathbf{S}_{st}(r,\theta,f)\,\mathbf{R}_\rho\,\mathbf{S}_{st}^H(r,\theta,f) = \sigma_s^2 \cdot \mathbf{s}(r,\theta,f)\,\mathbf{s}^H(r,\theta,f).$$

Here vector $N_t L \times 1$ vector $\mathbf{s}(r,\theta,f)$ is simply the concatenation of all L sub-arrays' steering vectors. The multi-rank processor de-generalizes into a traditional rank-1 beamformer. In general, the rank, the multi-mode filters and the associated weighting coefficients are determined by the spatial coherence existing in the operation environment. Therefore, the rank along with the eigen-structure of the inter-module spatial coherence matrix $\mathbf{R}_\rho$ determines the structure and performance of the passive ranging system.

As mentioned earlier, the spatial coherence loss is related to the distortion on the wavefront perceived by the distributes array. It is also related to a random phase or delay jitter or phase discontinuity of wavefronts shown among different modula arrays, which causes a smearing or dispersion effect on a point source. Such distortion has been studied and characterized by other researchers in DoA estimation related applications [8, 12, 11]. Under the assumption that the random phase jitter has a zero mean and symmetric probability density function (pdf), the spatial coherence matrix can be justified to follow a real-valued model. By introducing a zero-mean stochastic model on the random phase jitter to an array's steering vector,

$$\mathbf{s}_{steer}(r_s,\phi_s,f;\delta_\phi) = \mathbf{s}(r_s,\phi_s+\delta_\phi,f),$$

with (r_s,ϕ_s) being source's location with respect to a reference point, we can establish a possible model for such connection. Specifically, when modeling the random phase jitter with a symmetric probability density functions (pdf), using either a uniform pdf, $\delta_\phi \sim U(0,\phi_{max})$ or a Gaussian pdf $\delta_\phi \sim N(0,\sigma_\phi^2)$, we can come up with an approximate frequency dependent models for the real-valued inter-module coherence matrix,

$$\mathbf{R}_\rho^{(unif)}(n,m) = \text{sinc}\left(2\phi_{max}\frac{(m-n)L_t}{\lambda}\sin(\phi_s)\right)$$

and

$$\mathbf{R}_\rho^{(Gauss)}(n,m) = \exp\left\{-\left(\frac{\pi\sigma_\phi \sin(\phi_s)\cdot(m-n)L_t}{\lambda}\right)^2\right\}$$

where L_t is the inter-module spacing of the multi-module array; and ϕ_s is the source's bearing with respect to the baseline of the array.

In underwater environment, another commonly adopted model for the spatial coherence is the exponential model [5, 6, 8]. Based on such model, the inter-module coherence matrix among all L array modules (L_t-spaced) can be found having the Toeplitz form of,

$$\mathbf{R}_\rho = Toeplitz([1,\rho,\rho^4,\ldots,\rho^{(L-1)^2}]),$$

where the parameter $\rho = \exp\{-2(L_t/L_{coh})^2\}$ is a function of L_{coh} the coherence length of the wavefield.

To understand the eigen-structure of $\mathbf{R}_\rho$ from the spectral decomposition perspective, we can rewrite any Toeplitz structure coherence matrix precisely as,

$$\mathbf{R}_\rho = \int_{-1/2}^{+1/2} \mathbf{e}(\nu)P_\rho(\nu)\mathbf{e}^H(\nu)d\nu.$$

Here the vector $\mathbf{e}(\nu) = \begin{bmatrix} 1 & e^{j2\pi\nu} & \cdots & e^{j2\pi(L-1)\nu} \end{bmatrix}^T$ is simply the well known discrete-time Fourier transform vector. $P_\rho(\nu)$ is the spatial power spectral density (PSD) that describes the wavefield's power distribution in beamspace.

4. STRUCTURE OF THE MULTI-MODE FILTERS

When a large number of distributed sub-arrays are made available for WFC ranging, the asymptotic results developed in [13] provide design guidelines on how to choose the inter-module spacings, the multi-rank filters and the corresponding multi-rank combining coefficients, given the spatial coherence model, without resorting to the real-time large-size matrix eigen-analysis. When L is large enough, the following asymptotic spectral decomposition holds,

$$\mathbf{R}_\rho = \sum_{i=1}^{L} \lambda_i \mathbf{v}_i \, \mathbf{v}_i^H \approx \frac{1}{L} \sum_{i=1}^{L} P_\rho(\nu_i) \mathbf{e}(\nu_i) \, \mathbf{e}^H(\nu_i). \quad (7)$$

That is, for large L, the eigen-mode $\mathbf{v}_i \approx \frac{1}{\sqrt{L}} \mathbf{e}(\nu_i)$ becomes a normalized DFT vector and the eigenvalue $\lambda_i \approx P_\rho(\nu_i)$ becomes the spatial PSD, each being evaluated at the DFT bin $\nu_i = (i-1)/L, (i = 1, 2, \ldots, L)$. Note from the previous results on the inter-module coherence models that $\mathbf{R}_\rho$ is of real-valued. This implies that actual $\mathbf{v}_i$'s and λ_i's should be real-valued too. Further analysis reveals an interesting result on the structure of the multi-mode filters in I/Q balanced pair with increasing oscillating rates. This is accomplished by realizing the fact that the complex-valued DFT-based modes $\mathbf{e}(\nu_i)$ exhibits a conjugate behavior along the unit circle (within the digital spatial frequency region), and the spatial PSD $P_\rho(\nu)$ is of even-symmetry around the Nyquest spatial frequency. Hence, terms in eq. (7) can be combined together in conjugate pairs. Doing so results in the following real-valued eigen mode results,

$$\begin{aligned}
\mathbf{R}_\rho &= \tfrac{P_\rho(0)}{L} \mathbf{1} \cdot \mathbf{1}^T + \\
&\quad \sum_{i=2}^{\lceil L/2-1 \rceil} \frac{2P_\rho(\nu_i)}{L} \cdot Re\left\{\mathbf{e}(\nu_i)\mathbf{e}^H(\nu_i)\right\}, \text{ for odd } L, \\
&= \tfrac{P_\rho(0)}{L} \mathbf{1} \cdot \mathbf{1}^T + \tfrac{P_\rho(1/2)}{L} \mathbf{1}_- \cdot \mathbf{1}_-^T + \\
&\quad \sum_{i=2}^{L/2-1} \frac{2P_\rho(\nu_i)}{L} \cdot Re\left\{\mathbf{e}(\nu_i)\mathbf{e}^H(\nu_i)\right\}, \text{ for even } L,
\end{aligned} \quad (8)$$

where $L \times 1$ mode vector $\mathbf{1}$ contains all ones related to simply a delay-sum beamforming mode to pick up the broadside wavefield component; while the mode vector $\mathbf{1}_-$ contains $+1$'s and -1's in alternating signs, a chain of dipoles to pick up endfire wavefield component. The balanced I/Q mode vectors functioning to pick up the wavefield components from other spatial angles. This can be seen from a further simplification,

$$Re\left\{\mathbf{e}(\nu_i)\mathbf{e}^H(\nu_i)\right\} = \mathbf{c}(\nu_i) \cdot \mathbf{c}^T(\nu_i) + \mathbf{s}(\nu_i) \cdot \mathbf{s}^T(\nu_i),$$

with the in-phase and quadrature phase (I/Q) vectors $\mathbf{c}(\nu_i)$ and $\mathbf{s}(\nu_i)$ defined as,

$$\mathbf{c}(\nu_i) = \begin{bmatrix} \cos(2\pi\nu_i n) \\ \downarrow \\ n = 1, 2, \ldots, L \end{bmatrix}, \mathbf{s}(\nu_i) = \begin{bmatrix} \sin(2\pi\nu_i n) \\ \downarrow \\ n = 1, 2, \ldots, L \end{bmatrix}.$$

Shown in Fig.3 are a few typical multi-mode filters aiming at different beamspaces, where the first mode points towards broadside, other modes fan out in I/Q pairs along with the end-fire mode.

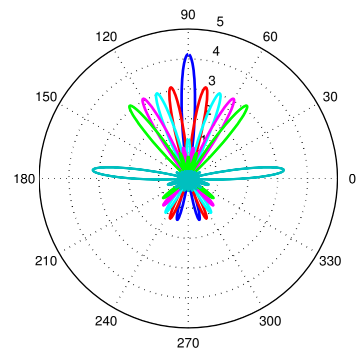


Fig. 3. A few multi-mode filters including the broadside mode, end-fire mode as well as the I/Q pairs, used in a multi-rank passive ranging system with $L = 20$.

Note that the ordering of the corresponding eigen-values occurs pair-wisely as follows,

$$\lambda_i = \left\{P_\rho(0), P_\rho(\nu_1), P_\rho(\nu_1), \cdots, P_\rho(\nu_{L/2}), P_\rho(\nu_{L/2})\right\}.$$

Their values depend on the spatial power spectral distribution along different beam-spaces, and depend on the actual inter-module spatial coherence present in the operation environment. The important message here is that for a large-scale array system, such power distribution can be obtained adaptively (using the estimated inter-module spatial coherence) by calculating power contained in the modular array beamformed data via a projection due to the fact,

$$\lambda_i = \mathbf{v}^H(\nu_i)\mathbf{R}_\rho \mathbf{v}(\nu_i).$$

The importance of the results is that given a pre-selected or an estimated inter-module spatial coherence model and the system parameters of a large-scale array network, the asymptotic eigen-analysis can be pre-calculated either analytically or numerically, and used for building the multi-rank processor for passive WFC ranging application.

5. CONCLUDING REMARKS

This work provides new results on the spatial coherence models and test existing in a distributed array system. The interesting I/Q balanced asymptotic results on the inter-module spatial coherence provide further understanding on the structures and roles played by the multi-mode filters in beamspace. Such filters are used in the multi-rank processor to rake in existing spatial coherence. The projection perspective of revealing the coherence levels existing in different beamspace leads to a simple way of obtaining the weighting coefficients for coherent or non-coherence combining to be used in the multi-rank passive WFC ranging system, operating in environments subject to spatial coherence loss.

6. REFERENCES

[1] R. M. Zeskind et al., "Accuracy of Linearized Performance Predictions for Wavefront Curvature Ranging Systems," *Proc. of ICASSP'83*, pp. 879-882, April 1983.

[2] D. Havelock, S. Kuwano, and M. Vorlander, "Handbook of Signal Processing in Acoustics," *Springer*, 2008.

[3] L. E. Kinsler, A. R Frey, A. B. Copper, and J. V. Sanders, "Fundamentals of Acoustics," *John Wiley & Sons Inc.*, New York, 2000.

[4] A. Pierce, "Acoustics: An Introduction to its Physical Principles and Applications," *McGraw Hill Inc.*, 1991.

[5] H. Cox, "Linear Array Performance When the Signal Coherence Is Spatially Dependent," *J. of Acoustic Society of America*, vol. 54, no. 6, pp 1743 -1746, 1973.

[6] W. Carey, "Measurement of Down-Slope Sound Propagation from a Shallow Source to a Deep Ocean Receiver," *J. of Acoustic Society of America*, vol. 79, no. 1, pp. 49-59, 1986.

[7] A. Paulraj and T. Kailath, "Direction of Arrival Estimation by Eigenstructure Methods with Imperfect Spatial Coherence of Wavefront," *J. of Acoustic Society of America*, vol. 83, no. 3, pp. 1034 - 1040, 1988.

[8] A. B. Gershman, C. F. Mecklenbrauker,and J. F. Bohme, "Matrix Fitting Approach to Direction of Arrival Estimation with Imperfect Spatial Coherence of Wavefronts," *IEEE Trans. on Signal Processing*, vol. 45, no. 7, pp. 1894-1899, 1997.

[9] H. Ge, and I. P. Kirsteins, "Multi-Rank Processing for Passive Ranging in Underwater Acoustic Environments Subject to Spatial Coherence Loss," *Proc. of ICASSP'11*, pp. 269–2695, May 2011.

[10] P. M. Schultheiss, "Optimum Range and Bearing Estimation with Randomly Perturbed Arrays," *J. Acoust. Soc. Am.*, vol. 68, pp. 167-173, 1980.

[11] Y. Meng, P. Stoica, and K. M. Wong, "Estimation of the Directions of Arrival of Spatially Dispersed Signals in Array Processing," *IEE Proc. Radar, Sonar Navig*, pp. 1-9, vol. 143, No. 1, Feb. 1996.

[12] G. Montalbano, and G. V. Serebryakov, "Optimum Array Signal Processing in the Presence of Imperfect Spatial Coherence of Wavefronts," *IEEE Conf. Oceans* 1998.

[13] H. Ge, and I. P. Kirsteins, "Asymptotic Results for Passive Wavefront Curvature Ranging using a Large-Scale Distributed Array System," *Proc. of ICASSP'12*, March 2012.

[14] V. I. Piterbarg, K. T. Wong, and Y. I. Wu, "Spatial Correlation Coefficient across a Receiving Sensor Array Accounting for Propagation Loss, *IEE Electronics Letters*, vol. 46, no. 19, 16th Sep. 2010.

[15] H. Ge, and I. P. Kirsteins, "Spatial Coherence Model for Passive Ranging using Distributed Arrays," *Proc. of IEEE Asilomar Conf. on Signals, Systems, and Computers*, Nov. 2012.