CONSTANT-ENVELOPE WAVEFORM DESIGN FOR OPTIMAL TARGET-DETECTION AND AUTOCORRELATION PERFORMANCES

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ABSTRACT

We propose an algorithm to directly synthesize in time-domain a constant-envelope transmit waveform that achieves the optimal performance in detecting an extended target in the presence of signal-dependent interference. This approach is in contrast to the traditional indirect methods that synthesize the transmit signal following the computation of the optimal energy spectral density. Additionally, we aim to maintain a good autocorrelation property of the designed signal. Therefore, our waveform design technique solves a bi-objective optimization problem in order to simultaneously improve the detection and autocorrelation performances, which are in general conflicting in nature. We demonstrate this compromising characteristics of the detection and autocorrelation performances with numerical examples. Furthermore, in the absence of the autocorrelation criterion, our designed signal is shown to achieve a near-optimum detection performance.

Index Terms— Constant-envelope waveform, time-domain synthesis, optimal detection, energy spectral density, autocorrelation function

1. INTRODUCTION

The problem of waveform design is becoming increasingly relevant and challenging to the modern state-of-the-art radar systems. For many years, the conventional radars have transmitted a fixed waveform on every pulse [1]; however, with the recent technological advancements in the fields of flexible waveform generators and high-speed signal processing hardware, it is now possible to generate and transmit sophisticated radar waveforms that are optimally adapted to the sensing environments on a periodic basis [2]-[6]. Such adaptation can lead to a significant performance gain over the classical (non-adaptive) radar waveforms.

Most of the salient research work on waveform design have addressed the problem of target detection and classification [7]-[11]. They can primarily be distinguished in terms of the employed optimality-criterion, e.g., signal-to-noise ratio (SNR), mutual information (MI), detection probability, as well as by the assumed modeling-conventions of the target and interference scenario, e.g., point or extended targets, deterministic or stochastic target-response, and signal-dependent or signal-independent interference. The commonality among them is in the design results which are provided in terms of the optimal energy-spectral-density (ESD) of the transmit signal. The actual time-domain synthesis of the transmit signal is carried out as a subsequent step of the optimal-ESD evaluation [12], [13].

In this work, we propose an algorithm to directly synthesize a constant-envelope transmit waveform in order to attain the optimal detection performance bypassing the requirement of the optimal-ESD computation. In addition, we emphasize that the designed waveform must have a very good autocorrelation property. Therefore, we formulate a bi-objective optimization problem that simultaneously optimizes the detection and autocorrelation performances. Our numerical results demonstrate that in the absence of the autocorrelation criterion the designed signal achieves a near-optimum detection performance. We also point out the conflicting nature of the optimal-detection and autocorrelation performances and show how difficult it is to achieve the optimal detection performance while maintaining a good autocorrelation property.

The rest of the paper is organized as follows. In Section 2, we briefly review the optimal radar detector and present the optimal-ESD expression. Then, in Section 3, we develop an algorithm to directly synthesize the transmit waveform. Numerical examples and conclusions are presented in Sections 4 and 5, respectively.

2. REVIEW OF OPTIMAL DETECTOR

In this section, we briefly discuss the modeling and performance characteristics of an optimal detector for an extended target in the presence of signal-dependent interference [8], [11]. Fig. 1 depicts a simple, schematic representation of a generic radar system. We denote the complex envelope of the transmitted signal by $s(t)$, which is nonzero only over $0 \leq t < T$ and has the total transmitted energy $E_{\mathrm{s}} = \int_0^T |s(t)|^2 dt$. The target and clutter scattering are modeled with two linear and time-invariant impulse responses $h_{\mathrm{T}}(t)$ and $h_{\mathrm{c}}(t)$, respectively. In addition, $e(t)$ represents the measurement error that in general includes the receiver thermal noise and any passive interferences (jamming).

We further assume that the clutter and noise processes are complex, wide-sense stationary (WSS), zero-mean Gaussian random processes with known power spectral densities (PSDs) $\mathcal{P}_{\mathrm{c}}(f)$ and $\mathcal{P}_{\mathrm{e}}(f)$, respectively. We model the target response as the Swerling-I type $h_{\mathrm{T}}(t) = x_{\mathrm{T}} g_{\mathrm{T}}(t)$ [14], where x_{T} is assumed to be a complex, zero-mean Gaussian random variable with variance σ_{x}^2, and $g_{\mathrm{T}}(t)$ is a known deterministic function that depends on the target orientation angle and radar aspect angle.

Next, we transform the problem into the frequency domain using the Fourier transform. Denoting $Y(f_n)$ as the frequency sam-

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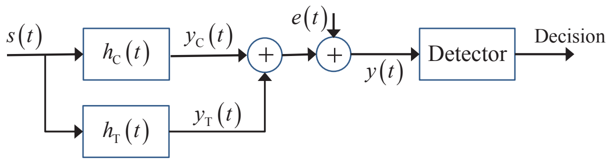


Fig. 1. A schematic block-diagram of a generic radar model.

ple of $y(t)$ at the nth frequency-bin with $f_n \in [-B/2, B/2]$ and frequency-spacing $\Delta f \equiv f_n - f_{n-1} = B/N$, we can write the detection problem as

$$\begin{aligned} \mathcal{H}_0: &\quad \boldsymbol{Y} = \boldsymbol{Y}_{\mathrm{c}} + \boldsymbol{E} \\ \mathcal{H}_1: &\quad \boldsymbol{Y} = x_{\mathrm{T}}\boldsymbol{Y}_{\mathrm{T}} + \boldsymbol{Y}_{\mathrm{c}} + \boldsymbol{E} \end{aligned}, \tag{1}$$

where $\boldsymbol{Y} = [Y(f_{(-N/2)}), \ldots, Y(f_{(N/2-1)})]^T$ and similarly $\boldsymbol{Y}_{\mathrm{T}}$, $\boldsymbol{Y}_{\mathrm{c}}$, and $\boldsymbol{E}$ include all the frequency samples of $(g_{\mathrm{T}}(t) * s(t))$, $(h_{\mathrm{c}}(t) * s(t))$, and $e(t)$ respectively. The statistical properties of the target, clutter, and noise processes mentioned earlier enable us to have $\boldsymbol{Y}|\mathcal{H}_0 \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_0)$ and $\boldsymbol{Y}|\mathcal{H}_1 \sim \mathbb{C}\mathcal{N}(\boldsymbol{0}, \boldsymbol{R}_1)$, where $\boldsymbol{R}_0$ asymptotically becomes a diagonal matrix for large BT with $[\boldsymbol{R}_0]_{(n,n)} = \mathcal{P}_{\mathrm{c}}(f_n)|S(f_n)|^2 + \mathcal{P}_{\mathrm{e}}(f_n)$, and $\boldsymbol{R}_1$ can be expressed as $\boldsymbol{R}_1 = \boldsymbol{R}_0 + \sigma_{\mathrm{x}}^2 \boldsymbol{Y}_{\mathrm{T}} \boldsymbol{Y}_{\mathrm{T}}^H$.

Under this circumstance, following the similar formulations as in [8], [14], we find that the probabilities of detection and false-alarm are related as $P_{\mathrm{D}} = P_{\mathrm{FA}}^{[1/(1+d^2)]}$, where

$$d^2 = \sum_{n=-N/2}^{N/2-1} \frac{\sigma_{\mathrm{x}}^2 |G_{\mathrm{T}}(f_n)|^2 |S(f_n)|^2}{\mathcal{P}_{\mathrm{c}}(f_n)|S(f_n)|^2 + \mathcal{P}_{\mathrm{e}}(f_n)} \Delta f. \tag{2}$$

Therefore, for a given P_{FA}, P_{D} can be made larger by increasing d^2, and thus an optimal waveform can be evaluated in terms of its ESD, $\mathcal{E}_{\mathrm{s}}(f) = |S(f)|^2$, by maximizing d^2 as [11]

$$\mathcal{E}_{\mathrm{s,opt}}(f_n) = \max\left(\frac{\sigma_{\mathrm{x}}|G_{\mathrm{T}}(f_n)|\sqrt{\mathcal{P}_{\mathrm{e}}(f_n)/\lambda} - \mathcal{P}_{\mathrm{e}}(f_n)}{\mathcal{P}_{\mathrm{c}}(f_n)}, 0 \right), \tag{3}$$

where λ is computed to satisfy a pre-defined total-energy ε, i.e., $E_{\mathrm{s}} = \sum_{n=-N/2}^{N/2-1} \mathcal{E}_{\mathrm{s,opt}}(f_n)\Delta f = \varepsilon$.

3. TRANSMIT WAVEFORM DESIGN

In this section, we describe an algorithm to directly synthesize the transmit waveform $s(t)$ in order to achieve the optimal detection-performance expressed in terms of d^2 in (2). We consider M constant-amplitude temporal samples of $s(t)$ over $0 \le t < T$ as $\{s(m) \equiv s(mt_{\mathrm{s}}), m = 0, 1, \ldots, M-1\}$ with $|s(m)| = A_{\mathrm{s}} \forall m$. Here, $t_{\mathrm{s}} = T/M$ denotes the sampling interval and $A_{\mathrm{s}} = \sqrt{\varepsilon/T}$ is the amplitude of the complex envelope. Then, the frequency response at the nth frequency-bin can be expressed as $S(f_n) = \boldsymbol{w}_n^H \boldsymbol{s}$, where $\boldsymbol{w}_n^H = \sqrt{\frac{t_{\mathrm{s}}}{B}} \left[e^{-j(2\pi/N)(n.0)}, \ldots, e^{-j(2\pi/N)(n.(M-1))} \right]$ is an $1 \times M$ vector representing a scaled version of the nth row of the discrete Fourier transform matrix, and $\boldsymbol{s} = [s(0), \ldots, s(M-1)]^T$ is an $M \times 1$ vector of the temporal samples. Substituting $S(f_n) = \boldsymbol{w}_n^H \boldsymbol{s}$ into (2), we get the optimal detection performance as

$$d(\boldsymbol{s})^2 = \sum_{n=-N/2}^{N/2-1} \frac{\boldsymbol{s}^H \left[\sigma_{\mathrm{x}}^2 |G_{\mathrm{T}}(f_n)|^2 \boldsymbol{w}_n \boldsymbol{w}_n^H \right] \boldsymbol{s}}{\boldsymbol{s}^H \left[\mathcal{P}_{\mathrm{c}}(f_n) \boldsymbol{w}_n \boldsymbol{w}_n^H \right] \boldsymbol{s} + \mathcal{P}_{\mathrm{e}}(f_n)} \Delta f. \tag{4}$$

Subsequently, we can obtain the optimal transmit signal by maximizing d^2 with respect to $\boldsymbol{s}$ as

$$\boldsymbol{s}^{(1)} = \arg \max_{\substack{\boldsymbol{s} \in \mathbb{C}^M \\ |s(m)| = A_{\mathrm{s}}}} d(\boldsymbol{s})^2 \quad \text{subject to } E_{\mathrm{s}}(\boldsymbol{s}) = \varepsilon, \tag{5}$$

where $E_{\mathrm{s}}(\boldsymbol{s}) = \sum_{n=-N/2}^{N/2-1} [\boldsymbol{s}^H \boldsymbol{w}_n \boldsymbol{w}_n^H \boldsymbol{s} \Delta f] = t_{\mathrm{s}} \boldsymbol{s}^H \boldsymbol{s}$.

Although the solution of (5) provides a constant-envelope waveform which is crucial in radar systems having the class C amplifiers, we do not have any control over the autocorrelation function (ACF) of the designed waveform. However, in a matched-filter radar receiver it is generally required to have a transmit signal with very good ACF properties, such as a narrow mainlobe and low sidelobes [15]. In the frequency domain, the ideal ACF criterion transforms into a flat ESD requirement over the entire bandwidth. We can satisfy this requirement by designing a transmit waveform whose ESD approximates a flat response in the mean square sense, i.e.,

$$\boldsymbol{s}^{(2)} = \arg \min_{\substack{\boldsymbol{s} \in \mathbb{C}^M \\ |s(m)| = A_{\mathrm{s}}}} S_{\Delta}(\boldsymbol{s})^2 \quad \text{subject to } E_{\mathrm{s}}(\boldsymbol{s}) = \varepsilon, \tag{6}$$

where $S_{\Delta}(\boldsymbol{s})^2 = (1/N) \sum_{n=-N/2}^{N/2-1} \left| \boldsymbol{s}^H \boldsymbol{w}_n \boldsymbol{w}_n^H \boldsymbol{s} - \varepsilon/B \right|^2 \Delta f$.

Noticing that the individual solutions of (5) and (6), we find that $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$ are very much conflicting in nature; because, the ESD of $\boldsymbol{s}^{(1)}$ would be similar to the optimal-ESD of (3) which is far from being a flat ESD corresponding to $\boldsymbol{s}^{(2)}$. Therefore, in this work, we propose to design a transmit waveform that simultaneously optimizes the detection and autocorrelation performances. We solve the bi-objective optimization problem according to the scalarization technique [16, Ch. 4.7] as

$$\begin{aligned} \boldsymbol{s}_{\mathrm{direct}}(\gamma) \quad = \quad \arg \min_{\substack{\boldsymbol{s} \in \mathbb{C}^M \\ |s(m)| = A_{\mathrm{s}}}} & -\gamma d(\boldsymbol{s})^2 + (1-\gamma) S_{\Delta}(\boldsymbol{s})^2 \\ & \text{subject to } E_{\mathrm{s}}(\boldsymbol{s}) = \varepsilon, \end{aligned} \tag{7}$$

where γ is a positive fraction representing the relative weight of the first objective (detection performance) with respect to the second (autocorrelation performance).

We compare the performance of the direct synthesis approach of (7) with those of a couple of existing indirect approaches that synthesize constant-envelope signals. The first steps of both of these indirect approaches are to apply (3) for computing the optimal ESD. Then, in the second step, [12] synthesizes the transmit signal by minimizing the least-square error between the designed frequency response and the square-root of the optimal-ESD, i.e.,

$$\begin{aligned} \boldsymbol{s}_{\mathrm{comp}} = & \\ & \arg \min_{\substack{\boldsymbol{s} \in \mathbb{C}^M \\ |s(m)| = A_{\mathrm{s}}}} \sum_{n=-N/2}^{N/2-1} \left| \sqrt{\mathcal{E}_{\mathrm{s,opt}}(f_n)} e^{j\angle S(f_n)} - S(f_n) \right|^2 \Delta f \\ & \text{subject to } E_{\mathrm{s}}(\boldsymbol{s}) = \varepsilon. \end{aligned} \tag{8}$$

Alternatively, in the second step, [13] proposes an optimality criterion as the minimization of the least-square error between the ESD of the design transmit waveform and the optimal ESD as

$$\begin{aligned} \boldsymbol{s}_{\mathrm{ms}} \quad = \quad \arg \min_{\substack{\boldsymbol{s} \in \mathbb{C}^M \\ |s(m)| = A_{\mathrm{s}}}} & \sum_{n=-N/2}^{N/2-1} \left[|S(f_n)|^2 - \mathcal{E}_{\mathrm{s,opt}}(f_n) \right]^2 \Delta f \\ & \text{subject to } E_{\mathrm{s}}(\boldsymbol{s}) = \varepsilon. \end{aligned} \tag{9}$$

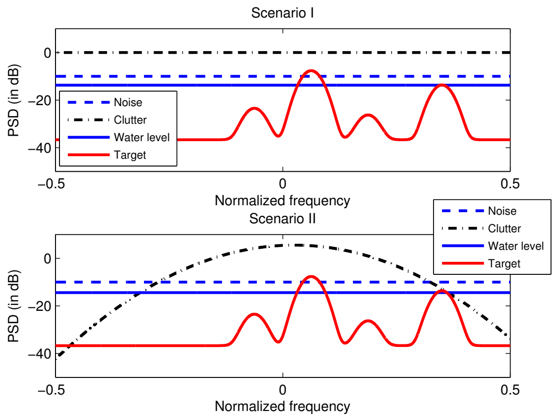


Fig. 2. Variations of the target, clutter, and noise powers in Scenarios I and II.

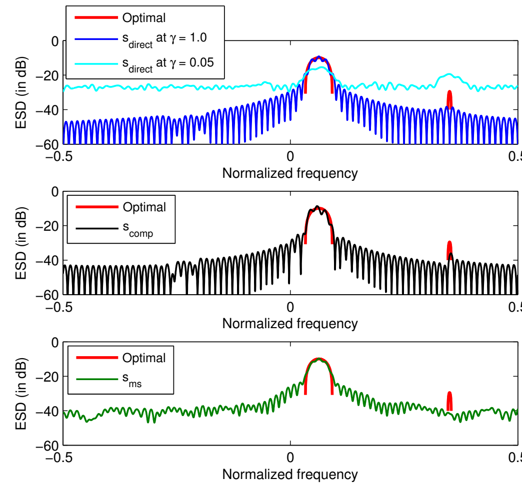
4. NUMERICAL RESULTS

In this section, we present the results of several numerical examples to demonstrate the performance of the designed waveforms. The target, clutter, and noise PSDs were simulated following the examples of [11]. For example, we considered the target signature as a Gaussian mixture: $|G_{\mathrm{T}}(f)|^2 = g_0 + \sum_{i=1}^{4} g_i e^{-(f-f_i)^2/(2\sigma_{\mathrm{T}}^2)}$, where $g_0 = 0.001$, $g_1 = 0.8$, $g_2 = 0.01$, $g_3 = 0.2$, $g_4 = 0.02$; the frequencies were $f_1 = 0.5$ kHz, $f_2 = 1.5$ kHz, $f_3 = 2.8$ kHz, $f_4 = -0.5$ kHz; and variance was $\sigma_{\mathrm{T}}^2 = 2 \times 10^4$. The noise PSD was assumed to be flat over the entire bandwidth of 8 kHz. We generated two different clutter-PSDs to simulate two scenarios. In Scenario I, we took a flat clutter-PSD $\mathcal{P}_{\mathrm{c}}^{(1)}(f) = 1$, for which the clutter return would be just a scaled version of the transmitted-signal ESD and this constitutes the worst-case scenario. In Scenario II, we modeled the clutter PSD to have a Gaussian shape: $\mathcal{P}_{\mathrm{c}}^{(2)}(f) = e^{-(f-f_{\mathrm{c}})^2/(2\sigma_{\mathrm{c}}^2)}$, where $f_{\mathrm{c}} = 0.2$ kHz and $\sigma_{\mathrm{c}}^2 = 8 \times 10^5$, and scaled it to ensure that the total clutter-energy remain the same, i.e., $\sum_n \mathcal{P}_{\mathrm{c}}^{(1)}(f_n) = \sum_n \mathcal{P}_{\mathrm{c}}^{(2)}(f_n)$. In addition, we scaled the target-signature and noise-PSD to satisfy the pre-defined clutter-to-noise ratio (CNR) of 10 dB and signal-to-noise ratio (SNR) of -10 dB. Fig. 2 depicts the variations of the target, clutter, and noise powers in Scenarios I and II, and the associated water levels $\lambda\mathcal{P}_{\mathrm{e}}(f)$.

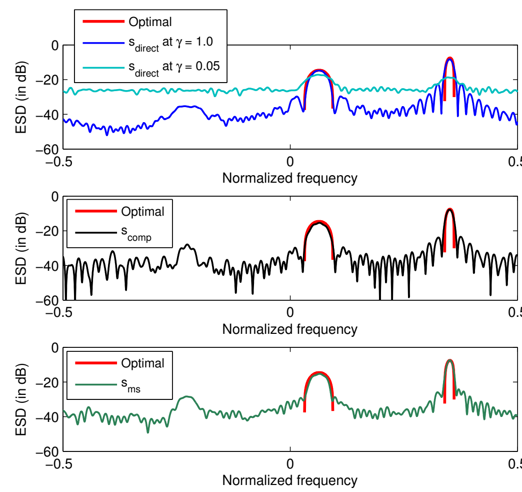
Fig. 3(a) shows the ESDs of the designed waveforms $\boldsymbol{s}_{\mathrm{direct}}(\gamma)$, $\boldsymbol{s}_{\mathrm{comp}}$ and $\boldsymbol{s}_{\mathrm{ms}}$ in Scenario I, along with the optimal ESD evaluated from (3). We noticed that the ESDs of all the three designed signals approximately matched with the optimal ESD at the normalized frequencies around 0.062, although the undulation of $\boldsymbol{s}_{\mathrm{comp}}$ was more than the other two signals. On the other hand, around the normalized frequency 0.35, only the ESD of $\boldsymbol{s}_{\mathrm{comp}}$ showed a coarse approximation of the optimal ESD. Overall, in terms of the detection performance, as shown in Table 1, $\boldsymbol{s}_{\mathrm{direct}}$ at $\gamma = 1$ was found to outperform the other two signals. In the same figure, we also plot the ESD of $\boldsymbol{s}_{\mathrm{direct}}$ at $\gamma = 0.05$ which was far from the optimal ESD and produced considerably deteriorating detection performance. In Scenario II, we found similar performance characteristics in terms of the ESDs of the designed signals and overall d^2 values, as can be noticed from Fig. 3(b) and Table 1.

Table 1. Detection performances of the designed waveforms

Waveforms	Values of d^2 in Scenario I	Values of d^2 in Scenario II
Optimal	22.63	14.93
$\boldsymbol{s}_{\mathrm{direct}}(\gamma = 1)$	21.92	14.20
$\boldsymbol{s}_{\mathrm{direct}}(\gamma = 0.05)$	11.55	7.66
$\boldsymbol{s}_{\mathrm{comp}}$	21.58	14.00
$\boldsymbol{s}_{\mathrm{ms}}$	21.57	13.85



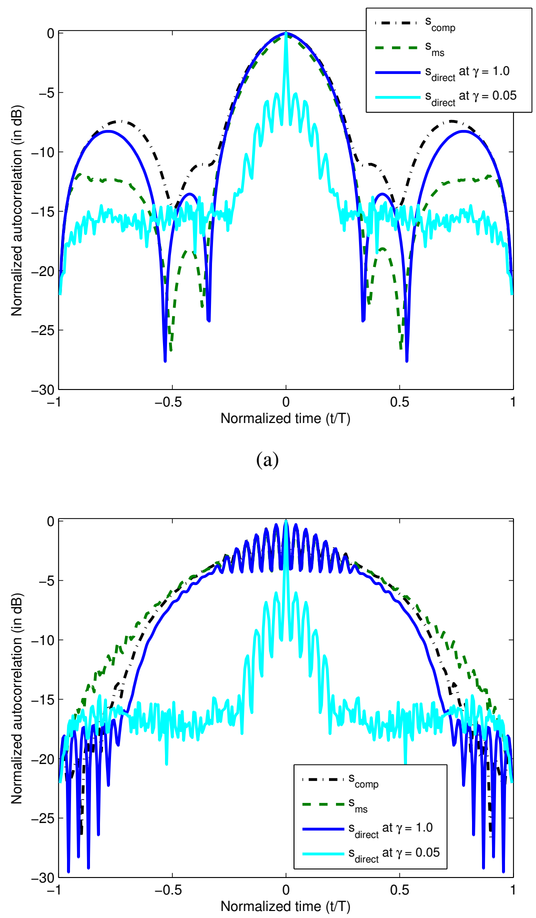
(a)



(b)

Fig. 3. Energy spectral densities of the designed waveforms in (a) Scenario I and (b) Scenario II.

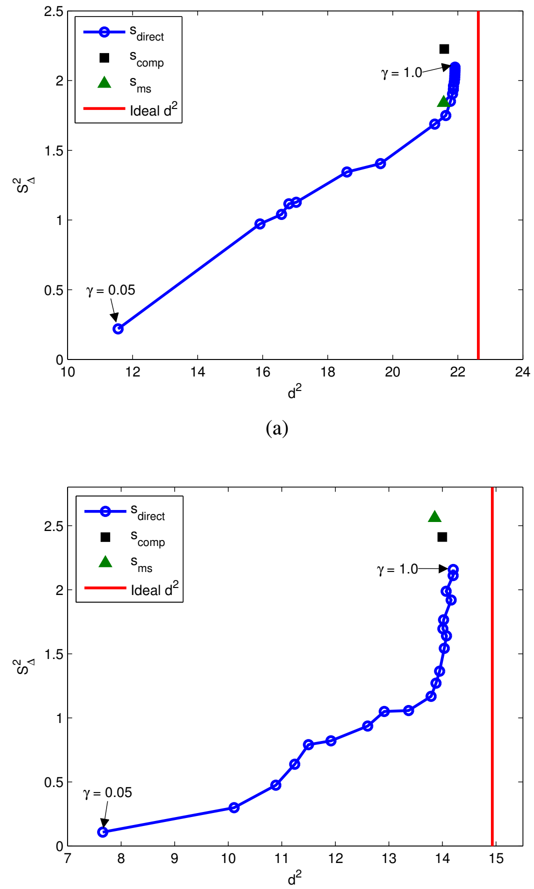
We plot the normalized ACFs of all the three designed waveforms for both the scenarios in Fig. 4. In Scenario I, all of the normalized ACFs corresponding to $\boldsymbol{s}_{\mathrm{direct}}(\gamma = 1)$, $\boldsymbol{s}_{\mathrm{comp}}$ and $\boldsymbol{s}_{\mathrm{ms}}$ demonstrated very wide mainlobes; the ACF characteristics became even worse in Scenario II. It is only the ACF of $\boldsymbol{s}_{\mathrm{direct}}$ at $\gamma = 0.05$



(a)

(b)

Fig. 4. Normalized autocorrelation functions of the designed waveforms in (a) Scenario I and (b) Scenario II.



(a)

(b)

Fig. 5. Performance characteristics of the designed waveforms in terms of the detection and autocorrelation measures in (a) Scenario I and (b) Scenario II.

that indicated a relatively improved autocorrelation performance.

The conflicting nature of the optimal-detection and autocorrelation performances were further evaluated by designing $\boldsymbol{s}_{\text{direct}}$ at various values of γ and the resultant performance characteristics are shown in Figs. 5(a) and 5(b) for both the scenarios. These sets of compromised solutions concurrently optimize both the objective functions in a Pareto-sense [17]-[19]. Along the x-axis we plot the detection performance in terms of d^2, which we want to maximize, and along the y-axis we show the autocorrelation performance in terms of S_Δ^2, which we aim to minimize. As the value of γ was decreased from 1 to 0.05, we observed the compromise in terms of the deterioration of the detection performance and simultaneous improvement of the autocorrelation performance. This implies that even when the target and interference characteristics are known we may not achieve the optimal detection performance while simultaneously ensuring a good autocorrelation property. In Fig. 5, we additionally depict the performances of $\boldsymbol{s}_{\text{comp}}$ and $\boldsymbol{s}_{\text{ms}}$ for fair comparison, and plot the optimal detection performance (independent of the autocorrelation criterion) as a vertical line. The gap between the optimal and the achieved detection performance of the designed signals suggests that there is still room for better signal synthesis.

5. CONCLUSIONS

In this paper, we developed a direct-form of constant-envelope waveform synthesis algorithm that achieved the optimal performance in detecting an extended target. Thus, we avoided the conventional indirect approaches that require to evaluate the optimal energy spectral density before synthesizing the transmit signal. In addition, to maintain a good autocorrelation property of the designed signal, we solved a bi-objective optimization problem that simultaneously optimized the detection and autocorrelation performances. We presented a comparative performance analysis of our designed signal with two other existing constant-envelope waveform design techniques. In the absence of the autocorrelation criterion, the proposed signal was found to outperform the other two waveforms in the overall detection performance. Furthermore, we demonstrated that it might not be possible to achieve the optimal detection performance and to obtain a good autocorrelation function at the same time, because these two objectives are in general conflicting in nature. In future, we intend to design a signal whose detection capability will be more close to the optimal performance.

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