JOINT LOW-RANK AND SPARSE LIGHT FIELD MODELLING FOR DENSE MULTIVIEW DATA COMPRESSION

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ABSTRACT

The effective representation of the structures in the multiview images is an important problem that arises in visual sensor networks. This paper presents a novel recovery scheme from compressive samples which exploit local and non-local correlated structures in dense multiview images. The recovery model casts into convex minimization framework which penalizes the sparse and low-rank constraints on the data. The sparsity constraint models the correlations among pixels in a single image whereas the global correlations across images are modelled with the low-rank prior. Simulation results demonstrate that our approach achieves better reconstruct quality in comparison with the state-of-the-art reconstruction schemes.

Index Terms— Low-rank matrix recovery, Compressive sensing, Sparsity, Compressive acquisition, Multiview imaging.

1. INTRODUCTION

Multiview imaging has received advent attention because of its wide range of applications such as virtual view synthesis, highperformance imaging and video processing [1]. Multiview systems are a network of visual sensors working together to execute a specific task, known as Visual Sensor Networks (VSN). The visual sensors are self-powered with limited on-board processing capability. The limitation of power is the main constraint in VSN which enforces a trade-off between the acquisition accuracy, computational power and battery life duration. These systems acquire highly overlapped images of the scene from different view points, thus the underlying data of such networks exhibit a specific structure. There are many issues that need to be addressed in designing a multiview imaging system. Specially, the large amount of data produced by multiview systems and more important is a low power mechanism to acquire a scene information.

In this paper, we focus on a VSN whose camera sensors acquire different images of the scene from a specific view points. Due to the spatial proximity of cameras, the obtained images have high correlations. However, cameras cannot collaborate in image acquisition. Therefore, compression should be performed locally at each camera and reconstruction is executed jointly to consider dependencies in the acquired data. In addition, the heart of compressive recovery problems lie on how to exploit the correlations of different parts of the underlying signal. Some compression schemes only consider local relationships on the assumption that the entries mainly have local dependencies. However, in problems such as multiview imaging entries also depend on far away values. Accordingly, for such problems it is necessary to employ tools to capture the global information of the data.

Recently, compressive sensing has been proposed as an interesting alternative to transform coding for power constrained systems [2, 3, 4, 5]. We introduce an independent compressive acquisition method similar to compressive sensing for multiview systems in which images are independently sampled by small number of random measurements. In order to address the reconstruction problems on how to efficiently consider the specific structures in multiview imaging, we propose a general framework in which many multiview imaging problems are addressed. Precisely, in addition to a non-collaborative measurement scheme, we introduce a joint recovery model for images from the ensemble of random measurements. We combine the sparsity of each image with a low-rank prior on the ensemble of the images to model the inter- and intra-correlations in a dense camera network. The sparsity prior intends to exploit the local dependencies in each image while the low-rank exploits the global, non-local structure of the multiview data.

2. BACKGROUND

2.1. Compressive Sampling

The compressive sampling methodology employs a linear measurement scheme to compress a signal with sampling rate much lower than Nyquist rate [2, 3]. The linear measurements are collected by projecting the signal onto a set of random vectors. Let $x \in \mathbb{R}^n$ represent the acquisition signal, the measurement vector $y \in \mathbb{R}^p$ is calculated by

$$y = \mathcal{A}(x) + z,\tag{1}$$

where $\mathcal{A}: \mathbb{R}^n \to \mathbb{R}^p$ is the sampling operator and $z \in \mathbb{R}^p$ is the measurement noise. The compressive sampling theory states that if the acquisition signal has a sparse representation in an orthogonal basis $\Phi \in \mathbb{R}^{n \times n}$, i.e. $x = \Phi \alpha, \alpha \in \mathbb{R}^n$, then the signal x can be robustly recovered from few measurements. The recovery problem solves the following convex optimization problem

$$\underset{\mathbf{x}\in\mathbb{R}^{\mathbf{n}}}{\operatorname{argmin}} \| \boldsymbol{\Phi}^{\mathbf{T}} \boldsymbol{x} \|_{1} \text{ subject to } \| \boldsymbol{y} - \mathcal{A}(\boldsymbol{x}) \|_{2} \le \epsilon,$$
(2)

where ϵ is a bound on the measurement noise. Recall that the l_q norm of a vector $\zeta \in \mathbb{R}^n$ is defined as $\|\zeta\|_q = \left(\sum_{i=1}^n |\zeta_i|^q\right)^{1/q}$. The compressive sampling scheme states that the linear map can be defined as a matrix generated at random from certain distributions (e.g. i.i.d. subgaussian distributions).

2.2. Low-rank Matrix Recovery

Affine rank minimization is a technique to reconstruct a low-rank matrix from linear samples using a linear map $\mathcal{A} : \mathbb{R}^{m \times n} \to \mathbb{R}^p$ [6, 7]. Let $\mathbf{X} \in \mathbb{R}^{m \times n}$ represent the low-rank matrix, the measurement vector $y \in \mathbb{R}^p$ is acquired by $y = \mathcal{A}(\mathbf{X}) + z$. If the number

of measurements is sufficient, we can find the exact low-rank matrix from the measurements. The key to recover the low-rank matrix is hidden in the behavior of the linear map. The recovery problem requires the linear map to be drawn at random from distributions such as i.i.d. subgaussian distributions. The low-rank matrix is recovered from the linear measurements using a convex optimization problem defined as

$$\underset{\mathbf{X}\in\mathbb{R}^{\mathbf{m}\times\mathbf{n}}}{\operatorname{argmin}} \|\mathbf{X}\|_{*} \text{ subject to } \|y-\mathcal{A}(\mathbf{X})\|_{2} \leq \epsilon.$$
(3)

In (3), the function $\|\cdot\|_*$ is the trace norm of a matrix, which equals to the sum of its singular values, and the rank of a matrix is equal to the number of non-zero singular values. It is verified that the trace norm is the tightest convex approximation of the matrix rank [8]; therefore, we can safely replace the matrix rank by the trace norm.

3. MULTIVIEW MEASUREMENT MODEL

Let $\mathbf{X} \in \mathbb{R}^{m \times n}$ represents the multiview image matrix, where m is the number of images and n represents the image resolution. The cameras image are vectorized and stored in the columns of \mathbf{X} . The multiview cameras do not collaborate during the signal acquisition, thus the signal acquisition is independent among cameras. The cameras collect $p \ll mn$ linear measurements from the scene in the measurement vector $y \in \mathbb{R}^p$ and $p = m\hat{p}$, where \hat{p} denotes the number of measurements collected per camera.

We can write the linear measurement operator in matrix form as $\mathcal{A}(\mathbf{X}) = \mathbf{A}\mathbf{X}_{\text{vec}}$, where $\mathbf{A} \in \mathbb{R}^{p \times (mn)}$ and $\mathbf{X}_{\text{vec}} \in \mathbb{R}^{mn}$ is the vectorized form of \mathbf{X} . Then the sensing model is defined through the equivalent expression $y = \mathbf{A}\mathbf{X}_{vec} + z$. As described the measurement matrix should be generated from a distribution such as Gaussian distribution. However, the large amount of data and practical limitations of the Gaussian random matrices highlights the importance of a computationally tractable measurement system. To tackle the limitations of the measurement system, several camera architectures have been proposed to acquire compressive measurements. As an example, the random convolution measurement model [9] convolves the light field of the scene with a random pattern and few random samples are acquired from the modulated light field. The random convolution can be extended to VSN signal acquisition, therefore each visual sensor should have the same acquisition scheme with a different random pattern. Subsequently, the measurement matrix A has a block-diagonal structure as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}_m \end{bmatrix},$$
(4)

where $\mathbf{A}_i \in \mathbb{R}^{\hat{p} \times n}$ is the random measurement matrix based on the random convolution scheme applied on the camera *i*. We can use the same random pattern for all cameras ($\mathbf{A}_1 = \ldots = \mathbf{A}_m$), which leads to a faster reconstruction. However, using different random patterns for cameras improve the efficiency of capturing more diverse information across cameras.

4. JOINT LOW-RANK AND SPARSE RECOVERY

We propose a specific scheme to recover dense multiview images from compressive samples. The multiview data is composed of images acquired from different view points of a common scene. Therefore, the ensemble of images are the samples of the light fields belonging to the underlying scene, which results in non-local correlations across images. The heart of the recovery scheme is to incorporate both local and non-local similarities of multiview data in the reconstruction algorithm. In order to consider both types of correlations, the method employs both sparse and low-rank prior on the image matrix. The sparse prior leads to exploit the local dependencies of the image matrix, i.e. the correlation inside a camera image. Moreover, the low-rank constraint on the whole set of images takes into account the non-local dependencies across the images. Therefore, the Joint Low-rank and Sparse Recovery (JLSR) model considers both inter- and intra-dependencies of the image matrix.

The sparsity domain of the image matrix **X** can be a specially designed dictionary [10, 11] or a basis. Since the images share similar contents, the multiview basis $\Gamma \in \mathbb{R}^{mn \times mn}$, where $\mathbf{X}_{vec} = \Gamma \Theta$ and $\Theta \in \mathbb{R}^{mn}$, has a block-diagonal structure, i.e. the same basis is used for each camera $\Phi \in \mathbb{R}^{n \times n}$. The sparsity domain of the image matrix is calculated by $\Gamma = \mathbf{I} \otimes \Phi$, where $\mathbf{I} \in \mathbb{R}^{m \times m}$ is the identity matrix and \otimes is the matrix Kronecker product.

Though the multiview images are highly correlated, because of cameras displacement a general multiview image matrix might not have a low-rank structure. Therefore, we need to use a linear operator $\mathcal{P}: \mathbb{R}^{m \times n} \to \mathbb{R}^{r \times s}$ that maps the image matrix into a low-rank matrix. The joint low-rank and sparse recovery scheme is defined as

$$\underset{\mathbf{X}\in\mathbb{R}^{\mathbf{m}\times\mathbf{n}}}{\operatorname{argmin}} \quad \|\mathcal{P}(\mathbf{X})\|_{*} + \lambda \|\mathbf{\Gamma}^{T}\mathbf{X}_{\operatorname{vec}}\|_{1} \text{ subject to } \|y - \mathbf{A}\mathbf{X}_{\operatorname{vec}}\|_{2} \leq \epsilon,$$
(5)

where λ is the regularization factor. Penalizing the prior constraints with a proper coefficient helps to reconstruct a solution which simultaneously satisfies both constraints with a proper factor.

4.1. Recovery Algorithm

We propose a convex optimization algorithm based on *Parallel Proximal Algorithm* (PPXA) [12] to solve the joint low-rank and sparse recovery model (5). PPXA is an iterative algorithm that looks for the minimizer of a sum of convex functions. The algorithm at each iteration computes the *proximity operators* of all functions and averages their results until convergence to the solution. Recall that the proximity operator of a convex function f and any $\nu \in (0, +\infty)$ is defined as

$$\operatorname{prox}_{\nu f}(\mathbf{X}) = \operatorname{argmin}_{\mathbf{Y} \in \mathbb{R}^{\mathbf{m} \times \mathbf{n}}} f(\mathbf{Y}) + \frac{1}{2\nu} \|\mathbf{X} - \mathbf{Y}\|_{\mathrm{F}}^{2}, \qquad (6)$$

where $\|\cdot\|_{\mathrm{F}}$ is the matrix Frobenius norm. The soft thresholding operator is defined as soft-thresh $(\beta, \nu) = \operatorname{sign}(\beta) \max(|\beta| - \nu, 0)$. In the joint low-rank and sparse problem (5), we define $f_1(\mathbf{X}) = \|\mathbf{\Gamma}^T \mathbf{X}_{\operatorname{vec}}\|_1$, $f_2(\mathbf{X}) = \|\mathcal{P}(\mathbf{X})\|_*$, and $f_3(x) = i_{\mathcal{B}_2}(\mathbf{X})$ where $\mathcal{B}_2 = \{\mathbf{X} \in \mathbb{R}^{m \times n} \mid \|y - \mathbf{A}\mathbf{X}_{\operatorname{vec}}\|_2 \le \epsilon\}$ and the indicator function of the set \mathcal{B}_2 is defined as

$$i_{\mathcal{B}_2}(\mathbf{X}) = \begin{cases} 0, & \text{if } \mathbf{X} \in \mathcal{B}_2; \\ +\infty, & \text{otherwise.} \end{cases}$$
(7)

As Γ is a tight frame (i.e. $\Gamma^T \Gamma = \tau \mathbf{I}$), the proximity map of $f_1(\mathbf{X}) = \|\Gamma^T \mathbf{X}_{\text{vec}}\|_1$ is the soft thresholding operator on the components of $\Gamma^T \mathbf{X}_{\text{vec}}$ composed by Γ

$$\operatorname{prox}_{\nu f_1}(\mathbf{X}) = \Gamma(\operatorname{soft-thresh}(\Gamma^T \mathbf{X}_{\operatorname{vec}}, \nu)).$$
(8)

Algorithm 1: Joint Low-rank and Sparse Recovery Algorithminput: $\gamma > 0$, A, Γ , y, λ .Initializations: $\Upsilon_1 \in \mathbb{R}^{m \times n}$, $\Upsilon_2 \in \mathbb{R}^{m \times n}$, $\Upsilon_3 \in \mathbb{R}^{m \times n}$, $\mathbf{X} \in \mathbb{R}^{m \times n}$;while not converged do $\mathbf{P}_1 = \mathcal{P}^* (\operatorname{shrink}(\mathcal{P}(\Upsilon_1), \gamma));$ $\mathbf{P}_2 = \Gamma (\operatorname{soft-thresh}(\Gamma^T \Upsilon_2, \lambda \gamma));$ $\mathbf{P}_3 = \operatorname{prox}_{i\mathcal{B}_2} (\Upsilon_3);$ $\mathbf{P} = (\mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3)/3;$ for $i = 1, \dots, 3$ do $\sqcup \Upsilon_i = \Upsilon_i + 2\mathbf{P} - \mathbf{X} - \mathbf{P}_i;$ $\mathbf{X} = \mathbf{P}.$

The proximity operator of the trace norm function is the shrinkage operator $\operatorname{prox}_{\nu \parallel \cdot \parallel_*}(\Xi) = \operatorname{shrink}(\Xi, \nu)$. This operator performs soft thresholding on the singular values of the matrix. $\Xi = \mathbf{U}\Sigma\mathbf{V}^T$ is the singular value decomposition of matrix Ξ with $\Sigma = \operatorname{diag}(\sigma_1, \ldots, \sigma_d)$ is the diagonal matrix containing the singular values of Ξ and $d = \min(m, n)$. We define $\hat{\Sigma} = \operatorname{diag}(\sigma_1 - \nu, \ldots, \sigma_d - \nu)$ as the shrunk diagonal matrix of singular values, then $\operatorname{shrink}(\mathbf{X}, \nu) = \mathbf{U}\hat{\Sigma}\mathbf{V}^T$. The proximity operator of $f_2(\mathbf{X})$ for a general operator \mathcal{P} can be calculated using a convex iterative algorithm explained in [13]. If \mathcal{P} is a tight operator, i.e. $\mathcal{P} \circ \mathcal{P}^* = \mathbf{I}$ and \mathcal{P}^* is the adjoint operator of \mathcal{P} , the proximity operator for f_2 is computed by

$$\operatorname{prox}_{\nu f_2}(\mathbf{X}) = \mathcal{P}^*(\operatorname{shrink}(\mathcal{P}(\mathbf{X}), \nu).$$
(9)

The proximity operator of $f_3(\mathbf{X})$ for a general measurement matrix **A** is an iterative method which can be calculated using the method explained in [13].

Having defined all proximity operators, we summarize the joint low-rank and sparse recovery in algorithm 1 when the low-rank operator \mathcal{P} is a tight frame. The parameter γ controls the speed of convergence and does not have a huge influence on the final result.

5. EXPERIMENTAL RESULTS

We evaluate the performance of our multiview approach on two different datasets. Image resolution of both datasets is cropped to $n = 256 \times 256$ to speed up the algorithm. The first dataset (Flower and Bowl) consists of m = 40 images and the second dataset (Park) is composed of m = 72 images. The cameras are approximately positioned along a line with a small distance so that the captured images are highly overlapped. Fig. 1 shows the scene of both multiview datasets.

The measurement scheme applies an independent random convolution measurement matrix [9] to each image to acquire p linear measurements as explained in section 3. The measurements are contaminated by additive Gaussian noise whose SNR is defined for each sampling ratio.

The sparsity domain for the image matrix is composed of the wavelet transform on each multiview image. For the compressive sensing scheme the image matrix is reconstructed as a whole using the multiview sparsity basis Γ . The linear operator \mathcal{P} plays a key role in the proposed recovery method. For a general multiview dataset, we need to estimate the cameras configuration in order to define the operator \mathcal{P} . However, in datasets consisting of highly overlapped images a proper matrix reshaping can transform the image matrix



(a) Flower and Bowl



(b) Park

Fig. 1: Original Scene.

to a low-rank matrix, though an operator based on cameras configuration results in a more structured matrix which can improve the reconstruction. In order to better model a low-rank matrix, the lowrank operator should reshape the image matrix into a matrix which is spread along both dimensions. Therefore, the linear operator \mathcal{P} reshapes the image matrix to a matrix of size $r \times s = (8n/2) \times (5n/2)$ for the first and $r \times s = (8n/2) \times (9n/2)$ for the second dataset. The joint low-rank and sparse recovery method (5) on multiview datasets is compared with the Compressive Sensing (CS) (2) and Low-Rank matrix Recovery (LRR) (3) schemes. The joint low-rank and sparse regularization factor is empirically set to $\lambda = 0.1$.

In order to asses the impact of the linear operator \mathcal{P} , we compare the low-rank matrix recovery method when the image matrix is not composed by the low-rank operator with the low-rank matrix recov-



Fig. 2: Reconstruction quality of different recovery methods for different sampling ratios and noiseless acquisition for the flower and bowl scene.



Fig. 3: Estimated multiview image for different scenes using JLSR recovery scheme for sampling ratio of 1/4 and different acquisition noise levels. (a)-(d) A reconstructed image of the park dataset for different noise levels. (e)-(h) A recovered image o of the flower and bowl dataset for different noise levels.

ery problem when the image matrix is composed by the linear operator for sampling ratio p/(mn) = 1/4 and noiseless acquisition. The reconstruction PSNR for the first scenario is PSNR = 7.6 dB and using the low-rank operator \mathcal{P} the reconstruction quality is increased to PSNR = 30.35 dB. Therefore, it is important to consider a proper operator that maps the image matrix to a low-rank structured matrix.

In another experiment, we compare different multiview recovery schemes for different sampling ratios when the acquisition is noiseless and the qualitative results are shown in Fig. 2. It is evident that the JLSR method outperforms the other models which reveals the importance of considering different types of correlations. Finally, in order to further evaluate the performance of the JLSR model, we compare this model with other recovery methods for both datasets. Table 1 compares the PSNR of different multiview recovery schemes for sampling ratio of p/(mn) = 1/4 and different acquisition noise levels. The reconstruction results reveal the better performance of the JLSR model, i.e. combining the low-rank and sparse constraints for the multiview scenario results in a better reconstruction quality. Fig. 3 demonstrates a qualitative comparison for different multiview dataset recovered by the JLSR scheme for different acquisition noise.

	Flower and Bowl			Park		
PSNR	∞ dB	30 dB	10 dB	∞dB	30 dB	10 dB
JLSR	36.49	35.10	26.86	36.78	36.55	31.54
CS	34.22	33.43	25.65	35.54	35.52	30.62
LRR	30.35	30.03	23.56	33.19	33.07	28.86

Table 1: Comparison of different multiview recovery methods for different noise levels and datasets with sampling ratio p/(mn) = 1/4.

6. PREVIOUS WORKS

This work is focused on compressive multiview imaging problem wherein an ensemble of cameras collect linear measurements from a scene. The recovery of a single image from the linear measurements is well studied subject in the framework of compressive sampling by exploiting the sparsity assumption on the image. While the separate recovery of each image can be applied to multiview images, an effective reconstruction methodology should exploit the similar structure in the multiview data. Therefore, most compressive multiview recovery algorithms relay on sparsity to exploit correlations in the multiview images [10, 11]. However, such algorithms do not address the global structure of the multiview images. [14] explains a method to exploit the global consistency by estimating the underlying cameras position. In our recovery model, we incorporate both inter- and intra-correlated structures by the sparse and low-rank constraints. The low-rank prior penalizes the global structure and the sparse constraint provides a method to penalize the local dependencies for the multiview images. Furthermore, we can incorporate the cameras information similar to [14] which results in a more appropriate selection of the low-rank linear map \mathcal{P} and can improve the reconstruction quality. However, including cameras information can increases the complexity of the algorithm as such information should be provided or estimated jointly with the underlying signal.

7. CONCLUSION

In this paper, the compressive acquisition and recovery scheme for capturing multiview images with a visual sensor network is addressed. It is shown that the local similarities of the multiview data are well modelled by a sparse constraint and a low-rank prior models the global structures in the data. Moreover, for multiview data to represent a low-rank matrix, the images need to be specifically combined which depends on cameras configuration. For dense multiview images the low-rank constraint composed by simple reshaping operator can exploit the non-local dependencies. However, the knowledge on the configuration of visual sensors can provide a more consistent low-rank operator which results in a better reconstruction quality. Furthermore, we have shown experimentally that the joint low-rank and sparse recovery model outperforms state-ofthe-art reconstruction schemes. In future research, we will consider different cameras configuration and incorporate the cameras position in the recovery scheme in order to adapt the low-rank operator to different cameras configuration.

8. REFERENCES

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