SIMULTANEOUS RECONSTRUCTION OF UNDERSAMPLED MULTICHANNEL SIGNALS WITH A DECAYED AND TIME-DELAYED COMMON COMPONENT

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ABSTRACT

This paper presents a new signal model in distributed compressed sensing (DCS) and a reconstruction algorithm to reduce the number of samples required to accurately reconstruct signals obtained via sensors. While the conventional signal models in DCS assume that all signals include exactly the same common component, our model exploits the attenuation and time delay of the common component. With this extension, one can deal with signals originated from a source located in the field. To reconstruct such signals, we developed a new algorithm based on the alternative direction multiplier method. The algorithm efficiently reconstructs computer-generated signals using a reduced number of samples. In addition, we demonstrate the superior reconstruction quality with our method by using real electromyographic signals.

***Index Terms*—** Distributed Compressed Sensing, Sensor Network, Alternative Direction Method

1. INTRODUCTION

A multichannel sensing scheme is an important signal processing tool in a wide range of fields. For instance, such a scheme is necessary for direction-of-arrival (DOA) estimation [1], and the detection of the abnormal propagation of electromyographic (EMG) signals [2]. Considering flexibility and construction costs, it is useful to employ a wireless network to make a multichannel sensing system. However, there are several challenges to implementing a wireless multichannel sensing system, such as network lifetime, computational ability, and bandwidth constraints. An efficient way to meet these challenges is to employ some low-complexity compression scheme in such a system.

Compressed sensing (CS) is a method that reduces a number of samples required for reconstructing a signal that is sparse in some basis (i.e., the number of non-zero elements is small relative to the frame length of the signal) [3, 4]. CS provides low-complexity compression of signals at the sensors and also achieves long network lifetime and reduces communication traffic. In addition, to deal with multichannel systems, CS has been extended to distributed compressed sensing (DCS) [5, 6, 7, 8]. The DCS scheme exploits correlations between channels and reduces the number of required samples more efficiently than single-channel CS. For these reasons, DCS has been studied to devise multichannel sensing systems [9]. One of the signal models in DCS, introduced by Baron *et al.* [5], is called joint sparse model 3 (JSM-3). It assumes that signals share a nonsparse common component and each signal has a sparse innovation component independently. JSM-3 has an advantage over other signal models because it can deal with nonsparse, or practical signals.

In this paper, we focus on JSM-3 to deal with time series signals originated from some source and obtained via sensors located in the field. In such a situation, it is natural to introduce attenuation and time delay into the common component in the each signal. For example, a speaker in a DOA application or a set of muscle-fibers in EMG measurement can be a source of such signals. To reconstruct such signals, we developed a new algorithm based on the alternative direction multiplier method (ADMM) [10], which has been recently applied in CS [11] and to the DCS problem [12], and the Fixed-point continuation (FPC) for compressed sensing [13, 14]. To represent attenuation and time delay between each channel, we introduce new constraints in the ADMM algorithm. We show the advantage of the devised method by recovering computer-generated signals and real EMG data.

Our work is related to conventional research in CS and DCS. There have been notable studies in the CS area that consider attenuation and time delay of signals [15, 16]; however, to the best of our knowledge, they treat *known* shape signals with *unknown* attenuation and time delay. In contrast, we consider signals with both an *unknown* common component shape and *unknown* attenuation and time delay. (Note that this does not mean that the signal class in our work is broader than in theirs.) There also notable work that consider multichannel signals linked between sparse filtering [17]. They derived a method that estimates both an unknown sparse filter in time domain and nonsparse signals. However, a time-delay filter is not sparse in the time domain when it includes sub sample delay. In our model, one can treat such delay. In addition to the above mentioned research, there have been several studies on the expansion of DCS model, where it was assumed that the sparse common components are not exactly the same but share their support (locations of nonzero elements) [9, 18].

The algorithms are effective in our case when the common component is sparse or approximately sparse. We compare the performance of our algorithm with that of one such algorithm, called precognition matching pursuit [9]. From viewpoint of algorithms, we should mention the work by Deng *et al.* [12], who proposed a DCS reconstruction algorithm based on ADMM. They focus on a model where all signals share their support. In contrast, we focus on a signal model with a common component and innovation component of each channel.

2. BACKGROUND

2.1. Distributed Compressed Sensing

In the CS literature, the signal of interest is assumed to be sparse in some sparsifying basis $\mathbf{\Phi} \in \mathbb{C}^{N\times N}$. That is, if we transfer the signal, $\mathbf{x} \in \mathbb{R}^N$, to another basis, few elements of the transferred vector, $\mathbf{s} = \mathbf{\Phi}^{-1}\mathbf{x}$, will be nonzero. If the number of nonzero coefficients of $\mathbf{s}$ is S, then we call such signal $\mathbf{s}$ a S-sparse signal in the domain $\mathbf{\Phi}$. We obtain transmitted data $\mathbf{y} \in \mathbb{C}^M$ that is generated by a few linear projections, $\mathbf{y} = \mathbf{\Psi}\mathbf{x} = \mathbf{\Psi}\mathbf{\Phi}\mathbf{s} = \mathbf{A}\mathbf{s}$, where $\mathbf{\Psi} \in \mathbb{C}^{M\times N}$represents the sensing matrix. The compressed sensing recovery procedure is performed to solve the ℓ_1-norm minimization problem to obtain an appropriate reconstructed signal. We describe the problem formulation in Problem 2.2.

DCS is an extension of CS and is a research area that is concerned with highly correlated multichannel signals [5, 6, 7, 8]. The problem is well-formulated by introducing the JSM proposed by Baron *et al*. [5]. In their model, a representation of the signal of interest is divided into two parts as follows:

$$\mathbf{x}_i = \mathbf{z}_\mathrm{c} + \mathbf{z}_{\mathrm{d},i}, \forall i \in \Omega \tag{1}$$

where $\mathbf{z}_\mathrm{c} \in \mathbb{R}^{N\times 1}$ and $\mathbf{z}_{\mathrm{d},i} \in \mathbb{R}^{N\times 1}$ is a common component and an innovation of channel i, respectively. In addition, $\Omega = \{1, 2, \cdots, N_c\}$ is a set of channel indices, and N_c is a number of channels of the system. In most DCS models, both common and innovation components or only innovation components are set sparse in the domain $\mathbf{\Phi}$. The latter case, Baron called the signal model JSM-3. The goal of DCS is to recover signals $\mathbf{x}_i$ from transmitted data $\mathbf{y}_i = \mathbf{\Psi}_i\mathbf{x}_i$. Similarly to distributed source coding [19], they show that in some cases signals can be reconstructed completely with a reduced number of observations compared to that in single-channel cases.

2.2. Alternative Direction Method Multipliers and Fixed-point Continuation

Here we describe the ADMM algorithm [10] and FPC algorithm for compressed sensing [13], which we applied to solve our problem. Let $\Gamma(\mathbb{K}^N)$ be a class of convex functions. ADMM is an iterative algorithm that approximates a solution of the following optimization problem.

Problem 2.1 *(A basic ADMM Problem)*

$$\min_{\mathbf{x},\mathbf{s}} \{f_1(\mathbf{x}) + f_2(\mathbf{s})\}, \text{s.t. } \mathbf{x} = \mathbf{\Phi}\mathbf{s}, \tag{2}$$

where $\mathbf{x} \in \mathbb{K}^N$, $\mathbf{s} \in \mathbb{K}^M$, $\mathbf{\Phi} \in \mathbb{K}^{M\times N}$, and $f_1 \in \Gamma(\mathbb{K}^N)$, $f_2 \in \Gamma(\mathbb{K}^M)$.

In ADMM, the above problem is translated into a function,

$$\mathcal{L} = f_1(\mathbf{x}) + f_2(\mathbf{s}) + \lambda^\mathrm{T}(\mathbf{x} - \mathbf{\Phi}\mathbf{s}) + \frac{\alpha}{2}\|\mathbf{x} - \mathbf{\Phi}\mathbf{s}\|_2^2, \tag{3}$$

where $\lambda \in \mathbb{R}^{N\times 1}$ and α are a Lagurange multiplier and a penalty parameter respectively and the function is minimized by the following algotithm.

Algorithm 1 ADMM

Require: $\mathbf{\Phi}$, set $k = 0$, choose $\mathbf{s}^{(0)}$ and $\lambda^{(0)}$.
while a stop criterion is not satisfied **do**
  $\mathbf{x}^{(k+1)} = \arg\min_\mathbf{x} \mathcal{L}(\mathbf{x}, \mathbf{s}^{(k)}, \lambda)$
  $\mathbf{s}^{(k+1)} = \arg\min_\mathbf{s} \mathcal{L}(\mathbf{x}^{(k+1)}, \mathbf{s}, \lambda)$
  $\lambda^{(k+1)} = \lambda^{(k)} + \alpha(\mathbf{x}^{(k+1)} - \mathbf{\Phi}\mathbf{s}^{(k+1)})$
  $k = k + 1.$
end while

Fixed-point continuation for the ℓ_1-norm minimization problem (Algorithm 2) is proposed by Hale *et al*. [13], to solve the following problem.

Problem 2.2 *(An ℓ_1-norm minimization problem)*

$$\min_\mathbf{s} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \bar{w}\|\mathbf{s}\|_1, \tag{4}$$

where $\mathbf{s} \in \mathbb{C}^N$, $\mathbf{y} \in \mathbb{R}^M$ $(M < N)$, and $\mathbf{A} \in \mathbb{C}^{M\times N}$. $\bar{w}$ is a weight that is calculated by noise power assumption.

Algorithm 2 Fixed-Point Continuation algorithm for CS

Require: $\mathbf{A}$,$\mathbf{y}$,α,$\bar{w}$, $stol$, $gtol$, set $k = 0$,$w = \|\mathbf{A}^*\mathbf{y}\|_\infty$, choose $\mathbf{s}^{(0)}$.
while $w \leq \bar{w}$ **do**
  while $\frac{\|\mathbf{s}-\mathbf{s}_p\|_2}{\|\mathbf{s}_p\|_2} > stol\sqrt{\frac{w}{\bar{w}}}$ or $\|g(\mathbf{s})\|_\infty - w > gtol$ **do**
    $\mathbf{s}_p = \mathbf{s}$
    $\mathbf{s} = \mathrm{S}_{\frac{w}{\alpha}}\left(\mathbf{s} - \frac{1}{\alpha}\mathrm{g}(\mathbf{s})\right)$
  end while
  $w = \max\{w/\eta, \bar{w}\}$
end while

Definitions of the above functions are as follows:

$$\mathrm{S}_{\frac{w}{\alpha}}(\mathbf{s}) = \mathrm{sgn}(\mathbf{s}) \odot \max\left\{|\mathbf{s}| - \frac{w}{\alpha}, \mathbf{0}\right\}, \tag{5}$$

and

$$\mathrm{g}(\mathbf{s}) = \mathbf{A}^*(\mathbf{A}\mathbf{s} - \mathbf{y}), \tag{6}$$

where $\odot$ denotes the element-wise product. In addition, $stol$ and $gtol$ are feasibility tolerances.

We can translate Problem 2.2 into the ADMM (Problem 2.1) by substituting $f_1(\mathbf{x}) = \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2$ and $f_2(\mathbf{s}) = \bar{w}\|\mathbf{s}\|_1$ and by adding an update step of weight w. Hence, we can solve the ℓ_1 minimization problem by ADMM (Algorithm 1).

3. PROPOSED MODEL AND ALGORITHM

3.1. JSM-3 with Amplitude and Phase Differences

As we mentioned in the introduction, we would like to deal with signals originated from a source and obtained via sensors located in the field. Hence, we propose a signal model expressed in the time domain as

$$\mathbf{x}_i = a_{i,\mathrm{c}}\mathbf{\Lambda}^{\theta_{i,c}}\mathbf{z}_\mathrm{c} + \mathbf{z}_{\mathrm{d},i}\forall \in \Omega, \tag{7}$$

where $\mathbf{\Lambda}^{\theta} \in \mathbb{R}^{N\times N}$ is a rotation matrix defined as

$$\mathbf{\Lambda}^{\theta} = [\mathrm{IDFT}]\mathrm{diag}[1\ \mathrm{e}^{\frac{2\pi}{N}\mathrm{j}}\ \mathrm{e}^{\frac{4\pi}{N}\mathrm{j}} \dots \mathrm{e}^{\frac{2\pi(N-1)}{N}\mathrm{j}}]^{\theta}[\mathrm{DFT}], \tag{8}$$

and $a_{i,\mathrm{c}}, \theta_{i,\mathrm{c}} \in \mathbb{R}$ are the amplitude and phase coefficients of the i th channel, respectively. In addition, [DFT] and [IDFT] are a discrete fourier transform (DFT) matrix and an inverse DFT matrix, respectively. The common component $\mathbf{z}_\mathrm{c}$ corresponds to the time series behavior of the source. Note that, if $a_{i,\mathrm{c}}$ equals one and $\theta_{i,\mathrm{c}}$ equals zero for all signals, the model defined by Eq. (7) is the same as JSM-3 (cf. Eq. (1)). Hereafter, the signal model is called JSM with amplitude and phase differences (JSMAP).

3.2. ADMM-based algorithm for reconstructing JSMAP Signals

Here we formulate a JSMAP reconstruction algorithm based on ADMM and FPC.

Problem 3.1 *(Reconstruction Algorithm of JSMAP Problem)*

$$\min_{\mathbf{x}_i,\mathbf{s}_i\ \forall i\in\Omega} \frac{1}{2}\sum_i \|\mathbf{y}_i - \mathbf{\Psi}_i\mathbf{x}_i\|_2^2 + \bar{w}\sum_i \|\mathbf{s}_{\mathbf{d},i}\|_1, \tag{9}$$

subject to,

$$\mathbf{x}_i = \mathbf{z}_{\mathrm{c},i} + \mathbf{z}_{\mathrm{d},i},\ \forall\ i\in\Omega, \tag{10}$$

$$\mathbf{z}_{\mathrm{c},i} = a_i\mathbf{e}_{\mathrm{c},i},\ \forall\ i\in\{1,2,\cdots,N_c-1\} = \Omega^- \tag{11}$$

$$\mathbf{e}_{\mathrm{c},i} = \mathbf{\Lambda}^{\theta_i}\mathbf{z}_{\mathrm{c},i+1},\ \ \forall\ i\in\Omega^-, \tag{12}$$

$$\mathbf{z}_{\mathrm{d},i} = \mathbf{\Phi}\mathbf{s}_{\mathrm{d},i},\ \ \forall\ i\in\Omega, \tag{13}$$

where $\mathbf{e}_i \in \mathbb{R}^{N\times 1}$ and $\mathbf{s}_{\mathrm{d},i} \in \mathbb{C}^{N\times 1}$.

The object function Eq. (9) and constraint equations are explained as follows:

- Eq. (10) corresponds to the basic DCS model Eq. (1).
- Eqs. (11) and (12) are obtained by conserning the JSMAP relation Eq. (7) of common component $\mathbf{z}_{\mathrm{c},i}$ and decompositing it; thus,

$$\mathbf{z}_{\mathrm{c},i} = a_i\mathbf{e}_{\mathrm{c},i}, \text{and}, \mathbf{e}_{\mathrm{c},i} = \mathbf{\Lambda}^{\theta_i}\mathbf{z}_{\mathrm{c},i+1},$$

- Eq. (13) corresponds to the basis transform of innovation component $\mathbf{z}_{\mathrm{d},i}\ \forall i \in \{1,2,\cdots,N_c\}$.

An inner loop of the ADMM algorithm for Problem 3.1 is constructed so as to minimaze a function defined in Eq. (14) for each variables, $\mathbf{x}_i$, $\mathbf{z}_{\mathrm{c},i}$, $\mathbf{z}_{\mathrm{d},i}$, $\mathbf{e}_{\mathrm{c},i}$, $\mathbf{s}_{\mathrm{d},i}$, a_i and θ_i, and update Lagrange multipliers $\lambda_i, \mu_{\mathrm{c},i}, \nu_{\mathrm{c},i}, \xi_{\mathrm{d},i} \in \mathbb{R}^{N\times 1}$, sequentially as Algorithm 1.

$$\begin{aligned}
\mathcal{H} = &\sum_{i\in\Omega}\frac{1}{2}\|\mathbf{y}_i - \mathbf{\Psi}_i\mathbf{x}_i\|_2^2 \\
&+ \sum_{i\ \in\Omega}\Big\{\lambda_i^\mathrm{T}(\mathbf{x}_i - (\mathbf{z}_{\mathrm{c},i} + \mathbf{z}_{\mathrm{d},i})) \\
&+ \frac{\alpha}{2}\|\mathbf{x}_i - (\mathbf{z}_{\mathrm{c},i} + \mathbf{z}_{\mathrm{d},i})\|_2^2\Big\} \\
&+ \sum_{i\ \in\Omega^-}\left\{\mu_{\mathrm{c},i}^\mathrm{T}(\mathbf{z}_{\mathrm{c},i} - a_i\mathbf{e}_{\mathrm{c},i}) + \frac{\beta}{2}\|\mathbf{z}_{\mathrm{c},i} - a_i\mathbf{e}_{\mathrm{c},i}\|_2^2\right\} \\
&+ \sum_{i\ \in\Omega^-}\Big\{\nu_{\mathrm{c},i}^\mathrm{T}(\mathbf{e}_{\mathrm{c},i} - \mathbf{\Lambda}^{\theta_i}\mathbf{z}_{\mathrm{c},i+1}) \\
&+ \frac{\gamma}{2}\left\|\mathbf{e}_{\mathrm{c},i} - \mathbf{\Lambda}^{\theta_i}\mathbf{z}_{\mathrm{c},i+1}\right\|_2^2\Big\} \\
&+ \sum_{i\ \in\Omega}\left\{\xi_{\mathrm{d},i}^\mathrm{T}(\mathbf{z}_{\mathrm{d},i} - \mathbf{\Phi}\mathbf{s}_{\mathrm{d},i}) + \frac{\sigma}{2}\|\mathbf{z}_{\mathrm{d},i} - \mathbf{\Phi}\mathbf{s}_{\mathrm{d},i}\|_2^2\right\} \\
&+ \bar{w}\sum_{i\ \in\Omega}\|\mathbf{s}_{\mathrm{d},i}\|_1,
\end{aligned} \tag{14}$$

where $\alpha, \beta, \gamma, \sigma$ are penalty constants. Note that the minimization steps of $\mathbf{s}_{\mathrm{d},i}$ are performed by the FPC algorithm (Algorithm 2).

Unfortunately, the above problem is not a convex problem but biconvex one. Hence, a solution to it may not be not globally optimal. However, the results of numerical simulation encourage us to use the algorithm for reconstructing JSMAP signals.

4. EXPERIMENTAL EVALUATION

4.1. Configuration

We applied our algorithm to computer-generated JSMAP signals to confirm its reconstruction performance for model-based signals. In addition, we evaluated the performance of the algorithm for real EMG signals and compared it with that of other conventional ones. For the computer-generated JSMAP signals, the experiment configuration was follows:

- set frame length $N = 100$
- generate nonsparse $\mathbf{z}_\mathrm{c}$ randomly, and generate $\mathbf{z}_{\mathrm{d},i}$ as sparseness $S = 8$ in Fourier space for each channel

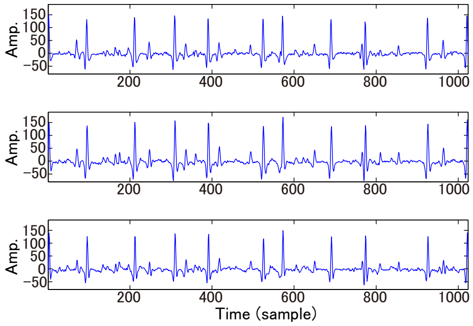

Fig. 1. Wave forms of channel $i = 1, 2, 3$

- set $a_{i,\mathrm{c}}$ and $\theta_{i,\mathrm{c}}$ randomly in ranges $0.8 < a_{i,\mathrm{c}} < 1.2$ and $-0.5 < \theta_{i,\mathrm{c}} < 0.5$
- set $\mathbf{y}_i = \mathbf{\Psi}_i(a_{i,\mathrm{c}}\mathbf{\Lambda}^{\theta_{i,\mathrm{c}}}\mathbf{z}_\mathrm{c} + \mathbf{z}_{\mathrm{d},i}) + \mathbf{n}_i = \mathbf{\Psi}_i\mathbf{x}_i + \mathbf{n}_i$
- set $w = 10^{-6}$, $\eta = 4$, $stol = 10^{-7}$, and $\alpha = \beta = \gamma = \sigma = 1/2$,

where $\mathbf{n}_i$ is an additive white noise, and $\frac{\sqrt{N}\|\mathbf{n}_i\|_2}{\sqrt{M}\|\mathbf{x}_i\|_2} < 10^{-6}$. The penalty constants are empirically-determined. We simulated 100 times for each number of observation per channel M. We measured the performance by the probability of successful reconstruction. We define the successful reconstruction when their maximum of relative errors of all channels is less than 10^{-3}.

To show the efficiency of our algorithm compared to the conventional ones for real signals, we employed $N_c = 3$-channel EMG signals [20]. An example of the signals is shown in Fig. 1, which are resampled at 2 kHz (the original sampling frequency is 10 kHz) and shifted to maximize cross-correlation functions. The configuration was as follows:

- set frame length $N = 1024$
- set $w = 10^{-1}$, $\eta = 2$, $stol = 10^{-4}$, and , and $\alpha = \beta = \gamma = \sigma = 1/2$,

We simulated 100 times (i.e. 100 EMG signals) for each M. We compare the performance of our algorithm and conventional one by the SNR defined as $\sum_i \frac{\|\mathbf{x}_i\|_2^2}{\|\mathbf{x}_i - \hat{\mathbf{x}}_i\|_2^2}/N_c$, where $\mathbf{x}_i$ and $\hat{\mathbf{x}}_i$ are the original signal and reconstructed signal of the i th sensor, respectively.

The obserbation matrices $\mathbf{\Psi}_i \in \mathbb{R}^{M\times N}$ for both experiments are what is called causal random sampling matrices and whose entries are all zeros except for the M entries in M different columns and rows.

4.2. Results

For the computer-generated signals, one can be sure our algorithm works in moderate condition (Fig. 2). One can also notice that the algorithm works more efficiently as the number of channels increases. Especially, one can successfully recover signals from about 50 % randomly sampled data in the 8 channels case. The results for our algorithm applied to real EMG signals (see Fig. 3) suggest that the developed

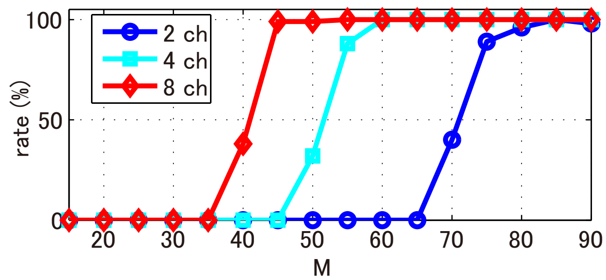

Fig. 2. Probability of perfect reconstruction vs. observation number M each channel with fixed sparseness $S = 8$. The results show feasibility of our algorithm for JSMAP signals reconstruction.

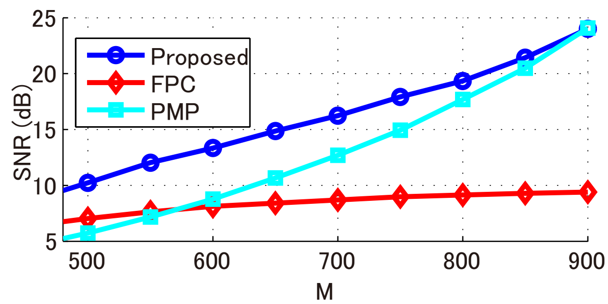

Fig. 3. Averages of SNR of the reconstructed real EMG signals. Our algorithm overcome other two conventional one.

method outperforms the conventional ones at the same number of measurements. Thus, our signal model and algorithm are effective when the obtained signals are originated from one source. With less than 60 % of randomly sampled data, one can obtain moderate SNR signals.

5. CONCLUSION

In this paper, we presented a new signal model in DCS and a reconstruction algorithm for the model to reduce the number of samples required for accurate observation. Our target is to deal with time series signals originated from some source and obtained via sensors located in the field, such as a speaker in DOA estimation application and a set of muscle-fibers in EMG measurement. To do this, we introduced additional coefficients that represent attenuation and time delay in JSM-3, and devised a reconstruction algorithm based on ADMM and FPC. We evaluated the performance of our algorithm for both computer-generated JSMAP signals and real EMG ones, and demonstrated the advantage of our method compared to conventional ones that do not consider attenuation and time delay in the reconstruction of the actual signals. Therefore, the developed method can be applied to low-complexity compression scheme in the multichannel sensing system for a DOA estimation or an EMG measurement application.

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