WORST CASE ROBUST DOWNLINK BEAMFORMING ON THE RIEMANNIAN MANIFOLD

Dana Ciochina, Marius Pesavento *

Technische Universität Darmstadt
Communication Systems Group
D-64283, Darmstadt, Germany

Kon Max Wong

McMaster University
Dept of Electrical and Computer Engineering
Hamilton, Ontario, Canada, L8S 4K1

ABSTRACT

In this paper we take a new perspective on the worst case robust multiuser downlink beamforming problem with imperfect second order channel state information at the transmitter. Recognizing that all channel covariance matrices form a Riemannian manifold, we propose to use a measure properly defined along this manifold in order to model the set of mismatched channel covariance matrices for which robustness shall be guaranteed. This leads to a new robust beamforming problem formulation for which a convex approximation is derived. Simulation results show a dramatically improved performance of the proposed scheme, both in terms of transmission power and constraint satisfaction, as compared to the previous methods.

Index Terms— robust downlink beamforming, imperfect CSI, covariance based CSI, Riemannian manifold

1. INTRODUCTION

When multiple antennas are available at a basestation (BS), beamforming techniques can be used to ensure a desired quality of service (QoS) to the users in the network [1]. However, the performance of beamforming techniques substantially depends on the quality of the channel state information (CSI) available at the transmitter. Since the errors in the CSI are inherent due to, e.g., erroneous CSI estimation [2], quantization and feedback delay [3], the design of robust beamformers that consider the channel imperfections is of major practical importance.

A common approach to ensure robustness in this context, generally referred to as worst case robust beamforming, is to construct beamformers that satisfy the QoS constraints for all possible channel states within a region suitably defined around the presumed CSI. For the slow fading case in which instantaneous CSI is available at the transmitter, robust beamformer designs have been proposed in [1]-[6], in which the Frobenius norm has been considered as a measure to limit the uncertainty in the CSI. Other approaches consider spectral norm bounds [7], rectangular bounds [8] or more generally convex uncertainty regions [9], [10]. When the CSI at the transmitter is based on second order statistics, as, e.g., in fast fading scenarios, the worst case robust beamforming approaches [1], [11]-[15] have considered Frobenius or weighted Frobenius norms to limit the mismatch in the presumed CSI. A different perspective is taken in the statistical approaches of, e.g., [16], [17], in which the QoS targets are satisfied with a certain predefined probability.

Our approach in this paper is in the context of worst case robust beamforming with second order channel information at the transmitter. With respect to the previously proposed approaches in [1], [11]-[14], our formulation is essentially different, in that we take into account the geometric structure of the space of the covariance matrices and measure the uncertainty region with a metric properly defined on this space. Our proposed approach is motivated by the observation that when second order statistical CSI is available at the transmitter, the region in which the mismatched CSI must be considered consists of positive definite matrices which form a Riemannian manifold [20]-[22]. Thus in order to correctly characterize this set, proper Riemannian distances are more appropriate than the previously used Frobenius norms, which are generally overly conservative. Therefore we formulate a new worst case robust beamforming problem, in which we use the Riemannian distance previously derived in [21] to define the uncertainty set around the presumed CSI, for which the signal-to-interference-and-noise (SINR) constraints are guaranteed. We note that a mathematically comparable bound was previously used in [15]. However, the motivation in [15] was merely computational and in the problem formulation the uncertainty sets are still bounded based on Frobenius norm. Moreover, due to the approximations used in [15], the techniques presented there are weaker competitors to our method, as we further show through simulations. With respect to the recently proposed approach in [19] where the uncertainty region is limited using trace bounds, our approach is more general in that it can be employed to a larger number of error models on the CSI.

The rest of the paper is organized as follows. After introducing the system model and the Riemannian distance in sec-

*This work was supported by the European Research Council (ERC) Advanced Investigator Grants program under Grant 227477-ROSE.

tion 2, we formulate the new worst case robust beamforming problem, to which we derive in Section 3 a convex approximation, based on Lagrange duality theory. Simulation results show that our new design significantly outperforms the existing methods in terms of constraint satisfaction and transmitted power. While in this paper our robust beamforming design is applied to a multiple-input single output downlink scenario, the ideas presented can be easily extended to more complicated scenarios including, e.g., cognitive radio [12], [18] or multigroup multicasting [19].

Notation: Throughout this paper we use $\mathrm{Tr}\{\cdot\}, \mathrm{E}\{\cdot\}$ to denote the trace and expectation of a square matrix. $(\cdot)^{+}, \mathrm{vec}(\cdot)$, $\|\cdot\|_F$ and $\mathcal{R}(\cdot)$, represent the pseudoinverse, vectorization, Frobenius norm and the range of the matrix, while $\otimes$, $\boldsymbol{I}_N$ and $\mathcal{U}$ stand for the Kronecker product, the identity matrix of dimension N and the set of unitary matrices respectively.

2. SYSTEM MODEL

Consider a BS with N antenna elements serving K single-antenna users. The signal transmitted at the BS is given by $\boldsymbol{x}(n) = \sum_{k=1}^{K} \boldsymbol{w}_k s_k(n)$, where $\boldsymbol{w}_k$, $s_k(n)$ denote the beamforming weight vector and the zero mean unit variance information symbol transmitted to the kth user at time instant n, respectively. The signal at the kth receiver can be written as $y_k(n) = \boldsymbol{h}_k^H \boldsymbol{x}(n) + z_k(n)$, where $\boldsymbol{h}_k \triangleq [h_{k,1}, \ldots, h_{k,N}]^T$ denotes the downlink channel vector of the kth user, with $h_{k,m}$ being the complex flat fading channel coefficient between the mth transmission antenna and the kth receiver. $z_k(n)$ is the additive complex circular Gaussian noise of zero mean and variance σ_k^2. At the transmitter, the available estimates of the channel covariance matrices can be written as $\hat{\boldsymbol{R}}_k = \boldsymbol{R}_k - \boldsymbol{\Delta}_k$, where $\boldsymbol{R}_k = E\left\{\boldsymbol{h}_k \boldsymbol{h}_k^H\right\}$ and $\boldsymbol{\Delta}_k$ represent the true covariance matrix and the estimation error for the kth user, respectively. The aim of the robust downlink beamforming problem is to design beamformers which minimize the total transmitted power while satisfying imposed SINR targets for all scheduled users in the network and for all possible mismatch matrices which lie within a predefined distance from the available channel covariance matrices. For a properly defined distance metric d and using $d_k \triangleq d(\hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k, \hat{\boldsymbol{R}}_k)$ to specify the distance between the true and estimated channel covariance matrices of user k, we write the worst case robust downlink beamforming as

$$\min_{\{\boldsymbol{w}_i\}} \sum_{i=1}^{K} ||\boldsymbol{w}_i||^2 \tag{1a}$$

$$\text{s.t.} \min_{\substack{\hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k \succ 0 \\ d_k^2 \leq \alpha_k^2}} \frac{\boldsymbol{w}_k^H \left(\hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k\right) \boldsymbol{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{w}_i^H \left(\hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k\right) \boldsymbol{w}_i + \sigma_k^2} \geq \gamma_k, \tag{1b}$$

$$k = 1, \ldots K.$$

where γ_k and α_k are the imposed SINR threshold and error bound for the kth user.

2.1. Measure Proposed to Characterize the Uncertainty Region

In order to characterize the uncertainty region for the channel mismatches, a proper distance must be introduced to measure the dissimilarity between the true and the estimated CSI. Since the Frobenius norm is known to be the shortest distance between two points in the Euclidean space, it has been widely considered as a reasonable choice for modelling the uncertainty sets around the presumed CSI. Indeed, the use of this norm is well justified when limiting the mismatches in the estimates of the instantaneous channels $\boldsymbol{h}_k$, available at the transmitter. This is because in this case the errors may occur arbitrarily inside a bounded set. This is however not the case when the available CSI is based on second order statistics. Due to their positive semidefiniteness property, the mismatched covariance matrices cannot be considered as free points in the Euclidean space. They form instead a Riemannian manifold, in which distances are not correctly characterized by Frobenius norms, but by properly defined Riemannian measures. Riemannian distances have been derived and succesfully used in different signal processing applications, e.g., in signal classification and feature detection [21], [22]. In this paper, in order to measure the dissimilarity between the true and mismatched covariance matrices on the Riemannian manifold, we adopt the Riemannian distance as derived in [21]:

$$d_R(\hat{\boldsymbol{R}}_k, \hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k) = \tag{2}$$

$$\sqrt{2\mathrm{Tr}\{\hat{\boldsymbol{R}}_\mathrm{k}\} + \mathrm{Tr}\{\boldsymbol{\Delta}_\mathrm{k}\} - 2\mathrm{Tr}\left\{\left(\hat{\boldsymbol{R}}_\mathrm{k}^{1/2}(\hat{\boldsymbol{R}}_\mathrm{k} + \boldsymbol{\Delta}_\mathrm{k})\hat{\boldsymbol{R}}_\mathrm{k}^{1/2}\right)^{1/2}\right\}}.$$

Defining $\boldsymbol{A}_k \triangleq \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{w}_i \boldsymbol{w}_i^H - \boldsymbol{w}_k \boldsymbol{w}_k^H$, and employing (2) the optimization sub-problem in (1b) can be written as

$$\min_{\boldsymbol{\Delta}_k} -\left(\mathrm{Tr}\left\{\boldsymbol{\Delta}_\mathrm{k} \boldsymbol{A}_\mathrm{k}\right\} + \mathrm{Tr}\left\{\hat{\boldsymbol{R}}_\mathrm{k} \boldsymbol{A}_\mathrm{k}\right\} + \sigma_\mathrm{k}^2 \gamma_\mathrm{k}\right) \geq 0 \tag{3a}$$

$$\text{s.t. } d_R^2(\hat{\boldsymbol{R}}_k, \hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k) \leq \alpha_{R,k}^2 \tag{3b}$$

$$\hat{\boldsymbol{R}}_k + \boldsymbol{\Delta}_k \succeq \boldsymbol{0}, \tag{3c}$$

where $\alpha_{R,k}$ is the kth bound on the Riemannian distance. Therefore, our proposed robust beamforming problem becomes:

$$\min_{\{\boldsymbol{w}_i\}} \sum_{i=1}^{K} ||\boldsymbol{w}_i||^2 \text{ s.t. } (3) \text{ being satisfied for } \mathrm{k} = 1 \ldots \mathrm{K}. \tag{4}$$

3. PROPOSED BEAMFORMING APPROACH

The problem in (4) is generally non-convex and therefore difficult to solve. In this section we derive a convex approximation of (4). To this aim, we rewrite the optimization subproblems (3) as closed form expressions, which can be reduced to convex reformulations. Due to the complicated form of the expressions (2), we first derive a simple approximation for the Riemannian distance, which we use in the following analysis.

Lemma 1: Let $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ be two positive definite Hermitian matrices. Then

$$\mathrm{Tr}\left\{(\boldsymbol{M}_1^{1/2}\boldsymbol{M}_2\boldsymbol{M}_1^{1/2})^{1/2}\right\}\geq \mathrm{Tr}\left\{\boldsymbol{M}_1^{1/2}\boldsymbol{M}_2^{1/2}\right\} \quad (5)$$

Proof: The term on the left hand side of (5) is the optimum of

$$\max_{\boldsymbol{U}_1,\boldsymbol{U}_2\in\mathcal{U}} \mathrm{Re}\left\{\mathrm{Tr}\left\{\boldsymbol{U}_1\boldsymbol{U}_2^{\mathrm{H}}\boldsymbol{M}_1^{1/2}\boldsymbol{M}_2^{1/2}\right\}\right\}, \quad (6)$$

which is attained when $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are right and left singular matrices of $\boldsymbol{M}_1^{1/2}\boldsymbol{M}_2^{1/2}$ [24]. Since $\boldsymbol{U}_1=\boldsymbol{U}_2=\boldsymbol{I}_N$ are also feasible solutions of (6), the inequality in Lemma 1 holds. ■

Applying the result of Lemma 1 on the expression in (2), it immediately follows that

$$d_R^2\left(\hat{\boldsymbol{R}}_k,\hat{\boldsymbol{R}}_k+\boldsymbol{\Delta}_k\right)\leq \mathrm{Tr}\left\{\left(\left(\hat{\boldsymbol{R}}_{\mathrm{k}}+\boldsymbol{\Delta}_{\mathrm{k}}\right)^{1/2}-\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)^2\right\}. \quad (7)$$

Using the upper bounds in (7), we can strengthen (3b) in (3) and obtain the inner approximations

$$\begin{aligned}&\min_{\boldsymbol{\Delta}_k} -\left(\mathrm{Tr}\left\{\boldsymbol{\Delta}_{\mathrm{k}}\boldsymbol{A}_{\mathrm{k}}\right\}+\mathrm{Tr}\left\{\hat{\boldsymbol{R}}_{\mathrm{k}}\boldsymbol{A}_{\mathrm{k}}\right\}+\sigma_{\mathrm{k}}^2\gamma_{\mathrm{k}}\right) && (8)\\ &\text{s.t } \mathrm{Tr}\left\{\left(\left(\hat{\boldsymbol{R}}_{\mathrm{k}}+\boldsymbol{\Delta}_{\mathrm{k}}\right)^{1/2}-\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)^2\right\}\leq\alpha_{\mathrm{R,k}}^2 \text{ and (3c)},\end{aligned}$$

whose optimal value is denoted by $\Upsilon_k^\star$. We have all elements to make the following statement.

Proposition 1: A sufficient condition for the worst case SINR constraints in (3) to be satisfied, i.e., $\Upsilon_k^\star \geq 0$ is that there exists a set of non-negative λ_k such that

$$\boldsymbol{X}_k \triangleq \begin{pmatrix} -\boldsymbol{I}_N\otimes\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_{N^2} & \boldsymbol{b}_k(\lambda_k)\\ \boldsymbol{b}_k^H(\lambda_k) & c_k(\lambda_k)\end{pmatrix}\succeq\boldsymbol{0}, \quad (9)$$

where

$$\boldsymbol{b}_k(\lambda_k)=-\lambda_k\mathrm{vec}(\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}), \quad (10a)$$

$$c_k(\lambda_k)=\lambda_k\mathrm{Tr}\{\hat{\boldsymbol{R}}_{\mathrm{k}}\}-\sigma_{\mathrm{k}}^2\gamma_{\mathrm{k}}-\lambda_{\mathrm{k}}\alpha_{\mathrm{R,k}}^2. \quad (10b)$$

Proof: Introducing $\boldsymbol{Q}_k\triangleq\left(\hat{\boldsymbol{R}}_k+\boldsymbol{\Delta}_k\right)^{1/2}$ problem (8) becomes equivalent to

$$\min_{\boldsymbol{Q}_k} -\mathrm{Tr}\left\{\boldsymbol{Q}_{\mathrm{k}}\boldsymbol{A}_{\mathrm{k}}\boldsymbol{Q}_{\mathrm{k}}^{\mathrm{H}}\right\}-\sigma_{\mathrm{k}}^2\gamma_{\mathrm{k}} \quad (11a)$$

$$\text{s.t.}\mathrm{Tr}\left\{\left(\boldsymbol{Q}_{\mathrm{k}}-\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)\left(\boldsymbol{Q}_{\mathrm{k}}-\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)^{\mathrm{H}}\right\}\leq\alpha_{\mathrm{R,k}}^2, \quad (11b)$$

$$\boldsymbol{Q}_k=\boldsymbol{Q}_k^H. \quad (11c)$$

Note, that in (11), the constraints (11c) are redundant since in their absence any optimal solution of (11) is still Hermitian. This can be proven as follows. If $\boldsymbol{Q}_k^\star$ is optimal for the problem formed by (11a) and (11b), then $\boldsymbol{Q}_k^{\star H}$ is also optimal and the first-order optimality conditions [23] imply that

$$\boldsymbol{Q}_k^\star\left(-\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_N\right)=\left(-\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_N\right)\boldsymbol{Q}_k^\star=\hat{\boldsymbol{R}}_k^{1/2}. \quad (12)$$

From (12) and further using [24, Th 1.3.12] it follows that $\boldsymbol{Q}_k^\star=\boldsymbol{Q}_k^{\star H}$. Therefore, with $\boldsymbol{q}_k\triangleq\mathrm{vec}(\boldsymbol{Q}_{\mathrm{k}})$, (11) can be equivalently written as

$$\begin{aligned}&\min_{\boldsymbol{q}_k} -\boldsymbol{q}_k^H(\boldsymbol{I}_N\otimes\boldsymbol{A}_k)\boldsymbol{q}_k-\sigma_k^2\gamma_k && (13)\\ &\text{s.t. } \boldsymbol{q}_k^H\boldsymbol{q}_k-2\mathrm{Re}\left\{\mathrm{vec}^{\mathrm{H}}\left(\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)\boldsymbol{q}_{\mathrm{k}}\right\}+\mathrm{Tr}\{\hat{\boldsymbol{R}}_{\mathrm{k}}\}-\alpha_{\mathrm{k}}^2\leq 0.\end{aligned}$$

Since (13) represents a quadratic program with a single quadratic constraint, strong duality holds if it admits a strictly feasible solution [23]. In the case of (13), strong duality can be claimed for any positive α_k, due to the strict feasibility of $\boldsymbol{q}_k=\mathrm{vec}\left(\hat{\boldsymbol{R}}_{\mathrm{k}}^{1/2}\right)$.

Using the notations in (10a) and (10b), the dual problem of (13) can be written as

$$\begin{aligned}&\max_{\lambda_k} c_k(\lambda_k)-\boldsymbol{b}_k^H(\lambda_k)\left(-\boldsymbol{I}_N\otimes\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_{N^2}\right)^+\boldsymbol{b}_k(\lambda_k) && (14)\\ &\text{s.t.} -\boldsymbol{I}_N\otimes\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_{N^2}\succeq 0\\ &\quad \boldsymbol{b}_k(\lambda_k)\in\mathcal{R}(-\boldsymbol{I}_N\otimes\boldsymbol{A}_k+\lambda_k\boldsymbol{I}_{N^2})\end{aligned}$$

Due to the strong duality property, it follows that, a non-negative optimum value for (14) guarantees the satisfaction of the worst case SINR constraints (3). Furthermore, since the objective of the dual is always smaller or equal than the primal for all feasible points, it is sufficient to find one λ_k in the feasible set of (14), such that the objective is non-negative.

Finally, using Schur complement, it is proven in [23, Appendix A5.4] that λ_k is in the feasible set of (14) and achieves a non-negative objective value if and only if $\boldsymbol{X}_k\succeq 0$, where $\boldsymbol{X}_k$ is defined in equation (9). ■

Therefore the worst case SINR constraints (3) in (4) can be replaced by (9). Moreover this allows for a convex reformulation, as follows. Introducing $\boldsymbol{W}_k\triangleq\boldsymbol{w}_k\boldsymbol{w}_k^H$, for $k=1,\ldots,K$ and using the semidefinite relaxation procedure [25], the initial problem (4) can be rewritten as

$$\begin{aligned}&\min_{\{\boldsymbol{W}_i,\lambda_i\}} \sum_{i=1}^K\mathrm{Tr}\{\boldsymbol{W}_{\mathrm{i}}\} && (15)\\ &\text{s.t. } (9),\ \boldsymbol{W}_k\succeq\boldsymbol{0},\ \lambda_k\geq 0,\ k=1,\ldots K\end{aligned}$$

If the resulting optimal matrices $\boldsymbol{W}_k$ exhibit a rank larger than one randomization techniques [25] can be used.

4. SIMULATIONS

We consider two scenarios in which different error models are assumed for the covariance matrices and show the performance of our algorithm in terms of transmission power and SINR satisfaction as compared to the methods in [13] and [15]. In order to illustrate the choice of the thresholds used in the comparison, we briefly present the measures employed in [13] and [15] to characterize the mismatches. In [13] the uncertainty region is bounded as $\|\boldsymbol{\Delta}_k\|_F\leq\alpha_{F,k}$ where $\alpha_{F,k}$ denotes the imposed uncertainty thresholds under Frobenius norm. In [15], the following approach is taken to simplify the positive semidefiniteness contraints on $\hat{\boldsymbol{R}}_k+\boldsymbol{\Delta}_k$. Assuming the estimated covariance matrix is decomposed as $\hat{\boldsymbol{R}}_k\triangleq\boldsymbol{P}_k\boldsymbol{P}_k^H$, then $\hat{\boldsymbol{R}}_k+\boldsymbol{\Delta}_k=\left(\boldsymbol{P}_k+\overline{\boldsymbol{\Delta}}_k\right)\left(\boldsymbol{P}_k+\overline{\boldsymbol{\Delta}}_k\right)^H$, where $\boldsymbol{\Delta}_k=\boldsymbol{P}_k^H\overline{\boldsymbol{\Delta}}_k+\overline{\boldsymbol{\Delta}}_k^H\boldsymbol{P}_k+\overline{\boldsymbol{\Delta}}_k^H\overline{\boldsymbol{\Delta}}_k$. Thereafter, Frobenius norm is used to bound both $\boldsymbol{\Delta}_k$ i.e., the mismatch on the available covariance matrix and $\overline{\boldsymbol{\Delta}}_k$, the mismatch on the square root of the erroneous CSI. If $\alpha_{F,k}$ is the threshold on $\boldsymbol{\Delta}_k$ and we denote by η_k the threshold on $\overline{\boldsymbol{\Delta}}_k$ it holds that[15]:

$$\alpha_{F,k}\leq 2\eta_k\|\boldsymbol{P}_k\|_F+\eta_k^2. \quad (16)$$

For our first simulation we use the scenario in [15], where

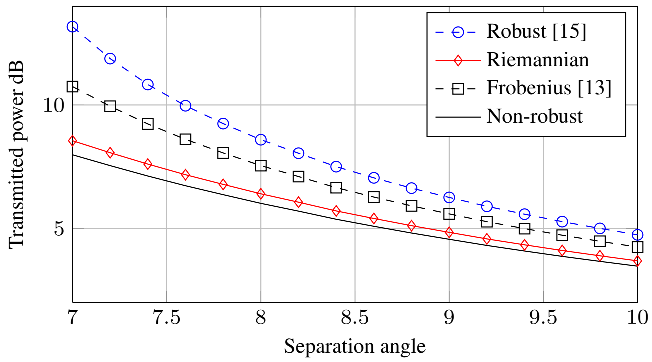

Fig. 1. Transmission power for uniform error on the covariance matrix

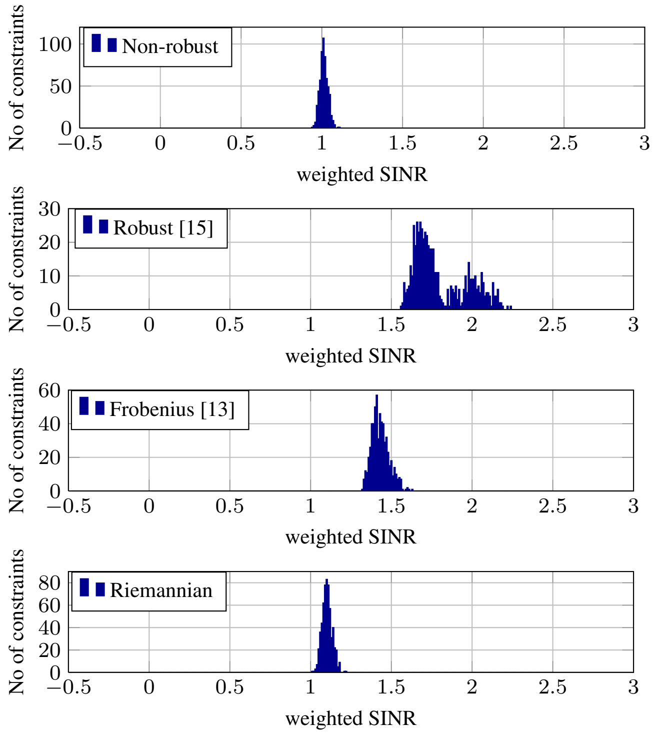

Fig. 2. Histogram of weighted SINR with error bound on the covariance matrix

each error matrix $\mathbf{\Delta}_k$ is generated uniformly within a sphere around the true covariance matrix.

This reflects the case when quantized covariance based CSI is available at the transmitter. The true covariance matrices are modelled as in [26],where the angular spread is $2°$ and users are positioned at $[10°,\ 10° + \theta_s,\ 10° + 2\theta_s]$, with θ_s denoting the separation angle. For an SINR level of 5dB for each user, and uncertainty thresholds $\alpha_{F,k}$=0.15 and $\alpha_{R,k}$ as well as η_k chosen according to (16), the transmission power is plotted in Figure 1. For further comparison, we plot in Figure 2 the histogram of the normalized QoS defined as

$$\frac{\boldsymbol{w}_k^H\left(\hat{\boldsymbol{R}}_k+\mathbf{\Delta}_k\right)\boldsymbol{w}_k}{\gamma_k\sum_{\substack{i=1\\i\neq k}}^{K}\boldsymbol{w}_i^H\left(\hat{\boldsymbol{R}}_k+\mathbf{\Delta}_k\right)\boldsymbol{w}_i+\gamma_k\sigma_k^2}. \tag{17}$$

We note from Figure 2 that our method respects all contraints

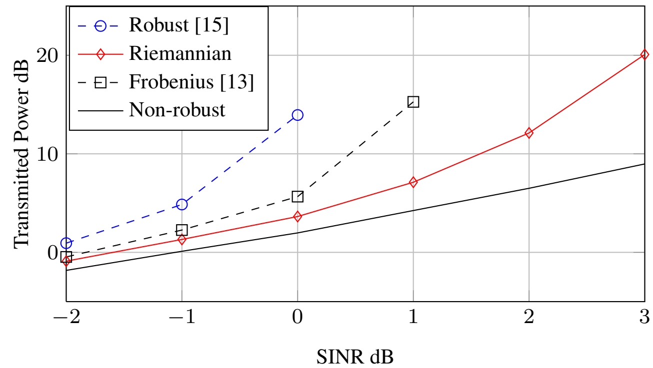

Fig. 3. Transmission power for the LS estimation and finite sampling case

without significant oversatisfaction as observed in the previous techniques of [13] and [15]. This further confirms the performance improvement obtained by our method in terms of transmission power.

In the second scenario we assume that the channels at the receiver are estimated using a Least Squares (LS) algorithm. In order to use realistic thresholds, we proceed as follows. We generate a large number of true and estimated channels, create the estimated covariance matrices and compute the Riemannian and Frobenius distances. The final thresholds are then chosen such that 95% of the distances are within their corresponding bounds. The covariance matrices are generated as described in the following. We use the true covariance matrices $\boldsymbol{R}_k$ as defined in [26] and assume a realization of the true channel as $\boldsymbol{h}_k = \boldsymbol{U}_k\mathbf{\Lambda}_k^{1/2}\boldsymbol{v}_k$, where $\boldsymbol{U}_k$ and $\mathbf{\Lambda_k}$ result from the eigenvalue decomposition $\boldsymbol{R}_k = \boldsymbol{U}_k\mathbf{\Lambda_k}\boldsymbol{U}_k^H$, while $\boldsymbol{v}_k$ are random Gaussian vectors. This corresponds to the general Rayleigh channel model with spatial correlation. The LS channel estimate in this case is given by $\hat{\boldsymbol{h}}_k = \boldsymbol{h} + \boldsymbol{T}^+\boldsymbol{e}_k$, where $\boldsymbol{T}$ is the training matrix that can, e.g., be chosen as a weighted DFT and $\boldsymbol{e}_k$ is the zero mean complex circular Gaussian error of the channel estimate for the kth user. Then the estimated covariance matrix is $\hat{\boldsymbol{R}}_k = 1/N_s\sum_{k=1}^{N_s}\hat{\boldsymbol{h}}_k\hat{\boldsymbol{h}}_k^H$, where N_s denotes the number of training snapshots. In our simulation 3000 true and estimated covariance matrices were used to obtain the thresholds of the mismatch set under Riemannian and Frobenius metrics. The remaining system parameters of the LS estimation are as follows: the number of training symbols is 6, the training power is 15dB, the variance of the estimation error is -20dB and $N_s = 512$. For these bounds, we plot in Figure 3 the required transmission power for a BS with 6 antennas to serve 3 users positioned such that the separation angles between them is $7°$. We note that our method not only requires less transmission power but also remains feasible for larger SINR values. Furthermore in all the simulations presented, we have obtained rank one solutions, therefore randomization techniques have not been required.

5. REFERENCES

[1] M. Bengtsson and B. Ottersten, "Handbook of Antennas in Wireless Communications," Optimal and Subop-

timal Transmit Beamforming: Boco Raton, FL:CRC, Aug. 2001.

[2] B. Hassibi and B. M. Hochwald, “How much training is needed in multiple-antenna wireless links?,” *IEEE Trans. on Inf. Theory*, vol. 49, no. 4, pp. 951-963, April 2003.

[3] D. J. Love, R. W. Heath, W. Santipach, and M. L. Honig, “What is the value of limited feedback for MIMO channels?,” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54-59, Oct. 2004.

[4] M. B. Shenouda and T. N Davidson, “Conic convex formulations of robust downlink precoder designs with quality of service constraints,” *IEEE J. Sel. Topics in Signal Process.*, vol. 1, pp. 714-724, Dec. 2007.

[5] J. Wang and D. Palomar, ‘Worst-case robust MIMO transmission with imperfect channel knowledge,” *IEEE Trans. in Signal Process.*, vol. 57, no. 8, Aug. 2009

[6] G. Zheng, K. K. Wong and T. S. Ng, “Robust linear MIMO in the downlink: A worst-case optimization with ellipsoidal uncertainty regions,” *EURASIP J. on Advances in Signal Process.*, pp. 1-15, Jun. 2008

[7] J. Wang and M. Payaro, “Is Transmit Beamforming Robust?, ” *in Proc. IEEE Int. Conf. on Commun. (ICC 2011)*, pp 1-5, Jun 2011

[8] N. Vucic and H. Boche, “Robust QoS-constrained optimization of downlink Multiuser MISO systems,” *IEEE Trans. in Signal Process.*, vol. 57, no. 2, Feb. 2009

[9] J. Wang, M Bengtsson, B. Ottersten, D. Palomar, “ Robust maximin MIMO precoding for arbitrary convex uncertainty sets” *in Proc. IEEE ICASSP*, pp. 3045-3048, March 2012

[10] A. Pascual-Iserte, D. P. Palomar, A. I. Perez-Neira, and M. A. Lagunas, ” A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization,” *IEEE Trans. in Signal Processing*, vol. 54, no. 1, pp. 346-360, Jan. 2006.

[11] L. Landau, R.C. de Lamare, L. Wang, M. Haardt, ”Low-complexity robust beamforming based on conjugate gradient techniques”, *Proc. International ITG Workshop on Smart Antennas (WSA 2011)*, Feb. 2011

[12] G. Zheng, K. K. Wong and B. Ottersten, “Robust cognitive beamforming with bounded channel uncertainties,” *IEEE Trans. in Signal Process.*, vol. 57, no. 12, Dec. 2009

[13] I. Wajid, Y. C Eldar, A. Gershman ” Robust downlink beamforming using covariance Channel State Information” *in Proc. IEEE ICASSP*, pp. 2285-2288 , April 2009

[14] M. Biguesh, S. Shahbazpanahi and A. B. Gershman, “Robust downlink power control in wireless cellular networks”, *EURASIP J. Wireless Communications and Networking*, no. 2, pp 261-272, Dec. 2004

[15] V. Sharma, I. Wajid, A. Gershman, H. Chen and S. Lambotharan, ‘Robust downlink beamforming using positive semidefinite covariance constraints,” *IEEE Int. ITG Workshop on Smart Antennas, WSA*, Feb. 2008

[16] D. Ponukumati, F. Gao, and M. Bode, ”Robust multicell downlink beamforming based on second-order statistics of channel state information”, *IEEE Global Telecomm. Conf (GLOBECOM 2011)*, Dec 2011, Houston, USA

[17] B. K, Chalise, S Shahbazpanahi, A. Czylwi and A. B. Gershman, ”Robust downlink beamforming based on outage probability specifications”, *IEEE Trans on Wireless Comm.*, vol. 6, no. 10, Oct. 2007

[18] I. Wajid, M. Pesavento, Y. C. Eldar, and A. B. Gershman, “Robust downlink beamforming for cognitive radio networks,” *Proc IEEE Global Communications Conf. (GLOBECOM’10)*, Dec 2010, Miami, FL, USA.

[19] K. Law, I Wajid and M. Pesavento, “ Robust downlink beamforming in multi-group mulicasting using trace bounds on the covariance mismatches” *in Proc. IEEE ICASSP*, pp. 3045-3048 , April 2011

[20] S. Gallot, D. Hulin and J. Lafontaine, *Riemannian Geometry*, Springer 3rd Ed., 2004.

[21] Y. Li, K. M. Wong and H, de Bruin “Electroencephalogram signals classification for sleepstate decision - A Riemannian geometry approach,” *IET Signal Processing*, vol. 6 , no. 4, pp 288-299

[22] Y. Li, K. M. Wong and H. de Bruin, “EEG signal classification based on a Riemannian distance measure”, *Proc. IEEE Science and Technology for Humanity (TIC-STH)*, pp 268-273, Toronto 2009

[23] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[24] R. A. Horn and C. Johnson, *Matrix Analysis*, Cambridge University Press, 2007.

[25] Z. Q. Luo, W. K. Ma, A M So, Y Ye, and S. Zhang“ Semidefinite relaxation of quadratic optimization problems” *IEEE Trans. in Signal Process.*, vol. 27, no. 3, May. 2010

[26] M. Bengtsson and B. Ottersten, “Optimal downlink beamforming using semidefinite optimization,” in Proc. 37th Annual Allerton Conf. Communications, Control and Computing, Sep. 1999, pp. 987- 996.