PARAMETER ESTIMATION AND CLASSIFICATION OF CENSORED GAUSSIAN DATA WITH APPLICATION TO WIFI INDOOR POSITIONING

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ABSTRACT

In this paper, we consider the Maximum Likelihood (ML) estimation of the parameters of a GAUSSIAN in the presence of censored, i.e., clipped data. We show that the resulting Expectation Maximization (EM) algorithm delivers virtually biasfree and efficient estimates, and we discuss its convergence properties. We also discuss optimal classification in the presence of censored data. Censored data are frequently encountered in wireless LAN positioning systems based on the fingerprinting method employing signal strength measurements, due to the limited sensitivity of the portable devices. Experiments both on simulated and real-world data demonstrate the effectiveness of the proposed algorithms.

***Index Terms*—** Indoor positioning, wireless LAN, signal strength, expectation maximization, censored data

1. INTRODUCTION

Achieving accurate positioning inside buildings is still a major technical challenge, due to the unavailability of GPS. Since the system should not come at the cost of an extra infrastructure solely for positioning purposes, solutions relying on already present devices, such as WiFi access points, are of particular interest. Smartphones and laptops are commonly equipped with WiFi sensors, such that the realization of an indoor localization functionality becomes only a matter of software.

However, IEEE 802.11 based positioning is a challenging task. Multipath propagation and missing line-of-sight renders time- or direction-of-arrival based algorithms unsuitable. As a result indoor positioning based on WiFi signal strength fingerprints have been proposed [1], where in an offline phase the statistics of the received signal strength index (RSSI) of access points (APs) measured at the target positions is gathered and the measurements during the online phase are compared to these statistics to come up with a decision on the user's location. The k-nearest neighbor (k-NN) rule with a Euclidian distance measure is used in the early RADAR system [1]. LOCATOR [2] clusters the locations of the k nearest neighbors and outputs the location of the most likely cluster center. The system in [3] employs k-NN with the Bhattacharyya coefficient between offline and online measurements, averaged over the strongest received APs, as a distance measure.

However, the sensitivity of RSSI sensors of typical smartphones is limited to a range from $-30\,\mathrm{dBm}$ to $-100\,\mathrm{dBm}$, resulting in loss or clipping of RSSI measurements, if access points are particularly strong or weak.

If this clipping is not accounted for in the training and classification, this results in loss of location accuracy. To improve performance, the problem of parameter estimation and classification in the presence of so-called censored, i.e., clipped data has to be considered. Recently, an Expectation Maximization (EM) algorithm has been derived for estimating the parameters of multivariate GAUSSIAN mixture models in the presence of censored measurements [4], other publications have also addressed this problems using maximum likelihood estimation (MLE) and analysed the CRAMER-RAO bound of the mean estimate of the GAUSSIAN [5, 6]. Utilizing the information of censored data to calculate the full likelihood has been discussed in [7] using sequential BAYESIAN estimation. We build upon the result from [4] and show that in the case of a single GAUSSIAN model, the estimation algorithm is biasfree and, moreover, achieves the CRB for both mean and variance estimates. We further derive how the convergence speed depends on the parameters of the GAUSSIAN and the clipping threshold. While this algorithm is used in the offline training phase, we also show how the online classification/localization stage has to be modified to account for clipped measurements.

The paper is organized as follows: in Section 2 we outline the derivation of the EM algorithm for univariate censored normally distributed measurements. Being somewhat different from the derivation in [4] it serves to introduce our notation. The properties of the algorithm are investigated in Section 3. Section 4 presents the classification algorithm, while Section 5 describes positioning experiments both on artificially generated data and on real measurements gathered inside an office building. The paper finished with conclusions drawn in Section 6.

2. PARAMETER ESTIMATION ALGORITHM

In the following derivation we consider GAUSSIAN measurements which are one-sided censored only, to simplify the exposition. An extension to the two-sided case is straightforward.

Let $\mathbf{y} = y_1, ..., y_N; y_i \in \mathbb{R}$ be the unobservable, non-censored data, where N is the number of measurements and where the y_i are i.i.d. with GAUSSIAN probability density function (PDF) $p_Y(y_i) = \mathcal{N}(y_i; \mu, \sigma^2)$. Observable are the data $\mathbf{x} = x_1, ..., x_N$, where $x_i = \max(y_i, c)$, with clipping threshold c. Our goal is to estimate the parameters $\theta = (\mu, \sigma^2)$ of the underlying GAUSSIAN.

Employing the EM algorithm we identify $\mathbf{y}$ and $\mathbf{x}$ to be the complete and the observed data. Thus the expected log-likelihood of the complete data is given by

$$Q(\theta; \theta^{(\kappa)}) = E\left[\ln\left(p_Y(\mathbf{y}; \theta)\right)|\mathbf{x}; \theta^{(\kappa)}\right] \tag{1}$$

$$= \sum_{i=1}^{N} \int_{-\infty}^{\infty} \ln\left(p_Y(y_i; \theta)\right) p\left(y_i | x_i; \theta^{(\kappa)}\right) dy_i \tag{2}$$

$$=: \sum_{i=1}^{N} f_i\left(\theta; \theta^{(\kappa)}\right), \tag{3}$$

where κ is the iteration index. A moment of thought shows

$$p(y_i|x_i;\theta^{(\kappa)}) = \begin{cases} \frac{\mathcal{N}(y_i;\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}, & \text{if } y_i \le c \\ \delta(y_i - x_i), & \text{if } y_i > c \end{cases}. \quad (4)$$

Here we have used the notation

$$I_j(\theta^{(\kappa)}) = \int_{-\infty}^{c} y^j \mathcal{N}\left(y;\theta^{(\kappa)}\right) dy \quad (5)$$

with $j = 0$.

Introducing the binary random variable Z_i with realization z_i, where $z_i = 0$ and $z_i = 1$ indicate that the i-th measurement is not censored or censored, respectively, the summand in (3) can be written as

$$f_i\left(\theta;\theta^{(\kappa)}\right) = \frac{z_i}{I_0\left(\theta^{(\kappa)}\right)} \int_{-\infty}^{c} \ln\left(\mathcal{N}(y_i;\theta)\right) \mathcal{N}\left(y_i;\theta^{(\kappa)}\right) dy_i + (1 - z_i)\ln\left(\mathcal{N}(y_i;\theta)\right). \quad (6)$$

The parameter estimates are obtained by computing the derivatives of (6) w.r.t. the elements of θ:

$$\frac{\partial}{\partial\mu} f_i\left(\theta;\theta^{(\kappa)}\right) = \frac{z_i}{\sigma^2}\left(\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} - \mu\right) + \frac{(1 - z_i)}{\sigma^2}(x_i - \mu) \quad (7)$$

$$\frac{\partial}{\partial\sigma} f_i\left(\theta;\theta^{(\kappa)}\right) = \frac{z_i}{\sigma}\left[\frac{1}{\sigma^2}\left(\frac{I_2(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} - 2\mu\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} + \mu^2\right) - 1\right] + \frac{1 - z_i}{\sigma}\left(\frac{(x_i - \mu)^2}{\sigma^2} - 1\right), \quad (8)$$

where we exploited the fact that $y_i = x_i$ in case $z_i = 0$. Setting the summation of the derivatives to zero, the estimates are readily obtained:

$$\mu^{(\kappa+1)} = \frac{1}{N}\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}\sum_{i=1}^{N} z_i + \frac{1}{N}\sum_{i=1}^{N}(1 - z_i)x_i \quad (9)$$

$$\left(\sigma^2\right)^{(\kappa+1)} = \left[\frac{I_2(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} - 2\mu^{(\kappa)}\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})} + \left(\mu^2\right)^{(\kappa)}\right]\frac{1}{N}\sum_{i=1}^{N} z_i + \frac{1}{N}\sum_{i=1}^{N}(1 - z_i)\left(x_i - \mu^{(\kappa)}\right)^2. \quad (10)$$

After convergence we have $\mu^{(\kappa+1)} \approx \mu^{(\kappa)} =: \hat{\mu}$ and $\left(\sigma^2\right)^{(\kappa+1)} \approx \left(\sigma^2\right)^{(\kappa)} =: \hat{\sigma}^2$. Using this in (9) and (10) and solving for the estimates, we arrive at

$$\hat{\mu} = \frac{M}{N}\frac{1}{M}\sum_{i=1}^{M} x_i + \left[1 - \frac{M}{N}\right]\frac{\int_{-\infty}^{c} y p_Y(y;\hat{\theta})dy}{\int_{-\infty}^{c} p_Y(y;\hat{\theta})dy}, \quad (11)$$

$$\hat{\sigma}^2 = \frac{M}{N}\frac{1}{M}\sum_{i=1}^{M}(x_i - \hat{\mu})^2 + \left[1 - \frac{M}{N}\right]\frac{\int_{-\infty}^{c}(y - \hat{\mu})^2 p_Y(y;\hat{\theta})dy}{\int_{-\infty}^{c} p_Y(y;\hat{\theta})dy}, \quad (12)$$

where we assumed w.l.o.g. that the first $M = \sum_i (1 - z_i)$ observations are the uncensored ones. These expressions lend themselves to the following interpretation: Mean and variance estimates are the weighted average between their ML estimates computed from the observed data and the mean and variance of the assumed truncated GAUSSIAN of the unobservable parts. The weights are the relative frequencies of the uncensored and censored measurements, respectively.

3. PROPERTIES OF ESTIMATES

3.1. Unbiasedness and Convergence Properties

In order to study the convergence properties we compute the expected values of the difference between the estimates, eqs. (9) and (10), and the true values of the parameters. First note that Z_i is a BERNOULLI random variable with

$$P(Z_i = 1) = \int_{-\infty}^{c} p_Y(y;\theta)dy = I_0(\theta)$$

and $P(Z_i = 0) = 1 - I_0(\theta)$. Thus: $E[Z_i] = E[Z_i^2] = I_0(\theta)$. Taking the expectation of (9) we have

$$E\left[\mu^{(\kappa+1)}\right] = \frac{1}{N}\sum_{i=1}^{N} E[(1 - z_i)x_i] + \frac{1}{N}\sum_{i=1}^{N} E\left[\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}z_i\right]. \quad (13)$$

The first expectation can be evaluated as follows:

$$E[(1 - Z_i)X_i] = \sum_{z_i=0}^{1}\int_{-\infty}^{\infty}(1 - z_i)x_i P(z_i|x_i)p_Y(x_i)dx_i = \int_{c}^{\infty} x_i \mathcal{N}(x_i;\theta)dx_i = \mu - I_1(\theta). \quad (14)$$

Here we have used that

$$P(z_i = 0|x_i) = \begin{cases} 1, & \text{if } x_i > c \\ 0, & \text{else} \end{cases}.$$

Using further $E\left[\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}z_i\right] \approx E\left[\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}\right] E[Z_i]$ and subtracting μ from either side of eq. (13) we obtain

$$E\left[\tilde{\mu}^{(\kappa+1)}\right] = -I_1(\theta) + I_0(\theta)E\left[\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}\right]. \quad (15)$$

where $\tilde{\mu}^{(\kappa+1)} = \mu^{(\kappa+1)} - \mu$. A similar equation can be found for $\left(\tilde{\sigma}^2\right)^{(\kappa+1)} = \left(\sigma^2\right)^{(\kappa+1)} - \sigma^2$.

In order to compute the remaining expectation, a Taylor series expansion around the true parameter values $\theta = (\mu, \sigma^2)$ is applied and truncated after the linear term:

$$E\left[\frac{I_1(\theta^{(\kappa)})}{I_0(\theta^{(\kappa)})}\right] \approx \frac{I_1(\theta)}{I_0(\theta)} + E\left[\tilde{\mu}^{(\kappa)}\right]\frac{\partial}{\partial\mu}\frac{I_1(\theta)}{I_0(\theta)} + E\left[\left(\tilde{\sigma}^2\right)^{(\kappa)}\right]\frac{\partial}{\partial\sigma^2}\frac{I_1(\theta)}{I_0(\theta)}. \quad (16)$$

Using this in (15) we arrive after a straightforward, however lengthy computation at

$$\begin{pmatrix} E\left[\tilde{\mu}^{(\kappa+1)}\right] \\ E\left[\left(\tilde{\sigma}^2\right)^{(\kappa+1)}\right] \end{pmatrix} \approx \begin{pmatrix} W_\mu & W_{\mu\sigma} \\ W_{\sigma\mu} & W_\sigma \end{pmatrix}\begin{pmatrix} E\left[\tilde{\mu}^{(\kappa)}\right] \\ E\left[\left(\tilde{\sigma}^2\right)^{(\kappa)}\right] \end{pmatrix} = \begin{pmatrix} W_\mu & W_{\mu\sigma} \\ W_{\sigma\mu} & W_\sigma \end{pmatrix}^{\kappa+1}\begin{pmatrix} E\left[\tilde{\mu}^{(0)}\right] \\ E\left[\left(\tilde{\sigma}^2\right)^{(0)}\right] \end{pmatrix}, \quad (17)$$

where

$$W_\mu = I_0(\theta)\frac{\partial}{\partial\mu}\frac{I_1(\mu)}{I_0(\mu)}, \quad W_{\mu\sigma} = I_0(\theta)\frac{\partial}{\partial\sigma^2}\frac{I_1(\theta)}{I_0(\theta)},$$

$$W_\sigma = I_0(\theta)\frac{\partial}{\partial\sigma^2}\frac{I_2(\theta) - 2I_1(\theta)\mu + I_0(\theta)\mu^2}{I_0(\theta)},$$

$$W_{\sigma\mu} = I_0(\theta)\frac{\partial}{\partial\mu}\frac{I_2(\theta) - 2I_1(\theta)\mu + I_0(\theta)\mu^2}{I_0(\theta)}.$$

Noting that the magnitudes of the eigenvalues of the matrix with the W entries are always less than one (approaching one for $c \to \infty$), we can conclude that the estimates are biasfree. Since, however, in practice μ has to be replaced by its estimate in the estimation of the variance, we exhibit the same bias as in ordinary ML estimation of the variance. In the applications considered here, the bias of the ML estimate of the variance can be neglected due to the large number of samples. Further, an estimate of the convergence speed of the EM algorithms can also be obtained from eq. (17), see Fig. 1. It can be seen that the number of iterations quickly rises once more than 50% of the data are clipped.

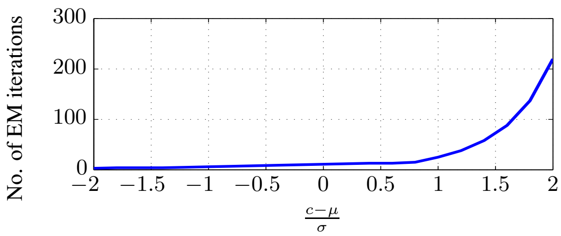

Fig. 1. Theoretical number of EM iterations κ required to reduce the estimation error to 10^{-4} of its initial value as a function of $(c-\mu)/\sigma$. Initial values $\mu^{(0)}$, $(\sigma^2)^{(0)}$ have been set to the ML estimates of μ, σ^2 computed from the unclipped observations only.

3.2. Precision

From the derivation of the EM algorithm it can be seen that the measurements consist of actually two types of data, the number M of noncensored observations and the observations themselves. While the first is binomially distributed, the second are drawn from a truncated GAUSSIAN. Further note that the draws from the GAUSSIAN are independent of the binomial random variable. Following [8] the log-likelihood is thus given by

$$L(\theta) = \ln\left(\frac{N!}{M!(N-M)!}\right) + (N-M)\ln\left(I_0(\theta)\right) - \frac{M}{2}\ln\left(2\pi\sigma^2\right) - \frac{1}{2}\sum_{j=1}^{M}\left(\frac{x_j-\mu}{\sigma}\right)^2. \quad (18)$$

From this the Fisher information matrix $\mathbf{I}$ can be readily computed. The CRAMER-RAO bound on the estimation error variance of the mean and variance estimates are then obtained by inverting $\mathbf{I}$.

4. CLASSIFICATION OF CENSORED DATA

Indoor localization can be formulated as a classification problem, where the classes are the positions from which RSSI measurements are taken during the offline training phase. For each position ℓ_k the parameters of a GAUSSIAN class-conditional density $p_Y(\mathbf{y}|\ell_k)$ of RSSI measurements are estimated using the EM algorithm of the last sections. During online classification, to estimate the user's location, first the posterior is calculated as follows

$$p(\ell_k|\mathbf{x}_{1:S}) = \frac{\prod_{s=1}^{S}\prod_{i=1}^{N_{AP}} p(x_{s,i}|\ell_k)P(\ell_k)}{\sum_{k'=1}^{K}\prod_{s=1}^{S}\prod_{i=1}^{N_{AP}} p(x_{s,i}|\ell_{k'})P(\ell_{k'})} \quad (19)$$

where K is the number of offline training locations, S is the number of online measurements, $x_{s,i}$ is the RSSI of i-th AP in the s-th measurement and N_{AP} is the total number of access points from which measurements are taken. Here we assumed independence of the measurements $\mathbf{x}_{1:S}$ and the RSSIs of different APs. Further, the prior $P(\ell_k)$ is assumed to be equal for all locations. Note that the test data are also subject to censoring. The likelihood $p(x_{s,i}|\ell_k)$ can be calculated as follows

$$p(x_{s,i}|\ell_k) = \int_{-\infty}^{\infty} p(x_{s,i}|y_i)p_Y(y_i|\ell_k)dy_i, \quad (20)$$

where

$$p(x_{s,i}|y_i) = \begin{cases} \delta(x_{s,i}-y_i), & \text{if } y_i > c \\ \delta(x_{s,i}-c), & \text{if } y_i \le c \end{cases}. \quad (21)$$

Plugging this into eq. (20) we arrive at

$$p(x_{s,i}|\ell_k) = \begin{cases} \mathcal{N}(x_{s,i}; \hat{\mu}_{\ell_k,i}, \hat{\sigma}^2_{\ell_k,i}), & \text{if } x_{s,i} > c \\ I_0(\hat{\mu}_{\ell_k,i}, \hat{\sigma}^2_{\ell_k,i}), & \text{if } x_{s,i} = c \end{cases}. \quad (22)$$

Here, $(\hat{\mu}_{\ell_k,i}, \hat{\sigma}^2_{\ell_k,i})$ are the estimated parameters of the i-th AP at location ℓ_k. In case all observations of the i-th AP at location ℓ_k are clipped, the mean estimate is set to a small value $\hat{\mu}_{\ell_k,i} \ll c$ and $\hat{\sigma}^2_{\ell_k,i}$ is set to an average value.

The set P of nearest neighbors is chosen among the offline locations by taking those with the largest posteriors. The final location estimate $\hat{\ell}$ is then obtained by the weighted average

$$\hat{\ell}(\mathbf{x}_{1:S}) = \frac{1}{\sum_{k\in P} p(\ell_k|\mathbf{x}_{1:S})}\sum_{k\in P}\ell_k p(\ell_k|\mathbf{x}_{1:S}). \quad (23)$$

5. EXPERIMENTAL RESULTS

5.1. Performance of EM Algorithm

Fig. 2 compares the CRAMER-RAO bound with the mean squared error (MSE) of the proposed estimators for mean μ and variance σ^2 obtained from a simulation. It can be seen that the estimator practically achieves the bound, with differences so small that they are no visible in the graph. We therefore conclude that the estimator is efficient. Further, for the limiting case of completely uncensored data the well-known results for ML parameter estimation from a normal population are obtained: $\mathrm{MSE}(\mu) = \sigma^2/N$; $\mathrm{MSE}(\sigma^2) = \sigma^4(2N-1)/N^2$.

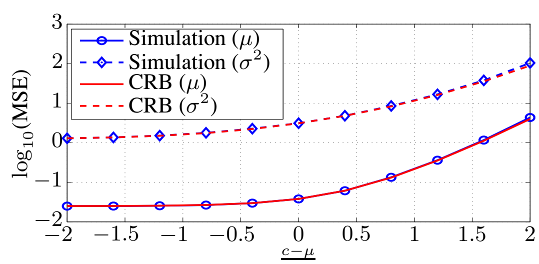

Fig. 2. Comparison of CRAMER-RAO bound for mean and variance with MSE obtained from simulation for $\sigma^2 = 25$ and $N = 1000$.

5.2. Classification on Artificial Data

In the following we evaluate the effectiveness of the EM algorithm for an indoor localization problem, first using artificially generated data: We consider a 2-class problem with $N_{AP} = 5$ access points. For each of the two locations $N = 1000$ training samples are drawn from a normal density with parameters according to Table 1 and then censored from the left with a threshold of $c = -100$ (dBm). A

Table 1. Mean and standard deviation of APs at 2 positions

AP index	AP1	AP2	AP3	AP4	AP5
$\mu_{1,i}$	-102	-103	-97	-89	-95
$\mu_{2,i}$	-105	-100	-99	-86	-101
$\sigma_{1,i}$	4.8	4.9	5.0	5.2	5.1
$\sigma_{2,i}$	5.0	4.8	4.8	5.4	5.0

Table 2. Classification error rate on artificial data

Method	Error rate (%)
Plain trng + recog	30.7
EM trng + plain recog	26.9
EM trng + censored recog	**22.5**
3-strongest APs	35.1
1-nearest neighbor	36.8

total of 200 test samples, 100 per location, are generated in the same manner.

We compared the classification error rate of the following schemes using one online measurement:

- Plain training (trng) + recognition (recog): ML parameter estimation is carried out assuming normally distributed, uncensored data. Also recognition is performed disregarding any censoring.
- EM trng + plain recog: ML parameter estimation in training accounts for the censored data using the proposed EM algorithm, while the presence of censored data is still disregarded in recognition.
- EM trng + censored recog: Training with the proposed EM algorithm and recognition employing eq. (22).
- 3-strongest APs: Select three strongest APs of each location in the training phase, then apply EM trng + censored recog.
- 1-nearest neighbor classification rule.

Table 2 clearly shows the superiority of the schemes which are aware of the censoring. Considering the presence of censored data in training improved the error rate from 30.7% to 26.9%, and a further improvement to 22.5% is obtained by accounting for censored data also in recognition. We can also see the important role of weak APs for the recognition accuracy: using only the three strongest APs raises the error rate to 35.1%.

5.3. Classification on Field Data

We conducted measurements on a floor of an office building consisting of 10 office rooms and a long aisle having an overall size of $12\,\mathrm{m}$ by $30\,\mathrm{m}$ (see Fig. 3). RSSI values were taken at 25 different positions, roughly evenly distributed, resulting in an average distance of $2.7\,\mathrm{m}$ between two locations. Two measurement campaigns were carried out using a smartphone, with 100 measurements taken per position per campaign. The data of the first were taken as training and the second for testing purposes. For the training data set, the percentage of unclipped observations, averaged over all APs which were observable at each location, was found to be 36.7%.

To compare our approach to a state-of-the-art system we implemented the algorithm from [3]. There, for each location ℓ_k the probability distributions of the 10 strongest APs are determined during the training phase and compared to those of the online measurements employing the Bhattacharyya coefficient. A 3-nearest neighbor rule

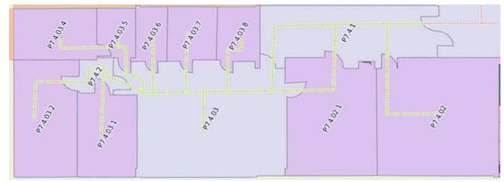

Fig. 3. Floor plan of area where field data has been conducted.

is then applied to decide on the user location. Further, we compared with the RADAR system [1], where classification is performed with a 3-nearest neighbor rule, employing the Euclidian distance. When applying our proposed algorithm, we used 5 online measurements and 3-nearest neighbors. Note that 5 online measurements were also employed in our implementation of the algorithm of [3].

Fig. 4 shows the cumulative distribution function (CDF) of the error as a function of the distance for each method. It is defined as the probability that the positioning error ϵ is lower than a certain distance d:

$$\mathtt{CDF}_\epsilon(d) = P(\epsilon \leq d) \quad d \geq 0. \tag{24}$$

The results in Fig. 4 show that the proposed method outperforms the other, especially for the 40% error quantile. Note, also, that the computational cost of the proposed method during the online phase is smaller than those of computing the Bhattacharyya distances between probability distributions [3] or the nearest-neighbor based [1] methods.

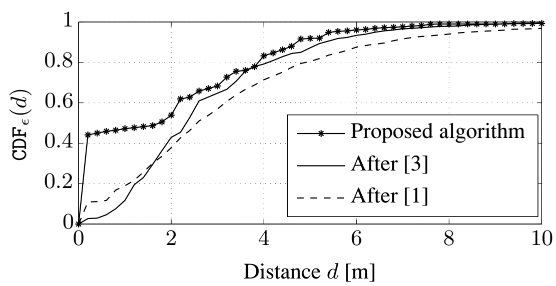

Fig. 4. CDF of the positioning error for different systems.

6. CONCLUSIONS AND RELATION TO PRIOR WORK

In this paper, an EM algorithm for estimating the parameters of a GAUSSIAN PDF in the presence of censored data was presented and analysed. Its convergence properties were studied and it was shown that the algorithm delivers unbiased and efficient estimates, achieving the CRAMER-RAO bound, which are novel results not addressed in prior work, such as [4] or [8]. Further, we have shown how classification has to be modified to account for censored data. The algorithm was then applied to WiFi-based indoor positioning. As far as we know, this is the first time a proper statistical treatment of RSSI values below the sensitivity threshold of the device has been carried out both in the offline training phase and the online classification stage.

The performance of the algorithms was first validated on artificially generated data and then on real field data of an experimental indoor positioning system. Improved positioning accuracy at low computational cost were observed, compared to other proposed algorithms.

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