MAP-ASSISTED KALMAN FILTERING

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ABSTRACT

We describe a method for incorporating map information to the Kalman filter that is commonly used in indoor and outdoor navigation systems. The map information is provided as a measurement to the Kalman filter to ensure the consistency of the Kalman estimate. The proposed method provides huge computational saving over common map matching algorithms that use the more computationally expensive particle filter. We show indoor navigation examples that highlight the efficiency of the proposed algorithm.

Index Terms— Kalman filter, estimation, GPS, navigation, constraints, map matching, WLAN, positioning.

1. INTRODUCTION

Navigation systems have recently witnessed significant progress with the widespread deployment of GPS positioning systems in civil and military applications [1]. The performance of outdoor positioning and navigation can even be enhanced by the incorporation of other satellite positioning systems, e.g., GLONASS and GALILEO. More recently, indoor positioning systems [2, 3] have also witnessed significant progress because of the wide range of applications, e.g., in location-based services and e-medicine. Indoor positioning systems typically use the wireless local-area network (WLAN) infrastructure because of the ubiquitous deployment of WLAN in business and commercial facilities. Positioning error in the order of few meters [4] is achievable for both indoor and outdoor positioning systems when sufficient resources are available.

A typical positioning/navigation system is illustrated in Fig. 1. It comprises an infrastructure (e.g., a satellite system in case of GPS, or access points in case of WLAN-based positioning) that provides reference signals, and a measurement engine (ME) that captures and processes the reference signals to provide ranging information to the positioning engine (PE). The positioning engine combines ranging information and sensor measurements to estimate the user position. The incorporation of sensors is enabled by the advances in MEMS technology and the availability of different sensors in many consumer electronics such as smart phones. Typical sensors that are used in positioning/navigation systems include accelerometer, magnetometer, and gyroscope. The PE typically incorporates constraints to guarantee consistent positioning estimate and/or smooth navigation trajectory. The constraints could be derived from the natural system limitations, e.g., maximum possible speed for pedestrian/vehicle, or from a priori position information, e.g., from a map or coarse positioning signals like radio broadcast.

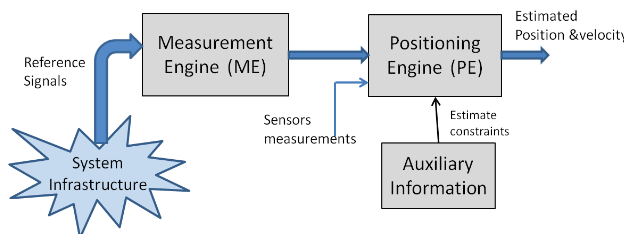

Fig. 1. A typical positioning/navigation system.

Typically, a Kalman filter [5] is used for the PE to combine the ranging information with sensor measurements. The states of the Kalman filter represent the position coordinate (with appropriate coordinate system), and the velocity components. In some cases, e.g., in GPS systems the extended Kalman filter is used to account for the nonlinear relation between the position and the ranging information. The incorporation of map information in positioning systems can be set as a postprocessing step where the estimate is refined to match the map constraints [6]. Recently, particle filters [7] have been used for map matching. Particle filters offer in general better performance than Kalman filtering but at the cost of a much higher complexity, which could be prohibitive in power-limited applications.

The incorporation of map constraints in the Kalman filter framework is more attractive in practical applications because of its reasonable complexity and because it is commonly used in most navigation applications for blending ranging and sensor measurements. It has been considered in some earlier work, e.g., [8]-[11] (the reader is referred to [12] for a comprehensive survey). In [8] the map matching measurements are considered only in route turns when the GPS signal is not available. The common factor between other related works [9]-[11] is to abstract the map to few segments and use a multiple hypothesis test to choose the most likely segment after the Kalman filtering. The general problem with this approach is the inconsistency in the computation of the covariance matrix of the Kalman estimate which impacts the processing of future observations.

In this work, we propose a new procedure that integrates the constraints posed by the map into the Kalman filter itself. These constraints are provided as measurements to the Kalman filter engine such that we have both consistent Kalman operation and satisfied map constraints. These map measurements are provided to the Kalman filter only if the Kalman estimate violates the map constraints. By properly choosing the measurement covariance, the Kalman estimate is refined to satisfy the map constraints and the corresponding covariance matrix is updated accordingly. The proposed procedure is shown to provide significant improvement over unconstrained Kalman filter at a marginal complexity cost.

2. KALMAN MODELING

The basic dynamic model of the Kalman filter has the form

$$\mathbf{x}_t = \mathbf{\Phi}_t \mathbf{x}_{t-1} + \mathbf{w}_t \quad (1)$$

$$\mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{v}_t \quad (2)$$

where $\mathbf{x}_t$ is the state vector and $\mathbf{z}_t$ is the measurement vector. $\mathbf{w}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$, and $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$, denote respectively the process and measurement noise at time t (where $\mathcal{N}(\mathbf{m}, \mathbf{A})$ denotes a normal distribution with mean $\mathbf{m}$ and covariance $\mathbf{A}$). $\mathbf{\Phi}_t$ is the state transition matrix and $\mathbf{H}_t$ is the measurement matrix at time t. The operation of the Kalman filter follows a standard form of equations [5] (which we do not include here because of the space limitation) that combine the old estimate from the model and the innovation from the new measurements. The amount of contribution of the new measurement to the Kalman filter is controlled by the covariance matrix $\mathbf{R}_t$ of the measurement. The Kalman filter is the minimum mean square estimator (MMSE) for the above problem, and it is the linear minimum mean square estimator (LMMSE) if the noise distributions are not normal [14]. In many cases, the measurements are not a linear function of the corresponding states, e.g., when the Time-of-Flight (TOF) is used for ranging information. The suboptimal extended Kalman filter is usually used to linearize the measurements around the old state estimate.

In positioning applications, the states of the Kalman filter always include the (x, y, z) position of the receiver and sometimes include the corresponding velocity and acceleration. The measurements contain the ranging information that depend on the system infrastructure. For example, the ranging information could be computed from the TOF to references with known position (e.g., satellites in GPS system) [1], by measuring the Angle-of-Arrival (AOA) to geometrically distributed references (e.g., in Radar applications)[13], or by measuring the strength of the received signal for known references as in indoor Wi-Fi positioning applications [3]. In all the above cases, the position estimation could be improved if a map is available to the user to refine the estimate according to the map information. In the following, we present a novel method for incorporating the map information into the Kalman Filter in a stable and computationally efficient way.

3. DIGITAL MAP ABSTRACTION FOR NAVIGATION

Digital maps, both indoor and outdoor, provide important information that could aid the positioning and navigation system. For positioning and navigation purposes, digital maps provide constraints on the position estimate, and in the absence of reference signals, e.g., in dead-reckoning scenarios, it could be combined with sensor information to compensate for the poor geometry. Consistency between the Kalman filter estimate (and the corresponding covariance matrix) and the map constraints is necessary for robust navigation.

In our model, the digital map is used to determine a feasible set of solutions for the Kalman estimate that is valid in most navigation scenarios. For example, in vehicular navigation buildings, shops, and parks are not in the feasible estimation set. For indoor navigation, if the sensors are indicating a moving user, then office spaces and labs are not in the feasible estimation set. Therefore, the first step for incorporating digital map information into Kalman-based navigation systems is to segment the map into two disjoint sets of feasible and infeasible areas. Because of the nature of the infeasible set in real maps, it is characterized by a set of non-connected areas. An example of this segmentation for a section of an indoor floor plan map is shown in Fig. 2. Each of the infeasible areas can be approximated by a polygon. Without loss of generality, these polygons can be assumed to be convex. A non-convex polygon can be approximated by more than one convex polygon. Note that, this segmentation is done offline

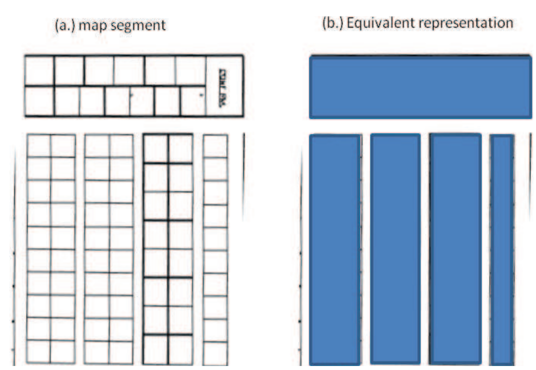

Fig. 2. Example of indoor map segmentation.

and each infeasible region is characterized the edge points of its perimeter, and the *line* equation for each edge.

For indoor navigation this map modeling can be done at the building level, i.e., using only exterior walls, if the indoor floorplan map is not available. This takes care of the error at the building boundary which is usually large due to the poor geometry of the access points [4].

4. MAP MATCHING

The map information is needed only if the Kalman estimate is in the infeasible region of the map. If the estimated position from the Kalman filter is in the feasible region, there is no need to do anything for map matching. If the estimated position is in the infeasible set, it needs to be projected onto the feasible set. This projection is set as an extra measurement to the Kalman filter. In the following discussion let P_i denote the i-th polygon of the infeasible regions, and B denote the set of infeasible points where

$$B = \bigcup_i \overline{P}_i \quad (3)$$

where $\overline{P}_i$ denotes the interior of P_i. The incorporation of map information into the Kalman filtering is done by the following steps:

1. Assume we have a map that contains the infeasible regions in the form of polygons. Each polygon is specified by its corners and its edges in the form of the line equation. The equation of the i-th edge of the k-th polygon (whose boundaries are $y_1^{(i,k)}$ and $y_2^{(i,k)}$) is expressed as
$$y = m_i^{(k)} x + c_i^{(k)} \quad (4)$$
and $y \in [y_1^{(i,k)}, y_2^{(i,k)}]$.

2. Assume that (x_t, y_t) is the position estimated at time t by the Kalman filter (based on the measurements, e.g., TOF, AOA, RSSI, .. etc).

3. If $(x_t, y_t) \notin B$, then output (x_t, y_t) as the final position and goto step 2 for the next time step.

4. If $(x_t, y_t) \in B$, identify the polygon $P_k \in B$ such that $(x_t, y_t) \in \overline{P}_k$.

5. Measure the distance between (x_t, y_t) and all the edges of P_k by projecting (x_t, y_t) onto all the polygon edges and pick the closest edge. Let the corresponding line equation be $y = m_j^{(k)} x + c_j^{(k)}$.

6. Input the line equation to the Kalman filter as a measurement. The measurement value is the constant $c_j^{(k)}$, and the corresponding measurement matrix ($\mathbf{H}_t$ in (2)) has all zeros except the entries that are multiplied by x and y in the state vector. The corresponding values in $\mathbf{H}_t$ are set to $-m_j^{(k)}$ and +1 respectively. Set the corresponding measurement covariance matrix to a very small value (compared to the covariance matrix of the Kalman estimate) to enforce the constraint.

7. Run the Kalman filter with the map measurement to obtain the refined estimation (x_t^+, y_t^+) after the map correction and output this estimation as the refined final position and move to the next time step.

Note that, the injection of the map measurement in step 6, with the appropriate scaling of the measurement covariance ensures the consistency of the Kalman processing while enforcing the map constraint. The updated covariance matrix of the Kalman estimate (x_t^+, y_t^+) at step 7, combines the confidence of the earlier measurement (x_t, y_t) and the map constraint.

Most of the complexity of the map matching algorithm is in steps 3 and 4, where we need to know whether or not the Kalman estimate is infeasible. For each $P_i \in B$, we need to check whether $(x_t, y_t) \in \overline{P}_i$. This can be done by using the ray casting algorithm [15]. The number of intersections with the polygon edges of a ray originating from the test point is counted. If the number of intersections is odd, then the point is inside, otherwise it is outside. To simplify computation, we choose a ray parallel to the y-axis which reduces the problem to direct substitution. The number of polygons that need to be checked could be significantly reduced by storing a lookup table with the centers of all the polygons, and run the ray casting algorithm only on the polygons that correspond to the closest centers to (x_t, y_t). Further, this list could by dynamically updated with new estimates such that the distance calculation to the centers is limited to the closest centers to the older estimate (x_{t-1}, y_{t-1}). If we have a total of M polygons with an average of N edges, then the overall complexity of the ray casting algorithm is $O(NM)$. This could be reduced to $O(N)$ by searching only over the local polygons in each step. The processing at each edge requires a single multiplication (in the line equation) and two comparisons with the edge boundaries.

5. EXAMPLES

The proposed map matching algorithm was integrated with the indoor algorithm positioning in [16] that uses RSSI measurement from the Wi-Fi access points for positioning estimate. In Fig. 3, we show two examples of the positioning performance with and without map matching. In the examples, the positioning is done using only the RSSI measurements (no sensors). The large swings in the raw Kalman estimation correspond to poor access point geometry which is more prominent at the border of the building. The proposed map matching algorithm provides significant improvement in the overall positioning estimation especially at segments of poor geometry. Further, it provides fine tuning of the estimate to improve the error margin in good geometry regions. The map matching procedure is most effective when the feasible set is much smaller than infeasible sets (which is often true in practice). On the down side, the map matching procedure exhibits under poor geometry a relatively long latency (up to few seconds) in updating the positioning estimate, which in general could be mitigated by incorporating sensor (e.g., accelerometer) measurements.

Fig. 3. Examples of incorporating the proposed algorithm in indoor navigation in office space.

6. CONCLUSION

We proposed a new algorithm for incorporating the map information in navigation systems. The map information is provided as correction measurements to the Kalman filter if the estimate does not satisfy the map constraints. This setup preserves the consistency the Kalman estimation and the corresponding covariance matrix. The proposed procedure offers more than an order of magnitude reduction in the complexity over the common particle filtering procedures which require the evaluation of thousands of particles over feasible map region.

Because of the proposed arrangement for map matching, it could be fine tuned to take advantage of the ecosystem of the positioning infrastructure without impacting the estimation consistency. For example, it could be turned off under good geometry or when the map is not available. The search space to determine the feasibility of the position estimate could be dynamically pruned based on the estimation history. Further, the procedure could also be extended to accommodate more than one Kalman filter by examining the closest N polygons of the infeasible set (rather than the closest one), and run multiple copies of the Kalman filter and choose the one with the smallest covariance matrix norm. This is a coarse approximation of particle filtering that provides a trade-off between complexity and accuracy which could be tailored to the computational resources of the underlying positioning system.

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