# DETECTION PERFORMANCE ANALYSIS OF AN ULTRASONIC PRESENCE SENSOR

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# ABSTRACT

An ultrasonic sensor that employs moving target processing based on differential echo processing and occupant tracking is considered for indoor occupant presence detection. We present a simple statistical model to analyze the presence detection performance of such a sensor. The probability distributions of the differential power signal are first obtained under vacancy and occupancy conditions. We then study the influence of occupant tracking and derive an upper bound on the probability of false alarm. Experimental data is used to verify that the presented analytical statistical models match actual distributions of the differential power.

*Index Terms*— Ultrasonic sensor, indoor presence detection, tracking, statistical detection model

# 1. INTRODUCTION

Ultrasonic sensors are commonly used for indoor occupancy sensing in lighting control systems. A grid of single-element ultrasonic sensors was used in [1] for occupant localization. An ultrasonic array sensor was described for presence detection [2] and occupant tracking [3]. In these systems, a pulsed sinusoidal waveform was transmitted and the received echoes were processed using moving target processing (based on differential echo processing) and occupant tracking to derive presence information. In this paper, we develop a statistical model characterizing the processed signal under occupancy and vacancy conditions, based on which the detection performance of such ultrasonic presence sensors is analyzed.

Typical metrics for quantifying detection performance under hypothesis testing are probability of detection and probability of false alarm. The performance of target detection systems, in particular radar systems, has been well studied [4] - [6]. The performance of a radar system depends strongly on the radar cross-section (RCS) characteristics of target and clutter. In the classic paper [4] by Swerling, simple statistical models for the RCS distribution of a target have been described. The four Swerling models for a fluctuating target correspond to the two choices for the probability density function (pdf) of target RCS and the two choices for echo fluctuation correlations over radar scans. An additional model is considered for the non-fluctuating target case. Clutter is typically characterized by a pdf with a given average RCS. Examples for modeling clutter are the Rayleigh, exponential, log-normal, and Weibull distribution [6, Chapter 2]. The pdf and average RCS have been empirically calculated for several types of clutter (e.g. land, sea) and targets.

We note that while several studies exist on radar statistical detection models, and their practical validity, these are largely considered for outdoor environments. To the best of our knowledge, similar analysis for determining the pdf and relevant parameters in an indoor environment does not exist, especially for ultrasonic occupant Frans M. J. Willems

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detection systems. In this paper, we obtain statistics based on a relevant parameter for the ultrasonic indoor presence sensor considered in [1] - [3]. A natural and relevant parameter for defining a pdf is the variance of the phase of the received signal. The variance of the phase can be easily used to model the movement of an occupant (e.g. small/large movement) and to model the stationarity of the environment (e.g. vibrations, air turbulence).

We describe the principle of operation of the ultrasonic presence sensor in Section 2. In Section 3, we obtain the pdf for the received differential power. Next, in Section 4, we analyze the false alarm probability under a typical family of tracking functions. In Section 5, we show results from data collected in a lab experiment to validate the analytical distribution models. Also, we analyze the impact of various design parameters on detection performance. Finally, in Section 6 we present conclusions.

# 2. ULTRASONIC PRESENCE SENSOR

We consider an ultrasonic presence sensor comprising of a transmitter, with center frequency  $f_c$ , and a single receiver at the same frequency, following the principle of operation as described in [2]. Over a transmission slot of duration  $T_s$ , the transmitter sends out pulsed sinusoids over a duration  $T_p$  followed by a quiet period. The waveform is repeated with period  $T_s$ , where  $T_s$  is a duration large enough so as to receive all echoes from within the detection region. Here we consider the two key processing steps at the receiver. The received echoes are processed by differential echo processing, i.e. echoes corresponding to two consecutive transmissions slots are subtracted. Based on the power of the differential echo signals, a coarse rangebin corresponding to moving objects is obtained. Finally an occupant tracking algorithm is used to improve the reliability of presence detection.

# 2.1. Pre-processing

The received signal is a continuous analog signal at frequency  $f_c$ , and is pre-processed by digitizing, filtering and down-mixing the signal to zero frequency. Let the sampling rate for digitizing the signal be  $f_s$  and the number of samples over which the signal is filtered be  $\Gamma$ . We will refer to each filtered group of samples as a range-bin  $\rho = 1, \ldots, R$ , where  $R = \left| \frac{T_s f_s}{\Gamma} \right|$ .

Let the received signal during the k-th cycle be given by  $u^{(k)}(\tilde{t})$  where  $\tilde{t}$  is the relative time with respect to the beginning of the k-th cycle. After pre-processing, the signal at range-bin  $\rho$  can be written as

$$r_{\rho}^{(k)} = \frac{1}{\Gamma} \sum_{\nu = (\rho-1)\Gamma+1}^{\rho\Gamma} u^{(k)} \left(\frac{\nu}{f_s}\right) \times e^{-2\pi i \frac{\nu f_c}{f_s}}, \ \rho = 1, \dots, R.$$

#### 2.2. Differential echo processing

In the differential echo processing step, we calculate the differential power for range-bin  $\rho = 1, 2, \dots, R$  and cycle k as

$$z_{\rho}^{(k)} = \left| r_{\rho}^{(k)} - r_{\rho}^{(k-1)} \right|^2.$$
(1)

# 2.3. Tracking processing

Assign to each range-bin  $\rho$ , a tracking score  $\Psi_{\rho}^{(k)}$  that indicates the confidence that there is an occupant at range-bin  $\rho$  and cycle k. Let us consider a family of tracking algorithms [3], [7] of the form

$$\Psi_{\rho}^{(k)} = f\left\{z^{(k)}\right\} + \beta \sum_{\eta} \tilde{G}_{\rho,\eta} \Psi_{\eta}^{(k-1)}$$
(2)

where  $f \{\cdot\}$  is a scoring function and  $0 \le \beta < 1$  is a tracking factor. Let  $\tilde{G}$  be a transition matrix with  $(\rho, n)$ -th element  $\tilde{G}_{\rho, \eta}$  indicating the probability of transition of the occupant from a given range-bin  $\eta$  to another range-bin  $\rho$ . Note that  $\tilde{G}$  is the assumed model for the occupant movement behavior.

For a given threshold value  $C_{th}$ , we define the false alarm probability as

$$P_{\text{fa}} = \lim_{k \to \infty} P\left(\max_{\rho} \left\{\Psi_{\rho}^{(k)}\right\} \ge C_{th} \left| H_{0}^{(k)}, H_{0}^{(k-1)}, \ldots\right) \\ = \lim_{k \to \infty} P\left(\bigcup_{\rho} \left\{\Psi_{\rho}^{(k)} > C_{th} \left| H_{0}^{(k)}, H_{0}^{(k-1)}, \ldots\right\}\right)$$
(3)

where  $H_0^{(k)}$  is the hypothesis that there is no occupant at cycle k. Now, let us consider that at cycle k+1 an occupant enters the room. We calculate the detection probability at cycle k+K (i.e. probability of detecting the occupant K - 1 cycles after he/she has entered the room). This probability is given by

$$P_{D}(K) = \lim_{k \to \infty} P\left(\max_{\rho} \left\{\Psi_{\rho}^{(k+K)}\right\} \ge C_{th} \\ \left|H_{1}^{(k+K)}, \dots, H_{1}^{(k+1)}, H_{0}^{(k)}, \dots\right),$$
(4)

where K-1 is the delay in detection and  $H_1^{(k)}$  is the hypothesis that there is an occupant at cycle k.

# 3. STATISTICAL DISTRIBUTION OF DIFFERENTIAL POWER

In this section, we present the pdf of  $z_{\rho}^{(k)}$  under two conditions: (i) clutter and noise; and (ii) single moving occupant, clutter and noise. The first condition corresponds to the situation that there is no presence or no movement from the occupant. The second condition considers the situation when there is a single occupant in the room.

#### 3.1. Probability distribution in clutter and noise

Let us consider that at cycle k and range-bin  $\rho$ , the ultrasonic sensor receives J echoes originating from the environment with a noise component  $n_{\rho}^{(k)}$ . The received signal can be written as

$$r_{\rho}^{(k)} = \sum_{j=1}^{J} A_j e^{i\theta_{k,j}} + n_{\rho}^{(k)}.$$
(5)

Here,  $A_j$  is the received amplitude of the *j*-th echo and  $\theta_{k,j}$  is the phase of the j-th echo received during the k-th cycle. Also,  $n_{\rho}^{(k)}$  is the complex noise component at cycle k and range-bin  $\rho$  given by

$$n_{\rho}^{(k)} = Q_n^{(k)}(\rho) + i I_n^{(k)}(\rho)$$

where  $Q_n^{(k)}(\rho)$  and  $I_n^{(k)}(\rho)$  are independent identically distributed (i.i.d.) with distribution  $\mathcal{N}(0, \sigma_n^2)$ .

Hence, from (1) and (5), the differential received power at cycle k and range-bin  $\rho$  is given by

$$z_{\rho}^{(k)} = \Big| \sum_{j=1}^{J} A_j e^{i\theta_{k,j}} + n_{\rho}^{(k)} - \sum_{j=1}^{J} A_j e^{i\theta_{k-1,j}} - n_{\rho}^{(k-1)} \Big|^2.$$

Let

$$Q_{c}^{(k)}(\rho) = \sum_{j=1}^{J} A_{j} x_{j}^{(k)} + Q_{n}^{(k)}(\rho) - Q_{n}^{(k-1)}(\rho) ,$$
  
$$I_{c}^{(k)}(\rho) = \sum_{j=1}^{J} A_{j} y_{j}^{(k)} + I_{n}^{(k)}(\rho) - I_{n}^{(k-1)}(\rho) , \qquad (6)$$

where  $x_{j}^{(k)} = \cos \theta_{k,j} - \cos \theta_{k-1,j}, y_{j}^{(k)} = \sin \theta_{k,j} - \sin \theta_{k-1,j}.$ 

We assume that for all  $j, A_j \approx A, x_j^{(k)}$  are i.i.d with mean  $\mu_x$ and variance  $\sigma_x^2$ , and  $y_i^{(k)}$  are i.i.d with mean  $\mu_y$  and variance  $\sigma_y^2$ This is a reasonable assumption because for a given range-bin, the echoes originate from the same large object (e.g. table, floor, etc). Then by using the central limit theorem, we have that

$$\sum_{j=1}^{J} A_j x_j^{(k)} \to \mathcal{N} \left( J A \mu_x, J A^2 \sigma_x^2 \right),$$
$$\sum_{j=1}^{J} A_j y_j^{(k)} \to \mathcal{N} \left( J A \mu_y, J A^2 \sigma_y^2 \right).$$
(7)

Furthermore, the phase of an echo originating from static object is almost identical between consecutive pulses. Hence, we assume that for all j and for any distribution of  $\theta_{k-1,j}$ , then  $\theta_{k,j}$  is distributed with distribution  $\mathcal{N}\left(\theta_{k-1,j}, \sigma_{\theta}^{2}\right)$ . Note that  $\sigma_{\theta}^{2}$  depends on the phase variation between consecutive pulses. We have that for small duration  $T_s$ , then  $\sigma_{\theta}^2 \ll 1$ . Thus, we have

$$\mu_x = \mu_y = 0 \text{ and } \sigma_x^2 \approx \sigma_y^2 \approx \frac{\sigma_\theta^2}{2}.$$
 (8)

Then, by using (7) and (8), we have that (6) becomes

$$Q_c^{(k)}(\rho) \to \mathcal{N}\left(0, \sigma_c^2\right), \text{ and } I_c^{(k)}(\rho) \to \mathcal{N}\left(0, \sigma_c^2\right),$$
 (9)

where  $\sigma_c^2 = 2\sigma_n^2 + \frac{JA^2\sigma_{\theta}^2}{2}$  is the clutter and noise power. Thus, using (9), we have the probability distribution of the differential power at cycle k and range-bin  $\rho$  given by

$$p_{cn}\left(z_{\rho}^{(k)}|\sigma_{c}^{2}\right) = \frac{1}{2\sigma_{c}^{2}}e^{-\frac{z_{\rho}^{(k)}}{2\sigma_{c}^{2}}}.$$
(10)

### 3.2. Probability distribution with moving occupant, clutter and noise

Now, let us consider that at cycle k and range-bin  $\rho$ , the sensor receives an echo from a moving occupant, J echoes originating from the environment and a noise component. Hence

$$r_{\rho}^{(k)} = Be^{i\phi_k} + \sum_{j=1}^{J} A_j e^{i\theta_{k,j}} + n_{\rho}^{(k)}$$

Here, B and  $\phi_k$  are the amplitude and phase of the received echo originating from the moving occupant, respectively.

Hence, we have that the differential received power at cycle k and range-bin  $\rho$  is given by

$$z_{\rho}^{(k)} = \left| Be^{i\phi_{k}} + \sum_{j=1}^{J} A_{j}e^{i\theta_{k,j}} + n_{\rho}^{(k)} - Be^{i\phi_{k-1}} - \sum_{j=1}^{J} A_{j}e^{i\theta_{k-1,j}} - n_{\rho}^{(k-1)} \right|^{2},$$
$$= \left( B \left[ \cos\phi_{k} - \cos\phi_{k-1} \right] + Q_{c}^{(k)}\left(\rho\right) \right)^{2} + \left( B \left[ \sin\phi_{k} - \sin\phi_{k-1} \right] + I_{c}^{(k)}\left(\rho\right) \right)^{2}.$$
(11)

Due to space limitations, we omit the derivation for the probability distribution  $p_{ocn}\left(z_{\rho}^{(k)}|B, p_{\Delta\phi}, \sigma_c^2\right)$  of the received power at cycle k and range-bin  $\rho$  due to the echo of a moving occupant with amplitude B and phase distribution  $p_{\Delta\phi}$ , and provide the final result,

$$p_{cn}\left(z_{\rho}^{(k)}|\sigma_{c}^{2}\right)E_{\Delta\phi}\left[e^{-\frac{4B^{2}\sin^{2}\left(\frac{\Delta\phi}{2}\right)}{2\sigma_{c}^{2}}}I_{0}\left\{\frac{\left|2B\sin\frac{\Delta\phi}{2}\right|}{\sigma_{c}}\frac{\sqrt{z_{\rho}^{(k)}}}{\sigma_{c}}\right\}\right].$$
(12)

Note that if for a given range-bin  $\rho$ , the clutter power is dominant, and thus  $\sigma_c^2 \approx \frac{JA^2 \sigma_{\theta}^2}{2}$ , then the term  $\frac{\left|2B\sin\frac{\Delta\phi}{2}\right|}{\sigma_c} \approx \left|\frac{4B}{A\sqrt{J}}\right| \frac{\sin\frac{\Delta\phi}{2}}{\sigma_{\theta}}$ . Further, we have that A and B are highly correlated because they both are proportional to the transmitted power and so the ratio  $\left|\frac{4B}{A\sqrt{J}}\right|$  is independent of the transmitted power. Hence, in clutter-dominant environments,  $p_{ocn}\left(z_{\rho}^{(k)}|B,p_{\Delta\phi},\sigma_c^2\right)$  is independent of the transmitted power.

### 4. TRACKING ALGORITHM AND PROBABILITY OF FALSE ALARM

In this section, we analyze the effect in the probability of false alarm of the tracking algorithm as defined in (2).

#### 4.1. Statistics of objects in the clutter

Consider the *R* possible range-bins. The probability that there is an echo from an object in a given range-bin  $\rho$  is assumed to follow a binomial distribution, i.e. the probability that there is an echo coming from an object at range-bin  $\rho$  is equal to  $p_{ec} (e_{\rho} = 1) = \tilde{p}_{ec}$  where  $e_{\rho} = 1$  indicates that there is an echo originating from range-bin  $\rho$ .

Furthermore, we assume that (i) the location distribution of objects in the clutter does not change with time and (ii) the echo of each object is received within a single range-bin. These assumptions are reasonable because (i) the room environment does not change substantially within short time durations and (ii) in general, a range-bin is chosen larger than the pulse duration  $T_p$ . Additionally, we assume a simplified model for the amplitude of the received echoes. Whenever there is an echo originating from range-bin  $\rho$ , the received amplitude  $A_{\rho}$  of this echo is given by  $A_{\rho} = A_0 \rho^{-\alpha}$  where  $A_0$  is the amplitude of the transmitted waveform and  $\alpha$  is the attenuation factor (e.g.  $\alpha = 1$ ).

### 4.2. Occupancy model

Let us consider a simple occupancy model, with initial vector  $\boldsymbol{g} = \begin{bmatrix} g_1 & g_2 & \dots \end{bmatrix}$  where  $g_{\rho}$  indicates the probability that an occupant enters in range-bin  $\rho$ . Let the actual transition probability matrix be denoted by  $\boldsymbol{G}$ , with  $(\rho, n)$ -th element  $G_{\rho,\eta}$  being the

probability of transition of the occupant from a given range-bin  $\eta$  to another range-bin  $\rho$ . We assume that the echo from the moving occupant is received within a single range-bin.

#### 4.3. Probability of false alarm

We choose a simple scoring function that is approximately proportional to the log-likelihood function [7] (i.e. logarithmic of ratio between (12) and (10)). Simplifying, we have

$$f\left\{z_{\rho}^{(k)}\right\} = \frac{z_{\rho}^{(k)}}{\sigma_c^2(\rho, e_{\rho})}$$

where

$$\sigma_c^2(\rho,e_\rho) = \begin{cases} 2\sigma_n^2 + \frac{JA_\rho^2 \sigma_\theta^2}{2} \ , \ e_\rho = 1 \\ 2\sigma_n^2 \ , \ \text{otherwise} \end{cases}$$

is the clutter and noise power at range-bin  $\rho$ . Further, we assume that  $\hat{G} = G$ , i.e. we have complete knowledge of the behavior of the occupant.

We can further upper bound (3) by

$$P_{\text{fa}} \le \lim_{k \to \infty} \sum_{\rho} P\left(\Psi_{\rho}^{(k)} > C_{th} \middle| H_0^{(k)}, \ldots\right)$$
(13)

where

$$P\left(\Psi_{\rho}^{(k)} \ge C_{th} \middle| H_{0}^{(k)}, \ldots\right) = \int_{\Psi_{\rho}^{(k)} \ge C_{th}} p\left(\Psi_{\rho}^{(k)} \middle| H_{0}^{(k)}, \ldots\right) d\Psi_{\rho}^{(k)}$$
(14)

and  $p\left(\Psi_{\rho}^{(k)}|H_{0}^{(k)},\ldots\right)$  is the probability distribution of tracking score at cycle k and range-bin  $\rho$ . This probability distribution can be calculated as

$$\int_{\Omega} p_{cn} \left( z_{\rho}^{(k)} | \sigma_c^2(\rho, e_{\rho}) \right) \prod_{s=0}^{k-1} \prod_{\eta} p_{cn} \left( z_{\eta}^{(s)} | \sigma_c^2(\eta, e_{\eta}) \right) \prod_{\varrho} p_{ec} \left( e_{\varrho} \right) d\Omega$$
(15)

where  $e_{\rho} = 1$  if and only if there is an echo originating from rangebin  $\eta$ , and  $\Omega$  is the region that satisfies

$$\Psi_{\rho}^{(k)} = \frac{z_{\rho}^{(k)}}{\sigma_c^2(\rho, e_{\rho})} + \sum_{s=0}^{k-1} \beta^{k-s} \sum_{\eta} \left[ \boldsymbol{G}^{k-s} \right]_{\rho,\eta} \frac{z_{\eta}^{(s)}}{\sigma_c^2(\eta, e_{\eta})} \ge 0.$$

Using the probability distribution defined in (10) and the change of variables

$$\begin{split} z_{\rho}^{(k)} &= \sigma_{c}^{2}(\rho, e_{\rho}) \Biggl( \Psi_{\rho}^{(k)} + \sum_{s=0}^{k-1} \beta^{k-s} \sum_{\eta} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta} \frac{z_{\eta}^{(s)}}{\sigma_{c}^{2}(\eta, e_{\eta})} \Biggr), \\ z_{\eta}^{(s)} &= z_{\eta}^{(s)}, \ \eta \neq \rho \text{ or } s \neq k, \\ e_{\eta} &= e_{\eta}, \ \forall \eta, \end{split}$$

with Jacobian equal to  $\frac{1}{\sigma_c^2(\rho, e_\rho)}$ , then we can rewrite (15) as

$$\int_{\Omega} \frac{e^{-\frac{1}{2}\Psi_{\rho}^{(k)} - \sum_{s=0}^{k-1}\sum_{\eta} \left(1 - \beta^{k-s} [\mathbf{G}^{k-s}]_{\rho,\eta}\right) \frac{z_{\eta}^{s}}{2\sigma_{c}^{2}(\rho,e_{\eta})}}}{2\prod_{s=0}^{k-1}\prod_{\eta} 2\sigma_{c}^{2}(\eta,e_{\eta})} \prod_{\varrho} p_{ec}(e_{\rho}) d\Omega$$

and so the probability in (14) becomes

$$\begin{pmatrix}
\sum_{s=0}^{k-1} \prod_{\eta} \frac{1}{1 - \beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta}} \\
- \left[ \sum_{s=0}^{k-1} \sum_{\eta} \frac{\beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta}}{1 - \beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta}} e^{-\frac{C_{th}}{2\beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta}}} \\
\times \prod_{v=0}^{k-1} \prod_{\varrho \neq \eta} \frac{\beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta}}{\beta^{k-s} \left[ \mathbf{G}^{k-s} \right]_{\rho,\eta} - \beta^{k-v} \left[ \mathbf{G}^{k-v} \right]_{\rho,\varrho}} \right]. \quad (16)$$

Note that the distribution in (16) is independent of the distribution of objects in the clutter and noise (i.e.  $\sigma_c^2(\rho, e_\rho)$ ). This is because of the definition of the tracking function in (2).

Finally, using (16), we have an upper bound on the false alarm probability from (13). Note that obtaining an expression for the detection probability using (12) is intractable and hence we do not provide analysis, but resort to simulations.

### 5. RESULTS

### 5.1. Experimental results

We first compare the analytical probability distribution of the differential power under noise and clutter, (10), with the probability distribution obtained from real measurements. These measurements were obtained in an experimental office lab using a prototype ultrasonic sensor [3] installed in a ceiling-mounted configuration. We collected the received signal at a given range-bin  $\rho = 16$  (a table was situated in this range-bin). The received signal was pre-processed to obtain  $r_{16}^{(k)}$  and the differential power,  $z_{16}^{(k)}$ . Note that the variance of the clutter,  $\sigma_c^2$ , increases with the separation between pulses. This is because the environment is more likely to change with larger spacing between pulses. Hence, we performed several experiments with different separation between pulses,  $T_s = 0.05, 0.1, 0.5$  and 1 seconds. In Fig. 1, we plot the distribution of the normalized differential power at range-bin 16,  $z_{16}^{(k)}/(r_{16}^{(k)})^2$ , and the distribution from (10) for each case. The standard deviation of the clutter,  $\sigma_c(16, 1)$ , was estimated from the measurements. We note that both distributions are quite close, validating our model distribution.



Fig. 1. Comparison between analytic and real pdf.

#### 5.2. Numerical results

We then performed a Monte Carlo simulation using different values for tracker parameters: K and  $\beta$ . We evaluate the performance for each set of parameters by using the false alarm probability and detection probability as performance metrics. The parameters chosen for our simulations are: 5 range-bins (i.e. R = 5); initial occupancy distribution,  $g = [0\ 0\ 0\ 0\ 1]$ ; transition probability matrix

$$\boldsymbol{G} = \begin{bmatrix} 0.75 \ 0.25 \ 0 \ 0 \ 0 \\ 0.25 \ 0.5 \ 0.25 \ 0 \\ 0 \ 0.25 \ 0.5 \ 0.25 \ 0 \\ 0 \ 0 \ 0.25 \ 0.5 \ 0.25 \\ 0 \ 0 \ 0 \ 0.25 \ 0.5 \ 0.25 \\ 0 \ 0 \ 0 \ 0.25 \ 0.75 \end{bmatrix};$$

clutter power,  $\frac{JA^2\sigma_{\theta}^2}{2} = 0.04$ ; noise power  $2\sigma_n^2 = 0.01$ ; amplitude of echo originating from moving occupant, B = 1;  $p_{\Delta\phi}$  follows distribution  $\mathcal{N}(0,1)$  (in radians); probability of an echo is independent of neighboring locations, and  $\tilde{p}_{ec} = 0.5$ .



Fig. 2. Performance comparison for different values of K.



Fig. 3. Performance comparison for different values of  $\beta$ .

We simulated  $10^6$  instances for each set of parameters. First, we fixed the values of k = 100 and  $\beta = 0.9$ . Note that when k = 100, then  $P_{\rm fa}$  converges to its maximum value. We analyzed  $P_D(K)$  and  $P_{\rm fa}$  for different delays K = 1, 2, 3 and 5. We can see in Fig. 2 that a larger K has a higher  $P_D(K)$  but also a larger delay (i.e. large K). We consider K = 3 cycles to be a reasonable delay for our system.

Finally, we consider the effect of different values of  $\beta = 0.2, 0.4, 0.8, 0.9$  for K = 3. It can be seen in Fig. 3 that the relation between  $P_D(3)$  and  $P_{\text{fa}}$  improves for large  $\beta$  (i.e. large memory of past observations).

# 6. CONCLUSIONS

We presented an analytical model for the pdf of the differential power in an ultrasonic presence sensor. We validated our model for an empty room using experimental data. We further analyzed the signal output under a simple class of tracking algorithms. The false alarm probability was analyzed, and by using Monte Carlo simulations, the effect of different tracker parameters on the relation between the false alarm probability and detection probability was studied. This analytical work may be extended to analyze detection performance of radar systems [8] - [10] for other indoor applications.

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