SOFT-THRESHOLDING ORTHOGONAL MATCHING PURSUIT FOR EFFICIENT SIGNAL RECONSTRUCTION

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ABSTRACT

In this paper, we propose a *Soft-thresholding Orthogonal Matching Pursuit* (ST-OMP) technique for efficient signal reconstruction in compressive sensing applications. The proposed ST-OMP recovers less significant signal elements using a low-complexity procedure without sacrificing much reconstruction quality. We apply the proposed ST-OMP in systems powered by non-deterministic renewable energy sources. The threshold of employing the efficient reconstruction is made dynamically adjustable according to the performance requirements and energy levels. Simulation results demonstrate that the ST-OMP can achieve good recovery performance while significantly reducing the energy consumption as compared to the original OMP implementation.

***Index Terms*—** Compressed sensing, Signal reconstruction, Energy efficiency, Soft-thresholding, Renewable Energy.

1. INTRODUCTION

Many embedded systems have to be operated under scarce physical resources. One effective way to achieve satisfactory performance is to reduce unnecessary/redundant data to be processed. Conventional sensing techniques acquire far more data than required, i.e., a large amount of data are irrelevant and thus can be thrown away without affecting the performance. For this consideration, Compressive Sensing (CS) [1] has gained significant interest recently due to its capability to process sparse signals, thereby reducing physical resources needed for data acquisition and transmission. Existing work include applying CS in wireless communications [2], data compression [3], imaging [4] and other related areas.

While showing great potential for embedded computing, CS-based techniques still face a critical problem of recovering original signals from relatively few measurements in a reliable and efficient way. Much research effort has been directed towards investigating reconstruction algorithms [5], [6] for this purpose. Two major approaches are the ℓ^1-minimization and greedy algorithms. Orthogonal Matching Pursuit (OMP) [7] as a greedy algorithm is getting popular for its relatively low complexity. However, most existing work focus mainly on algorithm optimization without considering domain-specific constraints. For example, one emerging area to apply CS is to exploit renewable energy sources in autonomous and distributed wireless sensor networks. Most renewable energy sources are non-deterministic due to the inherent uncertainties in environmental conditions. Therefore, it is critical to improve the energy efficiency of signal reconstruction.

In the iterative OMP algorithm, last rounds of iteration introduce large computational complexity as more complicated matrix operations are involved. However, these iterations usually recover less significant elements of original signal. On the other hand, many sensing applications do not require precise signal acquisition but are quite error-tolerant as long as sensing tasks can be fulfilled. Thus, there exist some interesting tradeoffs between computational complexity, performance, and cost in signal reconstruction. These tradeoffs can be exploited for applications that are operated under severe resource constraints (e.g., time, energy). In this paper, we propose a *Soft-thresholding OMP* technique (ST-OMP) that leverages these tradeoffs for efficient signal reconstruction under renewable energy. Since the last iterations are costly while only recovering less significant elements, we can replace these iterations with a low-complexity procedure to reduce recovery cost without sacrificing much quality. The threshold at which the algorithm switches is adjusted dynamically in accordance with performance requirements and available energy levels, which may be changing at runtime. The proposed ST-OMP allows efficient recovery of less significant but computation-intensive signal components while maintaining high reconstruction quality. This is a feature of great significance for realtime self-powered embedded systems.

2. PRELIMINARIES

Compressive sensing can be expressed mathematically by multiplying the original signal $X \in R^N$ with a measuring matrix $\Phi \in R^{M\times N}$ to obtain the measurement $Y \in R^M$, i.e.,

$$Y = \Phi * X, \tag{1}$$

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where X is assumed to have K non-zero components and matrix Φ is usually a random Gaussian or Bernoulli induced matrix [1]. Usually, $K < M \ll N$ for CS to make sense; that is, fewer measurements are taken while still guaranteeing reliable recovery of all K non-zero components in vector X.

One classical approach of signal recovery is Basis-Pursuit (BP) [5] that applies convex optimization in an over-complete dictionary. However, BP approach typically involves large computational complexity. Another approach is based on greedy algorithms, among which OMP-based reconstruction is getting popular because of its simple geometric interpolation and relatively low computational complexity. The procedure of OMP algorithm can be summarized as follows,

1. Initialize: $k = 1$; $I_0 = \emptyset$; $R_0 = Y$; $\alpha_0 = \Phi^T Y$;
2. $i = \text{argmax}_i(|\alpha_i|)$ and $I_k = I_{k-1} \cup i$
3. $\widehat{X}_k = \text{argmin}_z(\| Y - \Phi_{I_k} Z \|)$
4. $R_k = Y - \Phi_{I_k}\widehat{X}_k$ and $\alpha_k = \Phi^T R_k$
5. Increment k and return to step 2 if $k < K$

In the above algorithm, $I_k \in R^k$ and its complementary set $\widetilde{I}_k \in R^{N-k}$ stand for index set of selected and remaining columns in matrix Φ. At each iteration, the column most correlated with current residual R_k is chosen from matrix $\Phi_{\widetilde{I}_k}$ and its coordinate is added into I_k. Then, the algorithm minimizes residual error in step 3 by solving the equation [8] as

$$\widehat{X}_k = \Phi^\dagger_{I_k} Y, \qquad s.t. \quad \Phi^\dagger_{I_k} = (\Phi^T_{I_k}\Phi_{I_k})^{-1}\Phi^T_{I_k}, \qquad (2)$$

where $\widehat{X}_k$, $R_k \in R^M$, and $\alpha_k \in R^{N-k}$ are current estimate of signal X, residual vector, and inner product of $\Phi^T_{\widetilde{I}_k}$ and R_k, respectively. After K iterations, K elements are estimated with index set showing coordinates of these elements in X.

3. THE PROPOSED SOFT-THRESHOLDING OMP

3.1. Motivation

The computational complexity of OMP algorithm increases significantly as iteration order goes up [9]. The reason is that updating R_k and α_k needs more operations as I_k increases in size. Our past work [10] also observed this trend.

Interestingly, the last rounds of iteration usually recover less significant elements of the signal. This can be seen from the fact that the index vector I_k always selects the element with the largest absolute value from the current vector α_k (see step 2 of the OMP algorithm), which is the inner product of matrix Φ^T and residual vector R_k. Initially, the residual vector R_k is the measurement vector $Y = \Phi X$, thus $\Phi^T R_k = \Phi^T \Phi X$, where the measurement Φ is a random matrix following the Restricted Isometry Property [5], and the vectors in matrix Φ behave like an orthogonal basis. The matrix $\Phi^T\Phi$ is a symmetric matrix, whose diagonal elements are the dot product of same vectors, whereas the other elements are the dot product of two incoherent vectors. Thus the elements in the diagonal of $\Phi^T\Phi$ usually have the largest values, and $\alpha_k = \Phi^T\Phi X$ is likely to preserve the weight pattern of the elements in the original signal X. Consequently, the first iteration of the OMP algorithm will recover the element with the largest weight in the original signal X.

In the subsequent iterations, vector $R_k = \Phi Y - \Phi_{I_k}\Phi^\dagger_{I_k}\Phi X$ is updated according to index vector I_k. Consider vector

$$\Phi^T R_k = \Phi^T\Phi X - \Phi^T\Phi_{I_k}(\Phi^T_{I_k}\Phi_{I_k})^{-1}\Phi^T_{I_k}\Phi X. \qquad (3)$$

It is known that $\Phi^T_{I_k} \in \Phi^T$ and $\Phi^T_{I_k} R_k = \Phi^T_{I_k}\Phi X - \Phi^T_{I_k}\Phi X = 0$. As a result, selected elements in X will not be picked up again. On the other hand, $\Phi = \Phi_{\widetilde{I}_k} \bigcup \Phi_{I_k}$ and thus

$$\Phi^T_{\widetilde{I}} R_k = \Phi^T_{\widetilde{I}_k}\Phi X - \Phi^T_{\widetilde{I}_k}\Phi_{I_k}[(\Phi^T_{I_k}\Phi_{I_k})^{-1}\Phi^T_{I_k}\Phi_{\widetilde{I}_k}\bigcup I]X.$$

As $\Phi^T_{\widetilde{i}}$ is incoherent with all the vectors in matrix Φ_{I_k}, $\Phi^T_{\widetilde{I}_k}\Phi_{I_k}$ is a random matrix with much smaller diagonal elements than matrix $\Phi^T\Phi$. Therefore, $\Phi^T_{\widetilde{I}} R_k$ is mainly determined by $\Phi^T_{\widetilde{I}_k}\Phi X$ and remaining non-zero elements in signal X are likely to be recovered in the order of their weights.

To illustrate this observation, we run an experiment to recover a sparse signal with non-zero elements of values randomly chosen from 1 to 100. Results in Fig. 1 show how the signal is recovered with respect to iteration order. We can clearly see that elements with larger values are likely to be recovered earlier. For example, elements with a value of 90 are recovered before 20^{th} iteration, whereas elements with a value of 10 are most possibly recovered after 40^{th} iteration.

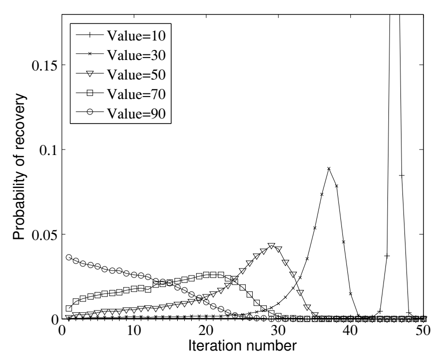


Fig. 1. Statistical results of signal recovery pattern.

3.2. Soft-thresholding OMP

Based on the above observation, we propose Soft-thresholding OMP (ST-OMP) for efficient signal recovery. Note that the proposed ST-OMP is a general approach that can be utilized to address various resource constraints. In this paper, we will focus on improving energy efficiency due to that energy is a major resource constraint in many embedded systems.

We consider a scenario where the signal reconstruction needs to be finished with limited energy supply. A challenging problem is that available energy may be dynamically changing if the computation is powered by renewable energy sources, in particular when available energy is insufficient to support signal reconstruction using OMP. The proposed ST-OMP is very effective to deal with this challenging problem.

At the beginning of each signal reconstruction task, the ST-OMP recovers one non-zero element in the signal X at each iteration using the same procedure in the OMP algorithm. Assume the signal to be recovered is K-sparse and its $(K-L)$ non-zero elements can be recovered given a certain renewable energy level. After that, the energy becomes insufficient to support the remaining iterations. Note that the energy consumption of each iteration increases with the order of iterations because the last iterations involve more computations as discussed in Section 3.1. The proposed ST-OMP switches to a low-complexity recovery algorithm, where iteration threshold L for making this switch is determined by the available energy, i.e., it is adjustable in accordance with renewable energy while trying to achieve the highest reconstruction quality as possible. This is expressed formally as

$$L = \text{argmin}_l (E_{OMP}(K-l) + \triangle E(l) \leq E_{avl}), \quad (4)$$

where $E_{OMP}(K-l)$ is the amount of energy consumed in the first $(K-l)$ rounds of iteration, $\triangle E(l)$ is the expected energy consumption of the low-complexity algorithm (see below), and E_{avl} is the available energy at the beginning of each signal reconstruction task. Note that E_{avl} is a time-dependent variable as in the case of renewable energy sources.

Once L determined in (4) is reached, we store the value of latest recovered element and α from the $(K-L)^{th}$ iteration,

$$\begin{cases} \widehat{X}_{temp} = \widehat{X}(K-L), \\ \alpha_{temp} = \alpha_{K-L}. \end{cases} \quad (5)$$

Signal reconstruction is then switched to a low-complexity algorithm. Instead of choosing only the largest element at each iteration like OMP algorithm, we choose the L largest elements (absolute values) in vector α_{temp} and add the corresponding coordinates into the index set $I_{(K-L)}$, i.e.,

1. Initialize: $count = 1$; $\alpha = \alpha_{temp}$
2. i = $\text{argmax}_{j \not\subseteq I}(|\alpha_j|)$
3. $I_{K-L+count} = I_{K-L+count-1} \cup i$
4. Increment $count$ and return to step 2) if $count \leq L$

By doing so, signal reconstruction avoids energy-consuming multiplication operations between matrix $\Phi^T \in R^{N \times M}$ and vector $R_k \in R^M$ to update α_k. Furthermore, there is no need to keep updating R_k, which involves matrix inversions. After the coordinates of the remaining L non-zero elements are determined, the corresponding entries in the estimated signal are assigned with the value of the latest recovered $\widehat{X}_{temp}$, i.e.,

$$\widehat{X}(I_{K-L+i}) = \widehat{X}_{temp}, \text{ for } i = 1 : L. \quad (6)$$

Note that the above low-complexity algorithm will introduce recovery errors (relative to OMP). This is because the recovered elements might not necessarily result in the minimal residual errors in signal reconstruction (see step 3 of the OMP algorithm in Section 2). However, as mentioned in Section 3.1, since the most significant elements of the signal have already been recovered in the previous $(K-L)$ rounds of iteration, the remaining signal elements recovered by the low-complexity algorithm are likely to be less significant, and thus the performance degradation is minimal. As more errors are introduced when the threshold L becomes larger (e.g., due to insufficient energy), a upper limit $L \leq L_{\text{MAX}}$ can be enforced for signal reconstruction. This sets up a lower bound on the recovery quality, which can be determined by the specific performance requirement of an application. In practice, signal reconstruction will fail if L obtained from (4) is larger than L_{MAX} due to insufficient renewable energy.

To apply the proposed ST-OMP, we need to know the available energy E_{avl} at the beginning of each signal reconstruction task, and energy consumption $E_{OMP}(K-l)$ and $\triangle E(l)$ related to the signal reconstruction. Existing work [11, 12] have shown that E_{avl} can be estimated by some prediction algorithms with sufficient accuracy. These algorithms operate at a much lower rate (e.g., once per hour) and thus the energy overhead can be ignored. On the other hand, $E_{OMP}(K-l)$ and $\triangle E(l)$ will depend upon a specific hardware implementation. It is usually very difficult (and in most cases unnecessary) to determine the analytical relationship between these energy components and the process of signal recovery. Rather, they can be estimated through simulations or pre-operation hardware measures, and the results can be stored in a look-up table (LUT) for runtime comparison with E_{avl}.

4. EVALUATION

We have synthesized the proposed ST-OMP using Synopsys Design Compiler for a 65nm CMOS process at a clock frequency of 500MHz. The original OMP was also implemented for the purpose of comparison. The computations in these implementations were verified by MATLAB simulation results.

We first evaluate the effect of soft thresholding on performance and computational complexity of signal reconstruction. Sparse signals with different numbers of non-zero elements (60, 80, 100 and 120) were recovered using the proposed ST-OMP by deliberately changing threshold L from 5 to 55. The experiments were repeated 10^5 times to obtain statistical results of recovery accuracy (i.e., identifying the non-zero elements) and computational complexity (in terms of the number of arithmetic operations needed to complete the recovery). As shown in Fig. 2, reconstruction accuracy shows almost no degradation when L is small. As L increases, less than 10% of performance degradation is observed, which is acceptable for most CS applications (see the case study below). On the other hand, ST-OMP achieves significant reduction in computational complexity (more than 60%), in particular when sparsity of signal is high because the low complexity procedure recovers more elements in such signals.

To visualize performance degradation of ST-OMP, a gray image of size 500×1000 pixels was recovered under different values of L. As shown in Fig. 3, the percentage in-

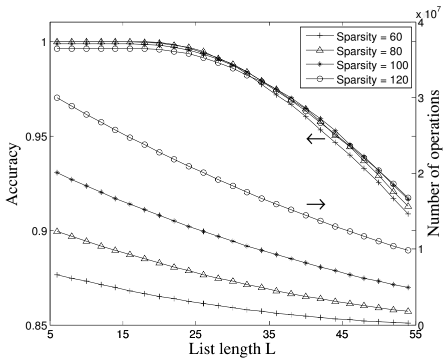
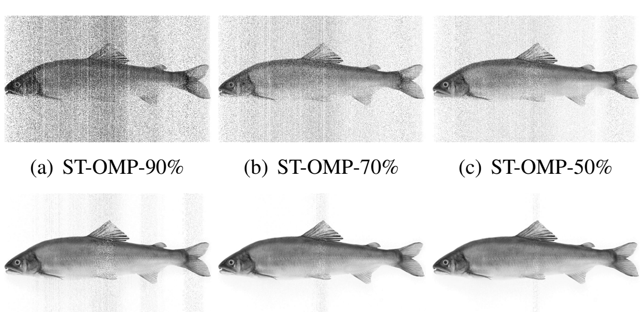


Fig. 2. Recovery accuracy and computational complexity for signals with different sparsities and threshold L.

Table 1. Hardware measurements of the proposed ST-OMP.

Algorithm	Recovery time	Energy consumption
ST-OMP-90%	0.16 ms	0.0205 mJ
ST-OMP-70%	1.50 ms	0.1944 mJ
ST-OMP-50%	4.48 ms	0.5822 mJ
ST-OMP-30%	9.52 ms	1.2382 mJ
ST-OMP-10%	16.95 ms	2.2031 mJ
OMP	21.57 ms	2.8046 mJ

dicates how many non-zero elements were recovered using low-complexity procedure in the second phase of ST-OMP. Even at a level of 90%, the quality of recovered image is still acceptable, i.e., the object can be sufficiently identified. Table 1 shows the results of reconstruction time and energy consumption obtained from the synthesized hardware implementation at clock frequency of 500MHz. These results further validate that ST-OMP can significantly reduce computational complexity and energy consumption while maintaining high reconstruction quality. This is very important for real-time energy-constrained embedded system.



(a) ST-OMP-90% (b) ST-OMP-70% (c) ST-OMP-50%

(d) ST-OMP-30% (e) ST-OMP-10% (f) OMP

Fig. 3. Image reconstruction via the proposed ST-OMP.

Finally, we evaluate the proposed technique in a self-sustained video monitoring system. We consider the system to be powered by solar energy and process compressively-sensed video (24 images/second with image size of 500×1000 pixels) in realtime. We adopt the commonly used solar energy model [13] to describe daily solar energy.

Since solar energy pattern is repetitive yet non-deterministic, the reconstruction algorithm needs to be adjusted dynamically to achieve the highest possible reconstruction quality under different energy condition. Fig. 4 shows the available solar power (as in the dashed columns, which also reflect the run-time adjustment of ST-OMP based on the available power) and the corresponding performance measured by Signal-to-Error Ratio (SER), where errors are defined as the difference between the recovered and original images. If energy is low, ST-OMP with a larger threshold L is selected to reduce the computational complexity, thereby reducing energy consumption in signal reconstruction. As expected, this will introduce some performance degradation. If we set the maximum allowable SER degradation to be 6dB, the corresponding L_{MAX} can be determined. The proposed ST-OMP only fails 8.3% of the time (i.e., threshold $L > L_{MAX}$ in 2 out of 24 time slots). In comparison, original OMP will fail 87.5% of the time due to its large energy consumption under insufficient solar energy supply. Therefore, through soft-thresholding the proposed technique is able to achieve better tradeoffs between recovery quality and energy consumption.

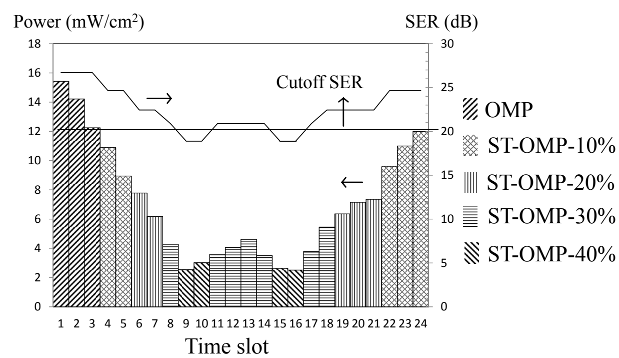


Fig. 4. Performance of the ST-OMP in a self-powered sensing system (solar cell panel area: 1.5cm^2).

5. CONCLUSIONS

The original iterative OMP algorithm involves large computational complexity in the last rounds of iteration while only less significant signal elements are recovered. Based on this observation, ST-OMP technique was proposed to improve the tradeoffs among computational complexity, performance, and cost in signal reconstruction. Recovering less significant elements with low-complexity computations can significantly reduce energy consumption without affecting recovery quality. This is a much-needed feature in many self-powered embedded systems. Further work is being directed toward developing a dedicated ASIC for the proposed ST-OMP.

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