SAR AUTOFOCUS BASED ON MINIMUM ENTROPY

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ABSTRACT

This paper proposes an autofocus method based on minimum-entropy criterion for synthetic aperture radar imaging. Through minimizing the entropy, an iterative method is derived to obtain the phase error, which corrupts the image and increases the image entropy. In addition, dominant-scatter areas are selected as the input of the phase-error estimation. Note that there is not any model assumption for the phase error and the dominant-scatter area isn't required to only contain one point scatter. The phase estimation accuracy and efficiency are also analyzed in this paper. Finally, the simulation and experimental results are used to validate the feasibility and effectiveness of the proposed method.

***Index Terms*—** SAR autofocus, entropy, iteration, phase estimation

1. INTRODUCTION

High resolution image is one of the most important reasons that synthetic aperture radar (SAR) has been widely focused and developed. However, the imaging quality is usually degraded by some undesired phase errors induced by the platform motion aberration, propagation effects and system phase instability [1]. Due to these unpredictable phase errors, it is necessary to use data-driven autofocus techniques.

The autofocus methods using minimum entropy have revealed their good performance and attracted more and more attentions. The algorithm in [2] is parametric and uses an adaptive-order polynomial model. However, if the phase error does not fit the polynomial model, like sinusoidal error, the polynomial order needs to be very high so that the computational burden is quite heavy.

In [3-5], the nonparametric algorithms are proposed and they do not use any model assumption for the phase error. However, there exists a problem that the restoration of SAR image may not mean its entropy is minimum. The theoretical derivation of the minimum entropy of SAR image with respect to the phase error is difficult and there is no closed-form solution. Nevertheless, it can be observed from simulations that the focused profile based on the minimum entropy more approaches to the restoration in the case of a small quantity of point scatters than in the case of many point scatters.

In this paper, dominant scatterers are selected from the SAR image as the input of the proposed autofocus method. There are two main reasons for the adoption of dominant scatterers. One is that signal noise ratio (SNR) is high in the area of dominant scatterers, which can directly cause the phase estimation accuracy is high. The other is it can be shown from the simulation that phase estimation error rises with the increase of the number of point scatters in each selected scatter area. The dominant scatterer contains fewer point scatters. Note that the proposed method does not restrict each selected dominant-scatter area contains only a strong point scatter. Then through minimizing the entropy of all selected areas, an iterative method is derived to obtain phase errors. This method does not limit any parametric model for phase errors and thus can be applied universally. Moreover, its phase estimation accuracy is also analyzed in this paper.

This paper is arranged as follows: First, Section 2 proposes an iterative minimum-entropy autofocus method for SAR image. In this section, the selection of dominant scatterers and the phase estimation accuracy are also described and analyzed. Section 3 shows the processing results of simulation and real experimental data to validate the proposed algorithm. Finally, Section 4 draws the conclusions.

2. ITERATIVE MINIMUM-ENTROPY AUTOFOCUS

SAR autofocus problem can be formulated as

$$s_l(n) = \frac{1}{N}\sum_{k=0}^{N-1} S_l(k)\exp\{j\varphi_k\}\exp\left\{j\frac{2\pi}{N}kn\right\} \tag{1}$$

where N is the azimuth sample number and $S_l(k)$ represents the azimuth Fourier transform of the selected lth dominant scatterer data after range alignment, range compression and rough azimuth compression. The indices n and k refer to azimuth time and azimuth frequency respectively and φ_k is used to cancel the phase errors.

It is generally acknowledged that better focus quality of SAR image has smaller entropy. Then, SAR autofocus based on minimum-entropy criterion is to determine φ_k to minimize

$$E = -\sum_{l=0}^{L-1}\sum_{n=0}^{N-1}\frac{\left|s_l\left(n\right)\right|^2}{C}\ln\frac{\left|s_l\left(n\right)\right|^2}{C} \tag{2}$$

where C is the total signal energy and is a constant. In addition, we assume there are L dominant scatterers.

Before the derivation of phase estimation, how to select dominant scatterers is presented. In this paper, contrast measurement is firstly adopted to pick out quality range cells, because high contrast indicates this range cell contains more prominent scatterers than low one. Then M brightest scatterers are selected from each quality range cell. For the convenience of further processing, these scatterers are shifted to the image center.

The next important step is windowing these shifted scatterers to preserve the width of the dominant blur while suppress the noise and neighboring clutter interference. The size of window can be determined by the average response of these shifted scatterers. This method is also used in conventional PGA [6]. Gaussian window is adopted in the whole processing.

Finally, quality scatterers is chosen from the windowed scatterers based on the criteria that the mainlobe energy is much higher than the background noise and the interference is negligible for both mainlobe and sidelobes [7]. The specific implement steps are performed as follows: 1) Estimate the mainlobe width utilizing the average response of selected scatterers; 2) Calculate the proportion of the mainlobe energy to the total signal energy; 3) Set a threshold and select dominant scatterers. The threshold should ensure that selected scatterers are prominent and not influenced by neighboring clutter interference. Next, these selected dominant scatterers will be used to estimate phase errors.

Next, an iterative method is derived to determine φ_k based on (2). The sign φ_k^i is denote as the ith estimate of φ_k. Then, (2) is expanded at φ_k^i to its Taylor series as follow and the cubic and higher order items are ignored.

$$\begin{aligned} E &= E\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}} \\ &\quad + E'\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}}\left(\varphi_k-\varphi_k^i\right) \\ &\quad + \frac{1}{2}E''\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}}\left(\varphi_k-\varphi_k^i\right)^2 \end{aligned} \tag{3}$$

The minimization of (3) with respect to φ_k is equivalent to solve $\partial E/\partial\varphi_k = 0$. Thus, the iterative equation can be written as

$$\varphi_k^{i+1} = \varphi_k^i - \frac{E'\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}}}{E''\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}}} \tag{4}$$

The proposed iterative method constructs a local quadratic curve to gradually approach the extremum of the objective function (2), which principle can be simply shown in Fig. 1. Moreover, in order to make the iteration converge towards a minimum point, these quadratic curves should be convex and namely,

$$E''\Big|_{\varphi=\{\varphi_0^i,\dots,\varphi_k^i,\dots,\varphi_{N-1}^i\}} > 0 \tag{5}$$

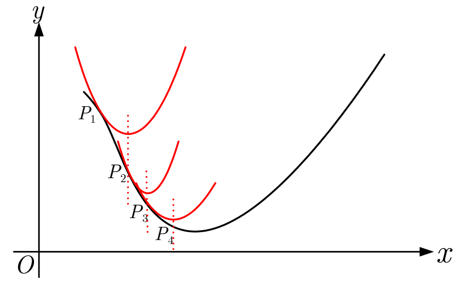


Fig.1. The schematic representation of iterative principle.

Note that if SAR image with the minimum entropy is equivalent to its restoration, the original well-focused SAR image without phase errors should satisfy at

$$\left\{\varphi_0,\dots,\varphi_k,\dots,\varphi_{N-1}\right\} = 0 \tag{6}$$

It can be found that the proposition above is tenable while $s_l\left(n\right)$ is an impulse function. However, for real SAR system, $s_l\left(n\right)$ can be approximately considered as a limited-width sinc function and it cannot meet (6). Without losing generality, we assume it will satisfy $\partial E/\partial\varphi_k = 0$ at

$$\left\{\varphi_0,\dots,\varphi_k,\dots,\varphi_{N-1}\right\} = \left\{\Delta_0,\dots,\Delta_k,\dots,\Delta_{N-1}\right\} \tag{7}$$

As thus, if SAR image is corrupted by phase error φ_k^E, phase estimation obtained based on minimum entropy will meet $\varphi_k - \varphi_k^E = \Delta_k$. Therefore, phase estimation error will be always higher than Δ_k. In addition, it can be observed from the simulations in next section that Δ_k is in direct proportion to the amount of the point scatters in each range cell. Consequently, the focused image obtained based on the minimum entropy more approaches to the restoration in the case of a small quantity of scatters than in the case of many scatters.

Note that for the iterative equation (4), the numerator and denominator of modified value are respectively the vector sum of the corresponding part of each selected dominant scatterer. It is widely known that the noises in the areas of different dominant scatterer usually can be considered as not correlative, and thus the summing can contribute to suppress the noise. Therefore, the phase estimation accuracy rises with the increase of the number L of selected dominant scatterers, which is also verified in next simulations. In addition, the areas of dominant scatterers relatively have higher SNR, which can directly cause the phase estimation accuracy is higher as well.

3. SIMULATIONS AND EXPERIMENTAL RESULTS

As analyzed above, for real SAR system, the phase estimation accuracy is higher than Δ_k, which satisfies $\partial E/\partial\varphi_k = 0$ for well-focused SAR image without phase errors. Next, we will demonstrate Δ_k is in direct proportion to the amount of the point scatters at each range cell. In the simulation, we randomly locate P point scatters in each range cell. Their magnitude and phase are random variables and independent of each other. Moreover, the magnitudes are Rayleigh distribution with the scale parameter of the one and the phases are independent and uniformly distributed between $-\pi$ and π.

Fig. 2 shows the variation of the root mean square (RMS) of phase estimation error with P for ideal SAR image without noise and phase errors. The Monte Carlo simulation is performed so as to obtain more accurate curve. It can be found that even for well-focused image, phase estimation based on minimum entropy is not equal to zero. Moreover, the estimation phase gradually rises with the increase of point scatters in each range cell although the variational trend becomes small with L increasing.

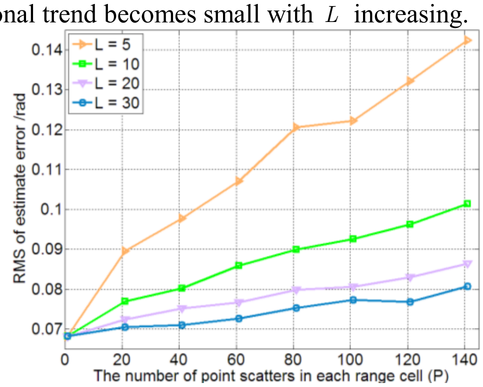


Fig. 2. RMS of estimated phase error versus the number of point scatters in each range cell.

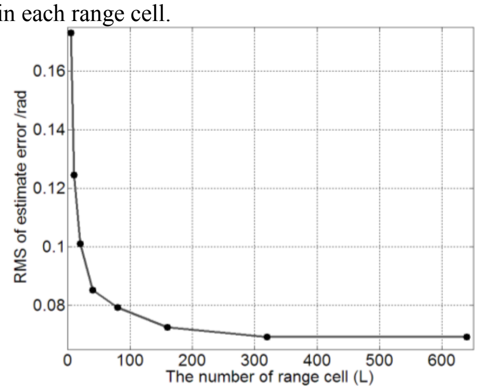
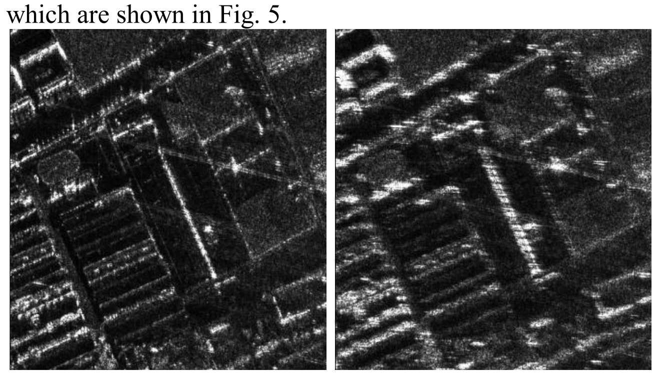


Fig. 3. RMS of estimated phase error versus the number of range cell.

Fig. 3 is mainly used to verify the phase estimation accuracy rises with the increase of range cells. In the simulation, we place the single point scatter in each range cell and both magnitude and phase are the random variables. The phase error is added to the phase history of data and SNR is set to 0 dB. The Monte Carlo simulation is also adopted. It can be seen that RMS of estimate error gradually decreases with increasing range cells and finally approaches to RMS of Δ_k 0.068 rad (as shown in Fig. 2), which is consistent with the analysis in Section 2.

Next, real SAR image (700×5632 pixel) is used to evaluate our algorithm and the original image is shown in Fig. 4(a). Then we simulate a random phase error function to the phase history of SAR image and it consists of an arbitrary curve and a white noise, which results in the corrupted SAR image shown in Fig. 4(b). Then, dominant scatterers are selected from this unfocused SAR image, which are shown in Fig. 5.



(a) (b)

Fig. 4. The original (a) and corrupted (b) SAR image.

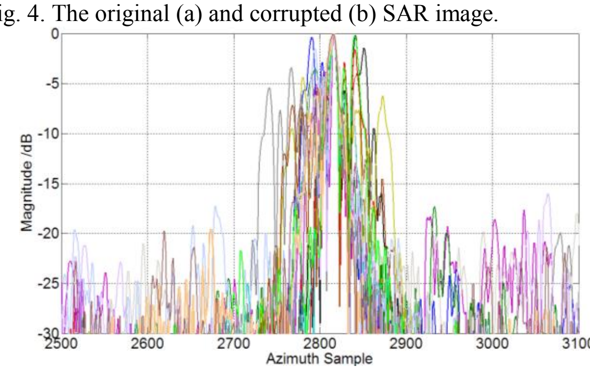


Fig. 5. The selected dominant scatterers.

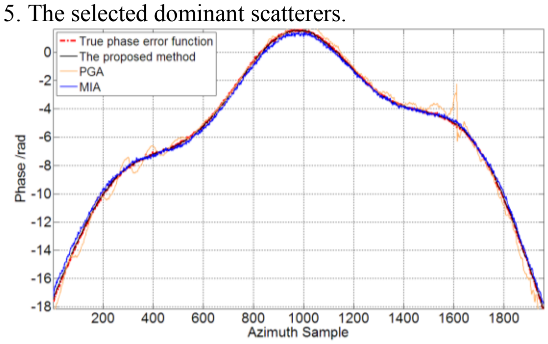


Fig. 6. True phase error function versus the estimate phase by the proposed method, PGA and MIA.

Through minimizing the entropy of these dominant scatterers, phase error estimation is obtained by our proposed method shown in Fig. 6 (black line) and RMS of estimate error is 0.0735 rad. The focused SAR image is shown in Fig. 7(d). It takes a total of 1.18 minutes with the final entropy of 10.84 while the entropy of the original SAR image is 10.83.

In order to test the performance of our proposed method, the estimated phases by the FFT-based monotonic iterative algorithm (MIA) in [5] and PGA in [6] are shown in Fig. 6 (blue and orange lines) and the focused SAR image are respectively shown in Fig. 7(b) and (c). PGA decreases the image entropy to 10.9 with taking only 9.17 seconds. However, its estimation accuracy is bad and RMS of estimate error is 0.48 rad. The possible reason is that its estimation accuracy more depends on the selected dominant scatterers. On the basis of its principle, the selected scatterer is supposed to be a strong point scatter. Nevertheless, it is difficult to pick such point scatter for the severely-unfocused image. It can be seen from Fig. 5 that each selected dominant scatterer contains a few of strong point scatters. Therefore, it is logical that the estimation accuracy decreases in this case.

In contrast, the autofocus method based on entropy criterion does not have such constraint. MIA achieves the final entropy of 10.83 with taking a total of 15.16 minutes through minimizing the whole SAR image. It can be found that the input of MIA is the whole SAR image, which has 700 range cells while the number of the selected dominants is only 19. Consequently, MIA spends more time. In addition, RMS of its estimate error is 0.24 rad which is higher than the one of our proposed method. It is consistent with the analysis in Section 2. Dominant scatterers have higher SNR, thus resulting in higher estimation accuracy. Moreover, it is verified in the simulation that the estimate error rises with the increase of point scatters in each range cell.

Therefore, from the results of both simulations and real SAR image, it can be found that selecting dominant scatterers is helpful to improve estimation accuracy and efficiency for the autofocus method based on entropy criterion. Meanwhile, the proposed iterative method also reveals good performance in the aspects of both estimation accuracy and efficiency.

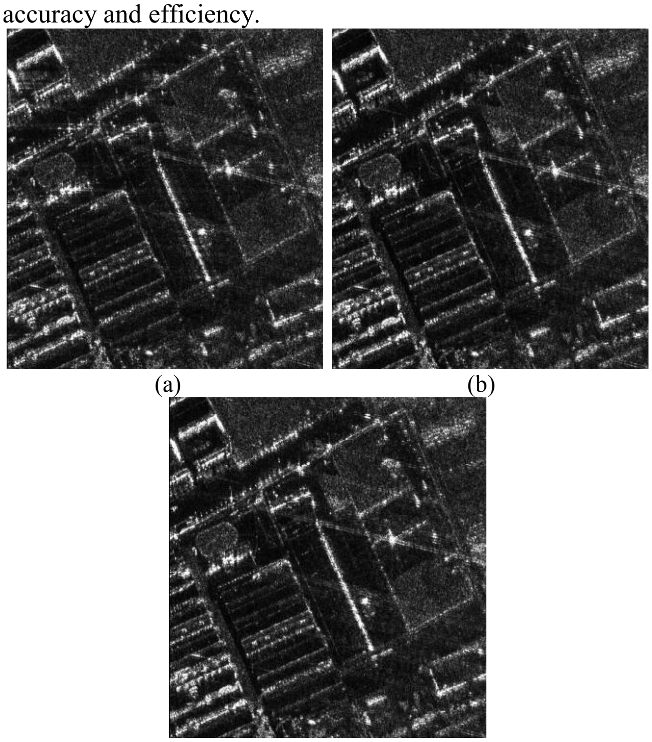


Fig. 7. The focused SAR image by PGA (a), MIA (b) and the proposed method (c).

4. CONCLUSIONS

This paper proposes an iterative autofocus method based on entropy criterion, which does not have any assumption for the phase error. Dominant scatterers firstly are selected and through minimizing the entropy of these dominant scatterers, phase estimation can be obtained. Both the results of simulations and real SAR image all show selecting dominant scatterers can conduce to improve estimation accuracy. Moreover, the proposed iterative method also reveals good performance in the aspects of both estimation accuracy and efficiency.

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