A CIRCLE DETECTION APPROACH BASED ON RADON TRANSFORM

O. Erman OKMAN, Gozde B. AKAR

Department of Electrical and Electronics Engineering, Middle East Technical University, 06531, Ankara, Turkey

ABSTRACT

In this paper a novel fast circle detection algorithm is proposed which depends on the spatial properties of the connected components on the image. Two 1-D transforms of each connected component is obtained by taking the Radon Transform of the image for two different directions, which are in fact the integrations of the image through horizontal and vertical directions. Circles are detected using the similarities of detected peaks on the transformed functions and the characteristics of the values in between those peaks. The success of the method is analyzed using synthetic images and the performance of the method is presented and compared with Modified Hough Transform (MHT) using synthetic images.

***Index Terms*—** Circle Detection, Radon Transform, Hough Transform

1. INTRODUCTION

Identification of shapes is one of the essential problems of pattern analysis and automatic target recognition (ATR). Detection of circles is an important step of this process, hence, this problem has been extensively studied in the literature and different approaches have been developed [1].

Hough Transform (HT) [2, 3] is a very popular method to detect circles and it is still widely used because of its robustness against any type of distortion. On the other hand, this method requires massive memory and excessive computation power. For efficient detection of a circle, many researches proposed modifications to the HT. In [4] it is proposed to accumulate only the points in the gradient direction of the edges. This revised method is standardized as Modified Hough Transform (MHT) where the information about the local edge direction is extracted using Sobel operator, [1]. There are many other studies trying to reduce the computational load or memory consumption of HT using different voting strategies, transform spaces or validation scheme [5-9]. However, the improvements in computation times of these methods are far from significant. Xu et. al. introduced Randomized Hough Transform [10], by which the computational performance of HT is improved drastically. They proposed to select n pixels randomly and map them to a point in the parameter space, instead of transforming each pixel into n-dimensional hyperspace. But this method still requires huge computation power to have successful results, especially for complicated scenes.

In [11], a size invariant method is proposed that mainly depends on detecting the edge pixel pairs at which the gradients are just in the opposite direction of each other and grouping the mid points of these pixel pairs. Similarly, Ho and Chen proposed an approach which utilizes the geometrical symmetry of the circles and ellipses, [12], where the pixels are classified to estimate the circle/ellipse parameters. However, these methods are examined only on synthetic data or on a limited type of data. Besides, the performances of the methods are not discussed based on the complexity of the scenes number of circular structures in the scene, i.e. the number of edge pixels that is an important factor in computational complexity. Peng and Rao suggested to use Radon Transform for detection of circles [13]. It is shown that for each of the angle, the Radon Transform of a circle has the same response due to the symmetry of the shape; hence a specific matched filter is designed in that study to make the detection possible after back projection of the filtered transformed signals. Despite higher SNR is achieved with respect to Hough Transform, the method still requires a search in radius and suffers from the forward and back projections of a various number of functions.

In this study, a new, fast and size invariant circle detection approach is presented that utilizes Radon Transforms of edges at two different directions. Circles are detected according to the similarities of the characteristics of these two 1-D functions. Since this approach reduces the search space a lot, the detection procedure becomes faster while still yielding similar performances of former approaches. In this context, this study has resemblances with [13] in terms of detection approach and is closely related to circle detection literature mentioned above. The proposed method is a suggestion to overcome high computation requirements of the circle detection methods. The performances are presented and compared with MHT using a set of synthetic images and some real world examples in terms of their detection rates and speeds.

In the following chapter, MHT and the proposed circle detection algorithms are explained. In Chapter 3 the results of the performed experiments are given and finally Chapter 4 concludes the paper.

2. CIRCLE DETECTION

2.1. Modified Hough Transform (MHT)

In the original HT, after extraction of the edges in the image, the positions of all possible center locations – namely, all points a distance r (radius of the anticipated circle) away from every edge pixel – are accumulated. In MHT, [4], in order to decrease the computational load, it is proposed to accumulate only the points in the gradient direction of the edges. Using the Sobel operation mask, two intensity gradient values g_x and g_y are obtained by which the magnitude and orientation of the edge gradient is obtained. Since it is not known whether inside the circle is dark and the background is light or vice versa, the accumulation is performed for the estimated center location both on the same direction and the opposite direction of the gradient vector as in Equation (1).

$$x_c = x \pm r\cos\theta,\ \ y_c = y \pm r\sin\theta \quad \textbf{(1)}$$

where (x, y) is the location of the edge and (x_c, y_c) is the location of the probable circle with radius r. Finally, local peaks of the accumulator data are detected and generally if their values are greater than a predefined threshold, they are determined as the center locations. In this study, we accumulate the histogram values in the 3x3 window around the local peaks to compare them with a threshold in order to be robust against the errors in the edge direction estimations and this threshold is determined as *kr* where *k* is a user defined constant. Note that, since the radius r is usually unknown, the whole procedure is repeated for a set of radius values (r_{min}, $r_{min}+r_{step}$, .. , r_{max} $-r_{step}$, r_{max}).

2.2. Proposed Method

The main idea of the proposed method is reducing the search space of the parameters in order to reduce the complexity of the algorithm by taking 1-D transforms of the images. For this purpose, Radon Transform is utilized since it has already shown that each angular profile obtained by Radon Transform is equivalent and has specific characteristics. Hence, in this study, the circles are proposed to be detected by use of the characteristics and the similarities of obtained profiles.

The Radon Transform is expressed as follows:

$$R_\theta(\rho) = \iint f(x,y)\delta(x\cos\theta + y\sin\theta - \rho)dxdy \quad \textbf{(2)}$$

where R_θ is the Radon Transform of function *f(x,y)* at angle θ and δ is the Dirac function. So, when $\theta = 0$ and $\theta = \pi/2$, simpler expressions can be obtained as given below:

$$R_0(\rho) = \iint f(x,y)\delta(x-\rho)dxdy = \int f(\rho,y)dy \quad \textbf{(3)}$$

$$R_{\pi/2}(\rho) = \iint f(x,y)\delta(y-\rho)dxdy = \int f(x,\rho)dx \quad \textbf{(4)}$$

From these equations, it is easily seen that $\rho=x$ for $R_0(\rho)$ and $\rho=y$ for $R_{\pi/2}(\rho)$. Then, we define them as $g_1(x)$ and $g_2(y)$, respectively, expressed by:

$$g_1(x) = \int_{-\infty}^{\infty} f(x,y)dy,\ \ g_2(y) = \int_{-\infty}^{\infty} f(x,y)dx \quad \textbf{(5)}$$

Assuming *f(x,y)* is an image that contains only one circle centered at (x_c, y_c) with radius r, it is expressed as:

$$f(x,y) = \begin{cases} \delta(x,y), & if\ (x-x_c)^2 + (y-y_c)^2 = r^2 \\ 0, & else \end{cases} \quad \textbf{(6)}$$

where (x_c, y_c) is the center and r is the radius of the circle. The integrations given in Equation 5 can be evaluated using infinitesimal part of the circle *ds* defined as below:

$$ds^2 = dx^2 + dy^2 \quad \textbf{(7)}$$

Then,

$$g_1(x) = \frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2},\ \ g_2(y) = \frac{ds}{dy} = \sqrt{1+\left(\frac{dx}{dy}\right)^2} \quad \textbf{(8)}$$

Using Equations (8), $g_1(x)$ and $g_2(y)$ are evaluated as given below, since *f(x,y)* is symmetric with respect to *x* and *y* axes:

$$g_i(t) = \begin{cases} 2\left(1 - \frac{(t-t_c)^2}{r^2}\right)^{-1/2} & , if\ |t-t_c| \le r \\ 0 & , otherwise \end{cases} \quad \left| \begin{array}{l} i \in \{1,2\} \\ t \in \{x,y\} \end{array} \right. \quad \textbf{(9)}$$

where $i \in \{1,2\}$ and $t \in \{x,y\}$.

As seen from Equation (9), $g_1(x)$ is minimum at $x=x_c$ and has asymptotes at $x=x_c \pm r$; $g_2(y)$ is minimum at $y=y_c$ and has asymptotes at $y=y_c \pm r$.

For discrete case, directional integrals can be defined as follows:

$$g_1[m] = \sum_n f[m,n],\ \ g_2[n] = \sum_m f[m,n] \quad \textbf{(10)}$$

which are evaluated by taking integral of $g_1(x)$ and $g_2(y)$ over [$m-0.5$, $m+0.5$] and [$n-0.5$, $n+0.5$], respectively.

Assuming x_c, y_c and r be integers, the maximum values of $g_1[m]$ are at $m=x_c \pm r$ and the maximum values of $g_2[n]$ are at $n=y_c \pm r$ evaluated by:

$$g_1[x_c \pm r] \approx 2\sqrt{r},\ \ g_2[y_c \pm r] \approx 2\sqrt{r} \qquad \text{as } r \to \infty \quad \textbf{(11)}$$

Equation 11 shows that ideally there should be two identical peaks in $g_1[m]$ and $g_2[n]$, but discretization, distorted shapes or noise on the image may cause other peaks to appear and/or desired peaks to be different from each other. Then, we propose to find two similar peaks in each profile to detect circles on the image by the following procedure:

1. Extract edge map of the image, E[m,n].
2. Find connected components, c_i's of E[m,n].
3. For each c_i,
   - 3.1. Evaluate $g_1[m]$ and $g_2[n]$ by Equations (10).
   - 3.2. Detect the global maxima of $g_1[m]$ and $g_2[n]$ by:

$$m_{max} = \arg\max_{m} \left\{ g_1\left[m\right] \right\}$$
$$n_{max} = \arg\max_{n} \left\{ g_2\left[n\right] \right\} \quad \textbf{(12)}$$

3.3. Skip to step 3 if the ratio of the peaks, $g_1[m_{max}]$ and $g_2[n_{max}]$, is below a threshold γ .

3.4. The local maxima in $g_1[m]$, m_j's, and $g_2[n]$, n_k's, are detected.

3.5. In order to maximize the detected circle, j_r and k_r indices are found using Equation (13) through the local peaks that satisfies the conditions given in Equation (14)-(17).

$$j_r = \arg\max_{j} \left\{ \left| m_{\max} - m_j \right| \right\},\ k_r = \arg\max_{k} \left\{ \left| n_{\max} - n_k \right| \right\} \quad \textbf{(13)}$$

$$\frac{g_1\left[m_j\right]}{g_1\left[m_{\max}\right]} \ge \alpha, \quad \frac{g_2\left[n_k\right]}{g_2\left[n_{\max}\right]} \ge \alpha \quad \textbf{(14)}$$

$$\beta \le \frac{g_1\left[m_{\max}\right]}{\sqrt{2\left|m_{\max} - m_j\right|}} \le \frac{1}{\beta}, \quad \beta \le \frac{g_2\left[n_{\max}\right]}{\sqrt{2\left|n_{\max} - n_k\right|}} \le \frac{1}{\beta} \quad \textbf{(15)}$$

$$\lambda \le \frac{\left|m_{\max} - m_j\right|}{\left|n_{\max} - n_k\right|} \le \frac{1}{\lambda} \quad \textbf{(16)}$$

$$\mu_{\min} \le \frac{1}{\pi\left|m_{\max} - m_j + 1\right|} \sum_{m=m_j}^{m_{\max}} g_1\left[m\right] \le \mu_{\max}$$
$$\mu_{\min} \le \frac{1}{\pi\left|n_{\max} - n_j + 1\right|} \sum_{n=n_k}^{n_{\max}} g_2\left[n\right] \le \mu_{\max} \quad \textbf{(17)}$$

where α, β and λ is defined in [0,1]; μ_{min} is smaller than 1 and μ_{max} is greater than 1.

3.6. If m_{jr} and n_{kr} are found, a circle is detected with parameters:

$$x_c = \frac{m_{\max} + m_{j_r}}{2}, \quad y_c = \frac{n_{\max} + n_{k_r}}{2}$$
$$r = \frac{\left|m_{\max} - m_{j_r}\right| + \left|n_{\max} - n_{k_r}\right|}{4} \quad \textbf{(18)}$$

3.7. If only m_{jr} is found, a circle is detected with parameters:

$$x_c = \frac{m_{\max} + m_{j_r}}{2}, \quad r = \frac{\left|m_{\max} - m_{j_r}\right|}{2}$$
$$y_c = n_{\max} + \operatorname{sgn}\left(g_2\left[n_{\max}+1\right] - g_2\left[n_{\max}-1\right]\right) \cdot r \quad \textbf{(19)}$$

3.8. If only n_{kr} is found, a circle is detected with parameters:

$$y_c = \frac{n_{\max} + n_{k_r}}{2}, \quad r = \frac{\left|n_{\max} - n_{k_r}\right|}{2}$$
$$x_c = m_{\max} + sgn\left(g_1\left[m_{\max}+1\right] - g_1\left[m_{\max}-1\right]\right) \cdot r \quad \textbf{(20)}$$

Note that, the condition in Equation (14) implies the similarity of the peak values in each profile; the one in Equation (15) implies the closeness of the peak values in the profiles to $2\sqrt{r}$; Equation (16) shows the similarity of the radii estimated from both axes and Equation (17) implies the closeness of number of edges in the connected component to the circle perimeter.

3. EXPERIMENTAL RESULTS

In this chapter, the detection performance of the proposed method is evaluated using synthetic images by comparing them with the outputs obtained by MHT and presenting results for some satellite images. Both of the methods are implemented in MATLAB R2009b and the experiments are done using a computer with Intel Core 2 Duo 2.4 GHz processor and 3 GB of RAM.

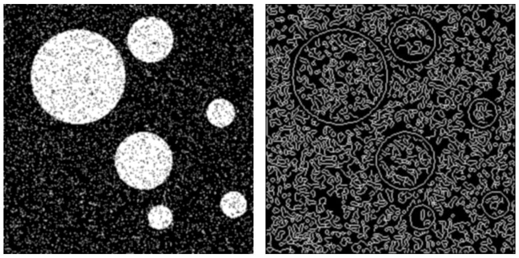
In the first experiment, 6 circles are generated in a 256x256 image with random radii between 10 and 50 at random locations and different amount of Salt & Pepper type noise is added to the image where the amount of noise is defined as:

$$L_{Noise} = \frac{\#\,of\ noise\,added\ pixels}{\#\,of\ all\,circles'\ pixels} \quad \textbf{(21)}$$

Then, for different noise levels 100 different randomly cases (see Figure 1 for a sample case) are generated, the experiments are repeated for each case to detect the circles by both the proposed method and MHT. For this experiment α, β and γ are set to 0.5, λ is set to 0.75, μ_{min} is set to 0.5 and μ_{max} is set to 1.5 for the proposed method; r_{min}, r_{max}, r_{step} and k are selected as 10, 50, 1 and 1.5 respectively for MHT. Finally, detection and false alarm probabilities, P_D and P_{FA}, respectively, are evaluated for the outputs of both methods. In this study, these probabilities are defined as:

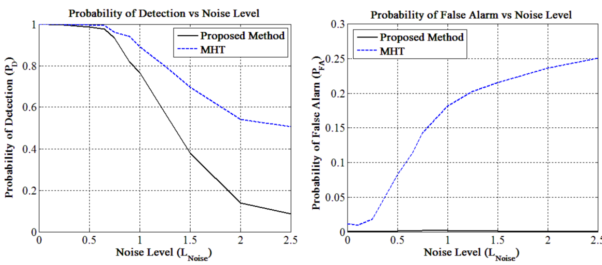
$$P_D = \frac{n_d}{n_d + n_m}, \qquad P_{FA} = \frac{n_f}{\text{Size of Image}} \quad \textbf{(22)}$$

where n_d is the number pixels of the circles in the original image that are also inside one of the detected circles, n_m is the number of pixels of the circles in the original image that are not inside one of the detected circles and n_f is the number pixels inside the detected circles but does not inside a circle in the original image. In Figure 2, the variation of the P_D and P_{FA} are given with respect o the change of the noise level. As seen from the graphs, by both of the methods high detection rates (over %95) can be produced up to L_{Noise}=0.75. The characteristics of the methods differ as the noise level is increased more. MHT method provides higher detection rates but also higher false alarm ratios as the noise level increases. On the other hand, the proposed method always produces very low false alarm rates but the detection probability decreases more rapidly than MHT.



(a) (b)

Figure 1 (a) A sample image with L_{Noise}=1 and (b) detected edge map from that image



(a) (b)

Figure 2 (a) Probability of detection and (b) probability of false alarm with respect to the noise level on the image for MHT and proposed method

Another experiment is performed to investigate the computational load of the approaches as the scene becomes more complex. For this purpose, L_{Noise} is set to 0 and the number of the circles in the image increased. Again the experiments are performed for 100 random cases and all the parameter set of the methods keep same as given for the first experiment. The mean computation durations of the methods with respect to the number of circles in the image is given in Figure 3. As can be seen from this figure, the proposed method is much faster than MHT and the increase in computation time with respect to scene complexity of the proposed method is lower. Note that for each case P_D is over 99% and P_{FA} is below 1%.

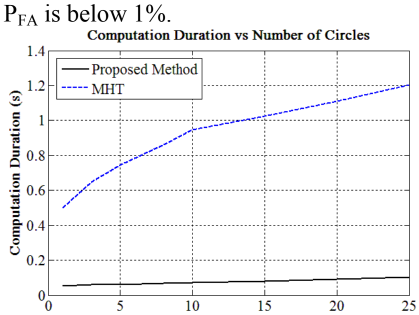


Figure 3 Computation durations of MHT and proposed method for different number of circles in the image

In this study, the methods are also analyzed for some remote sensing images. In Figure 4, two images captured from Google Earth[TM] are presented with the detected circles by MHT and proposed methods. As seen from the results, the outputs are very similar where the proposed method operates faster than MHT (see Table 1). Note that, edge map of the images are extracted after applying bilateral filter [14] where size of the Gaussian bilateral filter is 15, standard deviation of the filter is 5 in spatial domain and 1 in intensity domain. The parameters of the methods keep same with the previous examples. These experiments also show that the proposed method has a similar detection performance with MHT while producing less number of wrong circles.

(a) (b) (c) (d)

Figure 4 Detected circles in two satellite images using (a),(c) MHT; (b),(d) proposed method

Table 1 Computation durations of methods for the images given in Figure 4

	Computation Duration (s)	
	MHT	Proposed Method
Image in Figure 4 (a)	9.82	4.06
Image in Figure 4 (c)	4.61	1.79

4. CONCLUSION

In this study, a new and fast circle detection algorithm is proposed which is based on detection of peaks on two directional profiles of the segmented objects in the images. Unlike HT based approaches, the method can be performed without a priori knowledge about the size of the circles. By the experiments performed, its robustness is shown against noise, and its fastness is presented with respect to a Modified Hough Transform while preserving the detection performance. Experiments show that the proposed algorithm is robust to noise and faster than MHT while preserving the detection performance. Besides, the low false alarm probability of the method can be seen as another advantage of the method, since it makes this new approach very reliable. Outputs of some satellite images are also presented to show the applicability of the approach to realistic scenarios. The major drawback of this approach is its dependency on the connected components on the edge map. So, future studies focus on a procedure that connects the circular segments together to have better circular segments. Besides, this method can be extended for detection of different geometrical shapes like ellipse or quadrangle.

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