DICTIONARY LEARNING FOR INCOHERENT SAMPLING WITH APPLICATION TO PLENOPTIC IMAGING

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ABSTRACT

We propose a method for object reconstruction from images obtained by a plenoptic camera. Our approach exploits a plenoptic system model based on diffraction analysis in order to formulate an inverse problem for object reconstruction. To solve this inverse problem, we propose a dictionary learning algorithm for signal reconstruction from measurements obtained by a deterministic linear system. In contrast to prior work in Compressive Sensing, we do not impose constraints on the measurement matrix, but allow it to be defined by the properties of a specified camera system. Given the measurement matrix, the proposed algorithm learns a dictionary from a large database of examples and simultaneously minimizes the mutual coherence between the measurement matrix and the dictionary. We evaluate the performance of the algorithm on object reconstruction from plenoptic system measurements and show that it outperforms existing solutions.

Index Terms— Plenoptic imaging, dictionary learning, compressive sampling, mutual incoherence.

1. INTRODUCTION

In digital imaging systems we often encounter linear inverse problems, where we need to reconstruct certain properties of the original object (color, depth, etc.) from system measurements. For conventional digital cameras, such methods include denoising and deconvolution, which have been extensively studied in the literature. In computational imaging systems, however, the optical path is often modified such that the system cannot be modeled by a convolution with a conventional point spread function. A recently developed computational imaging system is a plenoptic camera, which provides additional functionalities compared to a standard camera, such as instantaneous multi-spectral imaging [1], refocusing [2] and 3D imaging [3]. This is achieved via optical modifications (insertion of a micro-lens array as shown in Fig. 1) and using advanced image processing algorithms.

Plenoptic systems provide those additional functionalities at the expense of spatial image resolution, reduced to the number of microlenses (lenslets) in the system. Several authors [4, 5, 6] proposed to model the process of image formation through a plenoptic system and to use this forward model for high-resolution object reconstruction. Shroff and Berkner [5] used diffraction analysis and derived the system response matrix, called the Pupil Image Function (PIF) matrix. By posing the object reconstruction problem as an inverse problem and solving it via non-linear fitting, they demonstrated recovery of high spatial frequency object information, up to the cut-off frequency of the main lens [6]. However, inversion represents a challenging problem because the PIF matrix is rank deficient, especially when the object is in focus on the microlens array plane. For typical levels of system noise, the quality of the image obtained in [6] is low compared to an image acquired by a conventional camera.

Similar linear inverse problems have recently been successfully solved by applying sparse priors and the theory of Compressive Sampling (CS). Almost all prior work in CS uses special random measurement matrices and well-known bases (or dictionaries) such as wavelets [7]. This is mostly driven by the existence of theoretical proofs of mutual incoherence between these measurement matrices and dictionaries, which is required for stable signal reconstruction. In contrast to these works, we address the case when we are given a deterministic measurement matrix specified by a physical system (e.g., a PIF matrix for a plenoptic camera) and propose an algorithm that uses dictionary learning [8] to train a dictionary that is incoherent with that system matrix.

Several authors have addressed dictionary learning from linear measurements. Isely et al. assume a Gaussian iid measurement matrix and show that the dictionary can be learned from such measurements, without a direct access to signals [9]. Gleichman and Eldar consider dictionary learning for a measurement matrix that is a union of orthogonal bases [10]. Both works assume specific measurement matrices. Such assumption is obviously too constraining for our case of pre-specified system matrices. Duarte-Carvajalino and Sapiro proposed an algorithm that simultaneously learns the measurement matrix $\mathbf{A}$ and the dictionary $\mathbf{\Phi}$ [11]. This is achieved by iterating between optimization of $\mathbf{A}$ and adaptation of $\mathbf{\Phi}$. They enforce a penalty on the mutual coherence between $\mathbf{A}$ and $\mathbf{\Phi}$, which is integrated in the optimization of $\mathbf{A}$. Thus, reducing the mutual coherence requires changing $\mathbf{A}$. This makes their algorithm unusable for incoherent learning with a fixed $\mathbf{A}$. Finally, we should mention the work of Yang et al. [12], who propose dictionary learning for im-

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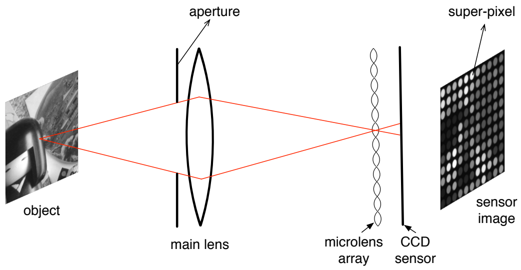

Fig. 1. Plenoptic (light field) camera system.

age superresolution. In their method, inference is done from the measurements, while learning is done on high-resolution images. They do not impose a coherence penalty, but hope that the learned dictionary will perform well in most cases. However, without the bound on the coherence between $\mathbf{A}$ and $\mathbf{\Phi}$, the reconstruction is not reliable.

The main difference of our approach to prior art is that we learn a dictionary that is both adapted to signal statistics and that satisfies a property necessary for reliable reconstruction: it has small mutual coherence with a given system matrix. We apply our method to high-resolution object reconstruction in plenoptic camera systems. Simulation results demonstrate a significantly improved object reconstruction quality compared to the previous solution in [13] and compared to a dictionary learning approach that does not minimize coherence.

2. PLENOPTIC SYSTEM MODEL

Plenoptic imaging can be achieved by placing a microlens array in front of an image sensor [2], as shown in Fig. 1. The effect of this modification is that we can acquire a 4D light field, capturing both the angular and spatial resolution of light reflected from an object in a scene. Moreover, we can insert a spectral filter array at the main lens aperture and obtain a multi-spectral camera. Data captured behind each microlens (lenslet) at the sensor is called a super-pixel and is comprised of several pixels capturing the light rays coming from the same point in space, but from different directions.

Because of these modifications, we do not have a one-to-one mapping between the sensor (pixel) space and the object space. Rather, object and sensor space are related by the system's response function called the pupil image function (PIF) [5], which is analogous to the point spread function in conventional cameras. For a given optical system and a local plane position, its PIF for different points in the local plane can be obtained using the principles of Fourier optics [5]. For a 3D object space, we can define PIFs for a set of local planes. The collection of all the PIF responses for different points in the object space comprises the system matrix $\mathbf{A}$. If we denote the image at the sensor as $\mathbf{y}$ (in a vectorized form) and the object points as $\mathbf{x}$, we can formulate the image acquisition process of a linear plenoptic system as: $\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$, where $\boldsymbol{\eta}$ represents system noise. The reconstruction problem is thus to find $\mathbf{x}$, given $\mathbf{y}$ and $\mathbf{A}$. However, the PIF matrix can be rank deficient and recovering a high resolution object data represents a difficult inverse problem. This is particularly the case when the main lens focuses onto the lenslet array, as in this case there is no parallax between light rays, and superresolution methods such as [4] are not applicable. The focused case is of particular interest for multi-spectral imaging systems.

To solve an under-determined linear system we need to incorporate some prior information about the signal or image, such as sparsity. This prior assumes that the signal is sparse in a certain dictionary $\mathbf{\Phi}$, i.e., that in the signal model $\mathbf{x} = \mathbf{\Phi}\mathbf{c}$ the vector of coefficients $\mathbf{c}$ has a small number of non-zero entries. Compressive Sensing (CS) theory addresses the problem of the reconstruction of sparse signals from linear measurements [7]. Following the introduced notation, the CS reconstruction problem is to find a sparse estimate for $\mathbf{c}$ from the measurements $\mathbf{y}$ such that $\mathbf{y} = \mathbf{A}\mathbf{\Phi}\mathbf{c} + \boldsymbol{\eta}$. In CS theory, matrix $\mathbf{A}$ is called the measurement matrix. When certain conditions on $\mathbf{A}$ and $\mathbf{\Phi}$ are met, it has been shown that sparse $\mathbf{c}$ (and hence $\mathbf{x}$) can be reconstructed by solving the following convex optimization problem:

$$\hat{\mathbf{c}} = \arg\min_{\mathbf{c}} \left[\|\mathbf{y} - \mathbf{A}\mathbf{\Phi}\mathbf{c}\|_2^2 + \lambda \|\mathbf{c}\|_1 \right], \tag{1}$$

where λ is a trade-off parameter between the level of sparsity and the fidelity of signal reconstruction [14]. The optimization problem (1) is convex and can be solved efficiently using interior point or gradient methods. However, CS theory requires that $\mathbf{A}$ and $\mathbf{\Phi}$ are mutually incoherent, i.e., that the coherence between them is small. The mutual coherence between $\mathbf{A}$ and $\mathbf{\Phi}$ is defined as [15]:

$$\mu(\mathbf{A}, \mathbf{\Phi}) = \max_{i,j} |\langle \mathbf{a}_i, \boldsymbol{\phi}_j \rangle|, \tag{2}$$

where $\mathbf{a}_i$ is the i-th row of $\mathbf{A}$, $\boldsymbol{\phi}_j$ is the j-th column of $\mathbf{\Phi}$ and $\langle \cdot \rangle$ denotes the inner product. To this date, only a few examples of the matrix $\mathbf{A}$ have been shown to satisfy this condition with high probability, for any given $\mathbf{\Phi}$. The most commonly used ones are Gaussian iid and Bernoulli iid matrices. However, in cases like ours where $\mathbf{A}$ is defined by deterministic system specifications, often there are no guarantees for incoherence. One can ignore the condition and still perform signal estimation, hoping for the best, but the reconstruction can then fail in some cases, making the whole system unreliable.

We propose to address this problem from a different perspective. Rather than trying to design a system such that its matrix $\mathbf{A}$ is incoherent with any existing dictionary, we want to find a dictionary $\mathbf{\Phi}$ that is incoherent with a given $\mathbf{A}$ and is well fitted to sparsely represent the measured signal. In order to do so, we propose a novel dictionary learning algorithm that trains a dictionary $\mathbf{\Phi}$ from a large set of image examples and simultaneously enforces incoherence between $\mathbf{A}$ and $\mathbf{\Phi}$. In the following, we refer to the proposed method as *Dictionary Learning for Incoherent Sampling* (DLIS).

3. DICTIONARY LEARNING FOR INCOHERENT SAMPLING (DLIS)

Most existing dictionary learning methods address the case when the signals or images are directly observed, i.e., when the measurement matrix is identity [16]. The signal model is then $\mathbf{X} = \mathbf{\Phi C}$, where $\mathbf{C}$ is a matrix whose columns are coefficient vectors for different signals. Since $\mathbf{C}$ is not known *a priori*, learning algorithms first estimate $\mathbf{C}$ by minimizing $\|\mathbf{Y} - \mathbf{\Phi C}\|_2^2 + \lambda\|\mathbf{C}\|_1$ (inference step) and then use those values to adapt the dictionary $\mathbf{\Phi}$ (learning step). The algorithm iterates between these two steps until convergence.

If the signals are not directly observed, but are measured through a measurement matrix $\mathbf{A}$, estimation of $\mathbf{C}$ during the inference step requires that matrices $\mathbf{A}$ and $\mathbf{\Phi}$ have small mutual coherence. This condition influences the dictionary learning process because we need to find a dictionary that not only well describes the data, but is also incoherent with the system measurement matrix $\mathbf{A}$. We propose a new dictionary learning algorithm that achieves such learning. It is also a two-step algorithm, but with two important modifications. First, since the goal of our dictionary learning is to find a dictionary from given measurements, we include the measurement matrix $\mathbf{A}$ in the inference step:

$$\text{(INFERENCE):} \qquad \hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \mathcal{J}_1,$$
$$\text{where} \quad \mathcal{J}_1 = \left[\|\mathbf{Y} - \mathbf{A\Phi C}\|_2^2 + \lambda\|\mathbf{C}\|_1\right]. \qquad (3)$$

Here, $\mathbf{Y}$ is a matrix whose columns are measurement vectors for different signal examples in the training set. Since the training data is usually huge, at each iteration we take a different subset of B randomly chosen examples. The objective function in Eq. (3) is convex and it can be easily optimized using gradient methods. The derivative of the objective is:

$$\frac{\partial \mathcal{J}_1}{\partial \mathbf{C}} = -2(\mathbf{A\Phi})^{\mathsf{T}}(\mathbf{Y} - \mathbf{A\Phi C}) + \lambda \text{sign}(\mathbf{C}). \qquad (4)$$

We define the sign function at zero to be equal zero.

Furthermore, we modify the learning step to include a penalty on the coherence between the measurement matrix $\mathbf{A}$ and the dictionary $\mathbf{\Phi}$. If we look at the definition of coherence in Eq. (2), we can see that it is $\mu = \|\mathbf{A\Phi}\|_\infty$, i.e., the infinity norm of $\mathbf{A\Phi}$. Since the infinity norm is not differentiable everywhere, we approximate it with the Frobenius norm $\|\mathbf{A\Phi}\|_F$, i.e., the ℓ_2 matrix norm. This norm is convex and differentiable everywhere, with a derivative that is fast to calculate. Alternatively, we can use an ℓ_p norm with $p > 2$ that would better approximate the infinity norm, but this would increase the computational complexity. Thus, the Frobenius norm represents a good trade-off between performance and complexity. We define the learning objective function as:

$$\text{(LEARNING):} \qquad \hat{\mathbf{\Phi}} = \arg\min_{\mathbf{\Phi}} \mathcal{J}_2, \quad \text{where}$$
$$\mathcal{J}_2 = \arg\min_{\mathbf{\Phi}} \left[\frac{1}{B}\|\mathbf{X} - \mathbf{\Phi}\hat{\mathbf{C}}\|_F^2 + \delta\|\mathbf{A\Phi}\|_F^2\right], \qquad (5)$$

where $\mathbf{X}$ is a matrix whose columns are training examples and δ is a trade-off parameter between approximation and coherence. Objective function $\mathcal{J}_2$ is convex and can be minimized using gradient methods. Its derivative over $\mathbf{\Phi}$ is:

$$\frac{\partial \mathcal{J}_2}{\partial \mathbf{\Phi}} = -\frac{2}{B}(\mathbf{X} - \mathbf{\Phi}\hat{\mathbf{C}})\hat{\mathbf{C}}^{\mathsf{T}} + 2\delta\left[\mathbf{A}^{\mathsf{T}}(\mathbf{A\Phi})\right]. \qquad (6)$$

Inference and learning steps are iterated until convergence. The proposed dictionary learning algorithm for incoherent sampling is summarized in Algorithm 1.

Once we have learned a dictionary $\mathbf{\Phi}$ that is incoherent with the measurement matrix $\mathbf{A}$, we can use it to reconstruct any signal $\mathbf{x}$ from measurements $\mathbf{y}$, by estimating its sparse coefficient vector $\mathbf{c}$ using (1).

Algorithm 1 Dictionary learning for incoherent sampling

Input: training data $\mathbf{X}_t$, measurement matrix $\mathbf{A}$, parameters $\sigma, \lambda, \delta, p, L$ (dictionary size), $B > 4L$
$[N, Q] = \text{size}(\mathbf{X}_t)$; $M = \text{size}(\mathbf{A}, 1)$
Initialize dictionary at random: $\mathbf{\Phi} \sim \mathcal{U}^{N\times L}(-0.5, 0.5)$
Run learning for p iterations (or until convergence):
for $i = 1 \to p$ **do**
  Randomly select B training signals: $\mathbf{X} = \mathbf{X}_t(:, s), s = \lceil t \rceil, t \sim \mathcal{U}^{B\times 1}(0, Q)$
  Generate noisy measurements: $\mathbf{Y} = \mathbf{AX} + \boldsymbol{\eta}$, $\boldsymbol{\eta} \sim \mathcal{N}^{M\times N}(0, \sigma)$
  Initialize coefficients: $\mathbf{C}_0 = \mathbf{0}$
  solve: $\hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \left[\|\mathbf{Y} - \mathbf{A\Phi C}\|_2^2 + \lambda\|\mathbf{C}\|_1\right]$
  solve: $\hat{\mathbf{\Phi}} = \arg\min_{\mathbf{\Phi}} \left[\frac{1}{B}\|\mathbf{X} - \mathbf{\Phi}\hat{\mathbf{C}}\|_F^2 + \delta\|\mathbf{A\Phi}\|_F^2\right]$
  normalize columns of $\hat{\mathbf{\Phi}}$: $\hat{\phi}_j := \frac{\hat{\phi}_j}{\|\hat{\phi}_j\|_2}, \forall j \in [1, L]$
  $\mathbf{\Phi} := \hat{\mathbf{\Phi}}$
end for
Output: $\mathbf{\Phi}$

4. APPLICATION TO PLENOPTIC IMAGING

We now present an application of the proposed algorithm to plenoptic imaging. Note, however, that this algorithm is general and applicable to any linear system whose measurement matrix is defined by the system configuration.

We have first simulated the PIF matrix for one on-axis lenslet using the wave-propagation analysis of the non-aberated plenoptic system [5]. This PIF simulates image formation for the case of a planar object that is in focus at the microlens array plane. Note that this object placement does not reduce the generality of the method, since the PIF matrix can be defined for any plane or for the whole volume. The training set $\mathbf{X}$ was a set of video frames from a natural movie database used in [17]. There was no pre-processing on the training set. We have learned a dictionary of atoms of size

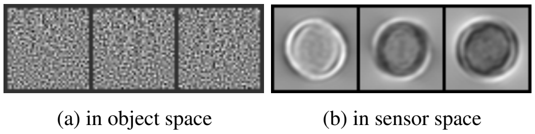

Fig. 2. Samples of learned atoms.

40×40 with $L = 1600$ atoms. This block size is chosen such that exactly one lenslet images one block, taking into account the sampling of the PIF (superpixel size 52×52). In each iteration we have selected a batch of 6400 blocks of size 40×40. Each block has been reshaped into a vector and placed into a column of $\mathbf{X}$. We have then simulated the plenoptic imaging process as $\mathbf{Y} = \mathbf{AX} + \boldsymbol{\eta}$, where $\mathbf{A}$ is the PIF matrix and $\boldsymbol{\eta}$ is white Gaussian noise of $SNR_L = 40dB$.

Learning has been initialized randomly and stopped after approximately 300 iterations, when there was no further decrease of the objective function. Fig. 2a shows three examples of learned atoms in the object space. We do not see much structure in these atoms; however, when projected on the sensor space (by multiplying with $\mathbf{A}$) the atoms exhibit circular structure corresponding to the superpixel structure (Fig. 2b). In Fig. 3 we show the DLIS reconstruction for the planar Doll object (Fig. 3a), when placed in front of the plenoptic system presented in [5]. We have divided the original object into blocks, according to the field of view of each lenslet in an 11×11 array, and simulated the superpixel at the sensor behind each lenslet (Fig. 3b). White Gaussian noise of $SNR_t = 60dB$ has been added to the sensor data. Object reconstructed using non-linear least square fitting, as proposed in [13], is shown in Fig. 3c. Using the inference step given by Eq. (3), we have estimated the sparse coefficient vectors $\hat{\mathbf{C}}$ and the reconstructed blocks as $\hat{\mathbf{X}} = \mathbf{\Phi}\hat{\mathbf{C}}$, shown in Fig. 3d. Peak Signal to Noise Ratio (PSNR) for the reconstructions using non-linear least square fitting is 22.5dB, while for the reconstruction using DLIS is 26.1dB. The visual quality is also better as shown in close-up images in Fig.3e and Fig.3f. One can notice some blocking artifacts due to per-superpixel processing of the sensor data. These can be removed by performing object reconstruction from 3×3 lenslet areas and averaging. This modification is one of our future work items.

We have also tried to reconstruct our object using dictionaries learned with conventional image sparse coding [8, 16], but the reconstruction failed in all tests. This is because plenoptic sensor images have completely different statistics than natural images. Finally, we have compared the reconstruction quality of DLIS for Doll and Lena images when the dictionary is learned: 1) with the coherence penalty ($\delta = 10^{-3} \neq 0$), yielding coherence value $\mu(\mathbf{A}, \mathbf{\Phi_1}) = 0.02$; and 2) without the coherence penalty ($\delta = 0$), yielding coherence value $\mu(\mathbf{A}, \mathbf{\Phi_2}) = 0.92$. Because the condition on mutual incoherence between $\mathbf{A}$ and $\mathbf{\Phi}$ ensures stable reconstruction, variations of PSNR for different noise realizations should be

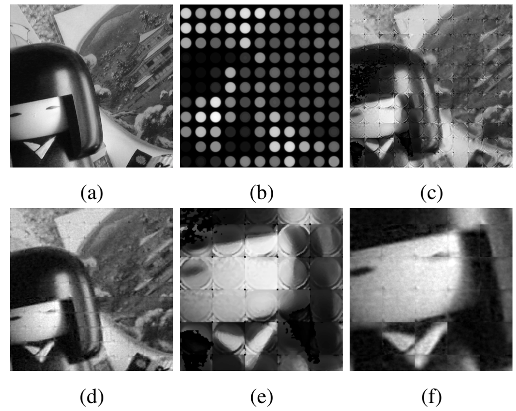

Fig. 3. Reconstruction results for the Doll object. a) original, b) image formed at the sensor, c) reconstruction using non-linear curve fitting, PSNR = 22.5dB, d) reconstruction using the DLIS, PSNR = 26.1dB, e) and f) close-up of c) and d).

SNR_t	40 dB		50 dB		60 dB		70 dB	
	PSNR [dB]		PSNR [dB]		PSNR [dB]		PSNR [dB]	
	m	σ	m	σ	m	σ	m	σ
Doll $\delta \neq 0$	21.8	1.5	24.2	0.3	25.9	0.3	26.4	0.5
Doll $\delta = 0$	20.5	2	23.7	1.5	24.0	2.7	25.4	1.5
Lena $\delta \neq 0$	21.3	1	23.5	0.3	24.9	0.3	25.5	0.1
Lena $\delta = 0$	18.7	3.7	22.9	1.8	24	1.6	24.3	1.3

Table 1. Doll and Lena reconstruction PSNR for different sensor noise SNR, each averaged over 50 runs (m denotes average PSNR, σ denotes standard deviation). The values are given for DLIS in two cases: 1) $\delta = 10^{-3} \neq 0$: dictionary is learned with the coherence penalty, and 2) $\delta = 0$: dictionary is learned without the coherence penalty.

smaller in the case 1) than in 2). Indeed, for 50 random noise realizations and SNR values of 40, 50, 60 and 70 dB, the case 1) always gives higher average PSNR and smaller standard deviation than the case 2), as shown in Table 1.

5. CONCLUSION

To the best of our knowledge, this is the first algorithm that achieves dictionary learning for a given measurement matrix while imposing a coherence penalty between $\mathbf{A}$ and $\mathbf{\Phi}$. The proposed algorithm provides an efficient way to reliably use CS in systems where the measurement matrix is defined. We have shown that the proposed method can be applied to 2D object reconstruction from plenoptic sensor measurements, yielding high-quality reconstructions. The proposed method is not limited to 2D objects and can be extended to 3D or multi-spectral objects, thus fully exploiting the advantages of plenoptic cameras. We leave this for future work, along with the convergence analysis of the learning algorithm and experiments on real plenoptic camera data.

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