AN IMAGE COLORIZATION ALGORITHM USING SPARSE OPTIMIZATION

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ABSTRACT

This paper proposes a new digital image colorization algorithm using the sparse optimization. We deal with the colorization problem where a grayscale image is colorized using a full color image with a similar composition, and formulate this problem as the sparse optimization problem. We also provide an iterative reweighted least squares (IRLS) algorithm to solve this problem approximately, and the full color image is obtained in practical time. Numerical examples show that the proposed algorithm colorizes a grayscale image well.

Index Terms— image colorization, sparse optimization, IRLS algorithm

1. INTRODUCTION

We consider a digital image colorization problem, where a full color image is obtained from a grayscale image. Various approaches have been proposed for this problem.

In [1, 2], the image colorization algorithms are proposed based on a texture recognition approach, where a grayscale image is colorized using known full color images which is similar to the grayscale image. Although these algorithms can colorize a grayscale image without any advance information about color of the grayscale image to be colorized, their results heavily depends on the quality of texture recognition.

In [3, 4, 5, 6], colorization algorithms are proposed based on the numerical optimization approach. These algorithms colorize a grayscale image with color data given in small regions. Because the colorization problem is ill-posed, they assume that grayscale images are converted from color images using a linear transformation and that the total variation (TV) norm of color images is small. Then the image colorization problem is formulated as the TV norm minimization problem. Although this approach can colorize the image efficiently if there are enough given color regions, it cannot recover the color of pixels far from the given color region.

In [6], a colorization algorithm is proposed based on the sparse optimization approach. This algorithm colorizes a grayscale image with color data given in very small regions. Because the colorization problem is ill-posed, it is assumed that the changing of each neighbor pixels is sparse. Then the image colorization problem is formulated as the l_0/l_1 norm minimization problem, the iteratively reweighted least squares (IRLS)[7, 8] algorithm is applied to recover the color image. Numerical examples indicate the efficiency of the algorithm.

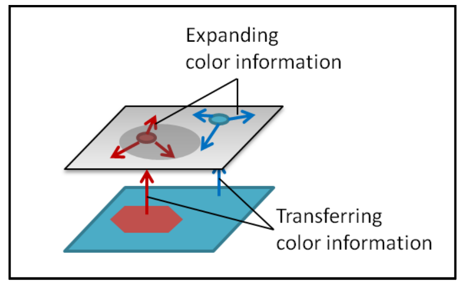

Fig. 1. Concept of the proposed algorithm.

In this paper, we propose a new colorization algorithm based on the sparse optimization using a reference color image whose composition is similar to a grayscale image. Assuming that the pixels of the same position of two images have the same color, the algorithm transfers the color information of proper pixels from the reference color image to the grayscale image and expands the color information in the same way as [6] to achieve the colorization as illustrated in Fig. 1. In order to transfer the proper color information from the reference image, the sparse optimization approach is utilized again. Numerical examples show that the proposed algorithm colorizes a grayscale image. The main contribution of this paper is to formulate the colorization problem using a reference color image as a sparse optimization problem and to propose an IRLS algorithm by modifying the algorithm proposed in [6].

2. MAIN RESULTS

2.1. Problem Formulation

This paper deals with the colorization problem where a color image is recovered from a grayscale image using a reference full color image. Let $I \in \boldsymbol{R}^{M\times N}$, $S^R \in \boldsymbol{R}^{M\times N}$, $S^G \in$

$\boldsymbol{R}^{M\times N}$ and $S^B \in \boldsymbol{R}^{M\times N}$ denote the intensity values of a grayscale image to be colorized, red, green and blue values of a reference color image, respectively. Then we consider the problem of recovering the red values $X^R \in \boldsymbol{R}^{M\times N}$, the green values $X^G \in \boldsymbol{R}^{M\times N}$ and the blue values $X^B \in \boldsymbol{R}^{M\times N}$ from the grayscale image I using S^R, S^G and S^B. We assume here that a grayscale image is converted from a color image by forming a weighted sum of the values of red, green and blue as follows,

$$I = a_r X^R + a_g X^G + a_b X^B, \tag{1}$$

where a_r, a_g and a_b are constants. Then the colorization problem considered in this paper is formulated as the following matrix completion problem,

$$\begin{array}{ll} \text{find} & X = [X^R\ X^G\ X^B] \in \boldsymbol{R}^{M\times 3N} \\ \text{subject to} & I = a_r X^R + a_g X^G + a_b X^B, \\ & [X^R_{i,j}\ X^G_{i,j}\ X^B_{i,j}] = f\left([S^R_{i,j}\ S^G_{i,j}\ S^B_{i,j}]\right), \\ & \forall (i,j) \in \mathcal{I}, \end{array} \tag{2}$$

where $A_{i,j}$ denotes the (i,j)-element of the matrix A, $\mathcal{I}$ denotes a given set of matrix indices corresponding to mapping pixels from a reference image, and f denotes a function which transfers the color information to a grayscale image from a reference image. In this problem $X = [X^R\ X^G\ X^B]$ is a design variable. This problem is obviously ill-posed, and therefore we usually provide the additional assumption that each color value changes smoothly between neighbor pixels if the grayscale intensity value changes smoothly.

If f in (2) is the identity mapping, the problem is equal to the image colorization problem with known color pixels considered in [6]. Hence we provide a colorization algorithm by modifying the algorithm proposed in [6]. This section makes a brief explain of the colorization algorithm proposed in [6]. Let

$$\boldsymbol{x}_R = \textbf{vec}\left(X^R\right),\ \boldsymbol{x}_G = \textbf{vec}\left(X^G\right),\ \boldsymbol{x}_B = \textbf{vec}\left(X^B\right),$$

$$\boldsymbol{s}_R = \textbf{vec}\left(S^R\right),\ \boldsymbol{s}_G = \textbf{vec}\left(S^G\right),\ \boldsymbol{s}_B = \textbf{vec}\left(S^B\right),$$

$$\boldsymbol{v} = \textbf{vec}(I),\ \boldsymbol{x} = [\boldsymbol{x}_R^T\ \boldsymbol{x}_G^T\ \boldsymbol{x}_B^T]^T \text{ and } \boldsymbol{s} = [\boldsymbol{s}_R^T\ \boldsymbol{s}_G^T\ \boldsymbol{s}_B^T]^T,$$

where **vec** denotes the function which converts a matrix to a vector by stacking the matrix columns successively. Let us define $U \in \boldsymbol{R}^{(M-1)\times M}$, $V \in \boldsymbol{R}^{M(N-1)\times MN}$, $\bar{U} \in \boldsymbol{R}^{3N(M-1)\times 3MN}$, $\bar{V} \in \boldsymbol{R}^{3M(N-1)\times 3MN}$, $D \in \boldsymbol{R}^{(6MN-3M-3N)\times 3MN}$, $C \in \boldsymbol{R}^{MN\times 3MN}$, $G \in \boldsymbol{R}^{3MN\times 3MN}$ and $J \in \boldsymbol{R}^{(6MN-3M-3N)\times(6MN-3M-3N)}$ as

$$U_{i,j} = \begin{cases} 1, & \text{if } i = j \\ -1, & \text{if } i+1 = j \\ 0, & \text{otherwise} \end{cases},\ V_{i,j} = \begin{cases} 1, & \text{if } i = j \\ -1, & \text{if } i+M = j, \\ 0, & \text{otherwise} \end{cases}$$

$$\bar{U} = \text{diag}(U,\ldots,U),\ \bar{V} = \text{diag}(V,V,V),\ D = [\bar{U}^T\ \bar{V}^T]^T,$$

$$C = \begin{cases} a_r, & \text{if } i = j \\ a_g, & \text{if } i + MN = j \\ a_b, & \text{if } i + 2MN = j \\ 0, & \text{otherwise} \end{cases}$$

$$G = \text{diag}(\bar{G},\bar{G},\bar{G}),\quad \bar{G}_{i,i} = \begin{cases} 1/v_i, & \text{if } v_i \neq 0 \\ 0, & \text{if } v_i = 0 \end{cases},$$

$$\text{and}\quad J_{i,i} = \begin{cases} 1, & \text{if } \left|(D[\boldsymbol{v}^T\ \boldsymbol{v}^T\ \boldsymbol{v}^T]^T)_i\right| \leq \nu_1 \\ 0, & \text{otherwise} \end{cases},$$

respectively, where $\text{diag}(A_1,\ldots,A_m)$ denotes a block diagonal matrix consisting of $A_1, \ldots, A_m$, v_i denotes the ith element of $\boldsymbol{v}$, $(\cdot)_i$ denotes the ith element of a vector and ν_1 is a given constant. The vectors $\boldsymbol{s}$ and $\boldsymbol{x}$ correspond to $[S^R\ S^G\ S^B]$ and $X = [X^R\ X^G\ X^B]$, respectively, and the matrices $\bar{U}$ and $\bar{V}$ denote vertical and horizontal difference operators. Then $D\boldsymbol{x}$ denotes the differences between the neighbor pixels of a whole image, that is, matrix J behave as the mask operation taking pixels in $\mathcal{J}$, where $\mathcal{J}$ is defined by

$$\mathcal{J} = \{(i,j) :\ |I_{i,j} - I_{i,j+1}| \leq \nu_1 \text{ or } |I_{i,j} - I_{i+1,j}| \leq \nu_1\}.$$

Assuming that each color value changes smoothly between neighbor pixels if the grayscale intensity value changes smoothly, we formulate the image colorization problem as the following sparse optimization problem,

$$\begin{array}{ll} \text{Minimize} & \mu_0\|JDG\boldsymbol{x}\|_0 + \mu_1\|(E-J)D\boldsymbol{x}\|_1 \\ \text{subject to} & \boldsymbol{v} = C\boldsymbol{x},\ x_i = f(s_i),\ \forall i \in \bar{\mathcal{I}}, \end{array} \tag{3}$$

where $\mu_0 \geq 0$ and $\mu_1 \geq 0$ are given constants, E denotes an identity matrix with a certain size, $\|\cdot\|_p$ denotes the l_p norm of a vector and $\bar{\mathcal{I}}$ denotes a given set of vector indices corresponding to $\mathcal{I}$. The first term of the objective function is to make the changes of neighbor pixels in $\mathcal{J}$ smooth, and the second term forces to colorize the pixels not in $\mathcal{J}$. Note that this problem is exactly equal to the colorization problem provided in [6]. The problem colorizes a whole image such that the pixels in the same region have the same color, and the details are written in [6].

Next, we consider the set $\mathcal{I}$ and the function f. The color information of pixels in $\mathcal{I}$ is mapped from the color reference image to the grayscale image using the function f. Although the grayscale image and the reference color image have the same composition, only some pixels of the same positions have the same RGB values. Therefore we choose the set $\mathcal{I}$ as

$$\mathcal{I} = \left\{(i,j) :\ |I_{i,j} - I^S_{i,j}| \leq \nu_2\right\}$$

for given $\nu_2 > 0$ and assume that the RGB values of pixels in $\mathcal{I}$ are given in proportion to the grayscale intensity value of a reference image, which can be described by the following equations,

$$\frac{X^R_{i,j}}{I_{i,j}} = \frac{S^R_{i,j}}{I^S_{i,j}},\ \frac{X^G_{i,j}}{I_{i,j}} = \frac{S^G_{i,j}}{I^S_{i,j}},\ \frac{X^B_{i,j}}{I_{i,j}} = \frac{S^B_{i,j}}{I^S_{i,j}},\ \forall (i,j) \in \mathcal{I}, \tag{4}$$

where, I^S is defined by $I^S = a_r S^R + a_g S^G + a_b S^B$. The equations (4) can be rewritten by $M(F\boldsymbol{s} - \boldsymbol{x}) = 0$.

$$F = \text{diag}(\bar{F},\bar{F},\bar{F}),\quad \bar{F}_{i,i} = \begin{cases} v_i/v_{Si}, & \text{if } v_{Si} \neq 0 \\ 0, & \text{if } v_{Si} = 0 \end{cases}.$$

$$M = \mathrm{diag}(\bar{M}, \bar{M}, \bar{M}), \quad \bar{M}_{i,i} = \begin{cases} 1, & \text{if } i \in \bar{\mathcal{I}} \\ 0, & \text{otherwise} \end{cases},$$

where $\boldsymbol{v}_S = \mathbf{vec}(I^S)$. Finally we formulate the colorization problem as follows,

$$\begin{array}{ll} \text{Minimize} & \mu_0 \|JDG\boldsymbol{x}\|_0 + \mu_1 \|(E-J)D\boldsymbol{x}\|_1 \\ \text{subject to} & \boldsymbol{v} = C\boldsymbol{x},\ M(F\boldsymbol{s} - \boldsymbol{x}) = 0 \end{array}. \quad (5)$$

However, experimental results indicate that the above approach does not recover the color image correctly. Because the set $\mathcal{I}$ has the large number of elements, the equality constraints of (5) are strict, and the objective function is not decreased enough, which causes the incorrect color recovery. In order to transfer necessary and sufficient color information of the reference color image, we reformulate the colorization problem using the l_0 norm of the vector of $M(F\boldsymbol{s} - \boldsymbol{x})$ as follows,

$$\begin{array}{ll} \text{Minimize} & \mu_0 \|JDG\boldsymbol{x}\|_0 + \mu_1 \|(E-J)D\boldsymbol{x}\|_1 \\ & \quad + \mu_2 \|M(F\boldsymbol{s} - \boldsymbol{x})\|_0 \\ \text{subject to} & \boldsymbol{v} = C\boldsymbol{x}, \end{array} \quad (6)$$

where $\mu_2 > 0$ is a given constant.

The problem (6) is well-posed. However, since the constraints are still strict, we relax the problem using the Lagrangian relaxation and propose the following problem,

$$\begin{array}{ll} \text{Minimize} & \mu_0 \|JDG\boldsymbol{x}\|_0 + \mu_1 \|(E-J)D\boldsymbol{x}\|_1 \\ & \quad + \mu_2 \|M(F\boldsymbol{s} - \boldsymbol{x})\|_0 + \lambda \|\boldsymbol{v} - C\boldsymbol{x}\|_2^2, \end{array} \quad (7)$$

where $\lambda > 0$ is a given constant.

2.2. Iterative Reweighted Least Squares Algorithm

Since the problem (7) is non-convex and difficult to solve exactly, this paper applies the iteratively reweighted least squares (IRLS) [7, 8]. The IRLS provides an approximate solution of the l_p norm minimization of $\boldsymbol{z} = [z_1\ z_2\ \ldots\ z_n]^T$ by solving the following least square problem iteratively,

$$\boldsymbol{z}^{(k+1)} = \arg\min_{\boldsymbol{z}} \|W^{(k)}\boldsymbol{z}\|_2^2,$$

where $\boldsymbol{z}^{(k+1)}$ denotes the solution at the $(k+1)$th iteration, $W^{(k)}$ is the diagonal matrix of weights whose diagonal elements are defined by $W_{i,i}^{(k)} = (|z_i^{(k)}|^{1-p/2} + \varepsilon)^{-1}$, and $\varepsilon > 0$ is a small constant.

Applying the IRLS to (7), the solution at the $(k+1)$th iteration is obtained as

$$\begin{aligned} &\boldsymbol{x}^{(k+1)} = \\ &\mathop{\arg\min}_{\boldsymbol{x} \in \boldsymbol{R}^{3MN}} \ \mu_0 \|W_0^{(k)} JDG\boldsymbol{x}\|_2^2 + \mu_1 \|W_1^{(k)}(E-J)D\boldsymbol{x}\|_2^2 \\ &\qquad + \mu_2 \|W_2 M(F\boldsymbol{s} - \boldsymbol{x})\|_2^2 + \lambda \|\boldsymbol{v} - C\boldsymbol{x}\|_2^2. \end{aligned} \quad (8)$$

Algorithm 1 IRLS algorithm for image colorization

Require: $\boldsymbol{v}$, $\boldsymbol{s}^R$, $\boldsymbol{s}^G$, $\boldsymbol{s}^B$, μ_0, μ_1, μ_2, ν_1, ν_2, λ and ε
  set $k = 0$
  set $W_0^{(0)}$, $W_1^{(0)}$ and $W_2^{(0)}$ as the identity matrices
  repeat
    calculate $\boldsymbol{x}^{(k+1)}$ using (10)
    update $W_0^{(k+1)}$, $W_1^{(k+1)}$ and $W_2^{(k+1)}$ using (9)
    $k \leftarrow k+1$
  until termination criterion is satisfied
Ensure: color pixels $\boldsymbol{x}^{(k+1)}$

In the above equation, $W_0^{(k)}$, $W_1^{(k)}$ and $W_2^{(k)}$ are diagonal matrices whose diagonal elements are calculated as

$$\begin{aligned} &{W_0^{(k)}}_{i,i} = (|p_i| + \varepsilon)^{-1}, \quad {W_1^{(k)}}_{i,i} = (\sqrt{q_i} + \varepsilon)^{-1}, \\ &\quad \text{and } {W_2^{(k)}}_{i,i} = (|h_i| + \varepsilon)^{-1}, \end{aligned} \quad (9)$$

respectively, where p_i, q_i and h_i are the ith element of the vectors $JDG\boldsymbol{x}^{(k)}$, $(E-J)D\boldsymbol{x}^{(k)}$ and $M(F\boldsymbol{s} - \boldsymbol{x}^{(k)})$. The least squares problem (8) can be solved simply as

$$\boldsymbol{x}^{(k+1)} = \begin{bmatrix} \mu_0 W_0^{(k)} JDG \\ \mu_1 W_1^{(k)}(E-J)D \\ \mu_2 W_2^{(k)} M \\ \lambda_1 C \end{bmatrix}^{\dagger} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mu_2 F\boldsymbol{s} \\ \lambda_1 \boldsymbol{v} \end{bmatrix}, \quad (10)$$

where $\mathbf{0}$ denotes the zero vector of size $6MN - 3M - 3N$, and $A^{\dagger}$ denotes the pseudoinverse of a matrix A. Finally this paper proposes the IRLS algorithm for the image colorization using sparse optimization as shown in Algorithm 1.

3. NUMERICAL EXAMPLES

This section shows colorization examples to demonstrate the effectiveness of the proposed algorithm. In almost all examples with the exception of some experiments, we use $\varepsilon = 10^{-4}$, $\mu_0 = 9.999 \times 10^{-1}$, $\mu_1 = 10^{-4}$, $\mu_2 = 0.5$, $\lambda = 100$, $\nu_1 = 15$ and $\nu_2 = 2$, which give the best colorization. We utilize the termination criterion $k = 10$, that is, the number of iterations is 10, because almost the same results are given by more iterations. The values of the constants (1) are adopted as $[a_r\ a_g\ a_g] = [0.29891\ 0.58661\ 0.11448]$, which is usually used in the color conversion from RGB to YCbCr according to ITU-R BT.601.

First we compare (7) with the Lagrangian relaxation of (5), which can be obtained by replacing $\|M(F\boldsymbol{s} - \boldsymbol{x})\|_0$ with $\|M(F\boldsymbol{s} - \boldsymbol{x})\|_2$ and solved by the same algorithm as Algorithm 1, in order to examine the effect of the replacement of the equality constraints by the l_0 norm. Figure 2 shows the results. we can see that proposed algorithm colorizes the grayscale image effectively while the problem (5) gives worse color images. Utilizing the sparse optimization, the color information of necessary and sufficient pixels of the reference color image is transferred to the grayscale image.

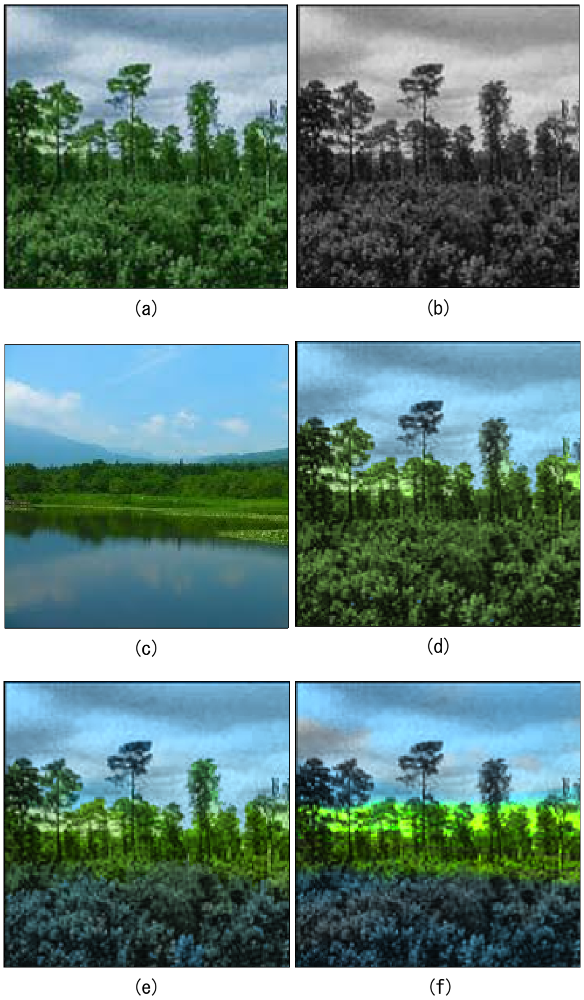

Fig. 2. (a) Original image (180 × 180 pixels). (b) Given grayscale image. (c) Given reference color image. (d) Result of the propped algorithm based on (7). (e) Result based on (5) with $\nu_2 = 2$ and (f) with $\nu_2 = 256$.

Next we apply the proposed algorithm to two examples in Fig. 3 and 4. As can be seen, the proposed algorithm can colorize the grayscale image efficiently using the reference image.

4. CONCLUSION

This paper proposes the sparse optimization for the image colorization. The image colorization problem is formulated as the sparse optimization problem, and we apply the IRLS algorithm in the problem. Numerical examples show that the proposed algorithm can colorize a grayscale image using a reference image.

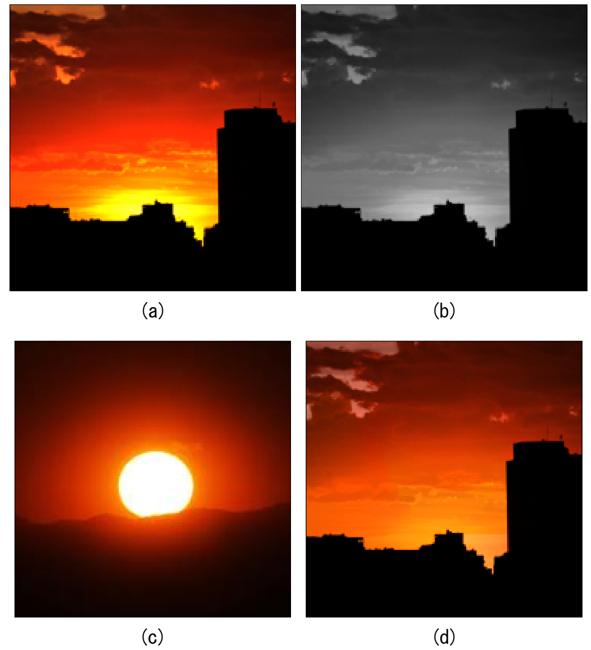

Fig. 3. (a) Original image (180 × 180 pixels). (b) Given grayscale image. (c) Given reference color image. (d) Result of the proposed algorithm.

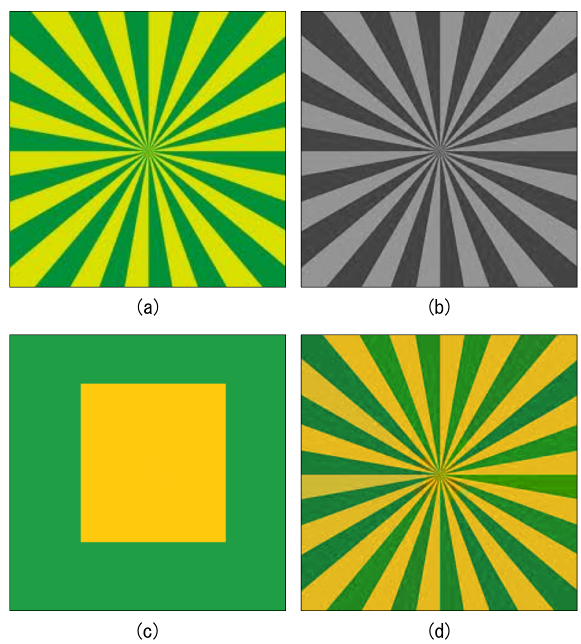

Fig. 4. (a) Original image (225 × 225 pixels). (b) Given grayscale image. (c) Given reference color image. (d) Result of the proposed algorithm.

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