AN EFFECTIVE FUZZY CLUSTERING ALGORITHM FOR IMAGE SEGMENTATION

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ABSTRACT

Fuzzy c-means (FCM) with spatial constraints has been considered as an effective algorithm for image segmentation. In this paper, we propose a new algorithm to incorporate the local spatial information with the consideration of mean template. Our algorithm is fully free of the empirically pre-defined parameters that are used in other FCM methods to balance between robustness to noise and effectiveness of preserving the image sharpness and details. Furthermore, in our algorithm, the prior probability of an image pixel is influenced by the fuzzy memberships of pixels in its immediate neighborhood to incorporate the local spatial information and intensity information. Finally, we utilize the mean template instead of the traditional hidden Markov random field (HMRF) model for estimation of prior probability. Compared to HMRF, our method is simple, easy and fast to implement.

Index Terms—Fuzzy C-Means, Image segmentation, Mean template, Spatial constraints

1. INTRODUCTION

Image segmentation is one of the most important and difficult problems in many applications, such as robot vision, object recognition and medical image processing. Although different methodologies [1-4] have been proposed for image segmentation, it remains a challenge due to overlapping intensities, low contrast of images, and noise perturbation. In the last decades, fuzzy segmentation methodologies, and especially the fuzzy c-means algorithms (FCM) [6], have been widely studied and successfully applied in image clustering and segmentation. Their fuzzy nature makes the clustering procedure able to retain more original image information than the crisp or hard clustering methodologies [5, 7].

Although the FCM algorithm usually performs well with non-noise images, it is still weak in imaging noise, outliers and other imaging artifacts. This may be caused by two aspects: one is the usage of the non-robust, Euclidean distance function, and the other does not pertain to any information about spatial context. Several attempts have been made to compensate for these drawbacks of FCM. For example, in [8-11], various more robust alternatives for the distance function of the FCM algorithm have been proposed. In [12-13], various FCM-type clustering schemes, incorporating spatial constraints into the fuzzy objective function, have been proposed. However, all the methods mentioned above have significant disadvantages such as limited robustness to outliers and high computational complexity.

To overcome the limitation of lacking spatial information, a wide variety of approaches has been proposed to incorporate spatial information into the image. A common approach is the use of the Hidden Markov Random Field (HMRF) Model [14, 15]. In the HMRF model, the spatial information in an image is encoded through the contextual constraints of neighboring pixels, which are characterized by conditional HMRF distributions. Parameter estimation in HMRF models usually relies on Maximum Likelihood (ML) or Bayesian methods. Besag [16] introduces the idea of the pseudo likelihood approximation when ML estimation is intractable.

In this paper, we first use Kullback-Leibler (KL) divergence information to regularize the fuzzy objective function with the consideration of the spatial information simultaneously. Moreover, we add weighting for distant pixels in order to distinguish among the contributions of different pixels, as the weighted parameters decrease with increasing distance. In our model, the distance function is measured by multivariate Gaussian function instead of the traditional Euclidean distance (L_2 norm) in the standard FCM algorithm. HMRF is a common way as the spatial constraints. However, the main drawback of HMRF models is that they are computationally expensive to implement, and require the additional parameter β to control the degree of image smoothness. The chosen parameter has to be both large enough to tolerate the noise, and small enough to preserve image sharpness and details. Thus, the parameter is noise dependent to some degree and selected generally based on experience. In our algorithm, the prior probability of an image pixel is influenced by the fuzzy membership of pixels in its immediate neighborhood with the help of a mean template. Different from the HMRF model, our model is fully free of the empirically adjusted parameter β.

2. FUZZY C-MEANS ALGORITHM

In the standard FCM algorithm [6, 17], the fuzzy objective function that needs to be minimized is given by

$$J_m = \sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij}^m d\left(y_i, \mu_j\right) \tag{1}$$

where y_i, i=(1,2,…,N), denotes the data set in the D-dimensional vector space, N is the total number of data points, K is the number of clusters, u_{ij} is the degree of membership of y_i in the j-th cluster, m is the weighting exponent on each fuzzy membership function u_{ij}, μ_j is the prototype of the center of cluster j, and $d(y_i, \mu_j)$ is a distance measure between point y_i and cluster center μ_j, called distance function. The Euclidean distance is usually used in standard FCM.

Ichihashi *et al*. [18] introduced another FCM variant, using a regularization to modify standard fuzzy objective function by KL information. Under this consideration, the modified fuzzy objective function becomes

$$J_\lambda = \sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij} d\left(y_i, \mu_j\right) + \lambda \sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij} \log\left(\frac{u_{ij}}{\pi_j}\right) \tag{2}$$

where λ is the model's degree of fuzziness of the fuzzy membership values and π_j is the prior probability of the j-th cluster.

More recently, Ahmed *et al*. [4] proposed another modification of the FCM, to allow the labeling of a pixel to be influenced by the labels in its immediate neighborhood, for image segmentation. The modified fuzzy objective function is defined as follows:

$$J_m = \sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij}^m \left(d\left(y_i, \mu_j\right) + \frac{a}{N_R}\sum_{m\in\mathcal{N}_i} d\left(y_m, \mu_j\right)\right) \tag{3}$$

where y_m represents the neighbor of y_i, $\mathcal{N}_i$ is the neighborhood of the i-th pixel including the i-th pixel itself, and N_R represents its cardinality. The parameter a is used to control the effect of the neighbor's term.

3. PROPOSED METHOD

It is noticed that (3) is equivalent to the usage of mean template for distance function d. In this paper, we also apply mean template on prior probability π_j to incorporate local spatial information and component information. To demonstrate our algorithm, we first combine (2) and (3), with some modification for consideration of spatial constraints, to generate a new objective function,

$$\begin{aligned} J = &\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij}\left(d\left(y_i, \mu_j\right) + \sum_{m\in\mathcal{N}_i}\frac{w_m}{R_i} d\left(y_m, \mu_j\right)\right) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{K} u_{ij}\log\left(\frac{u_{ij}}{\pi_{ij}}\right) \end{aligned} \tag{4}$$

where the distance function d in (4) is different from that in (2) and (3), which is defined by the multivariate Gaussian distribution p as follows:

$$d\left(y_i, \mu_j\right) = -\log p\left(y_i \mid \theta_j\right). \tag{5}$$

Here, θ_j represents the mean μ_j and covariance Σ_j of multivariate Gaussian distribution. Then, we have

$$d\left(y_i, \mu_j\right) = \frac{1}{2}\left(y_i - \mu_j\right)^T \Sigma_j^{-1}\left(y_i - \mu_j\right) + \frac{D}{2}\log 2\pi + \frac{1}{2}\log\Sigma_j . \tag{6}$$

In (4), $\mathcal{N}_i$ is the neighborhood of the i-th pixel, including the i-th pixel. w_m is the weighting to control the influence of the neighborhood pixels depending on their distance from the central pixel. R_i is the normalized factor, defined as

$$R_i = \sum_{m\in\mathcal{N}_i} w_m . \tag{7}$$

A simple choice of the weighted parameter w_m in (4) is that w_m=1 for all m-th pixels, and R_i equals the number of pixels in the neighborhood window. However, to incorporate the spatial information and pixel intensity value information, the strength of w_m should decrease as the distance between pixel m and i increases. For this reason, we define w_m as the function of L_{mi}, which is the spatial Euclidean distance between pixels m and i.

$$w_m = \frac{1}{\left(2\pi\delta^2\right)^{1/2}}\exp\left(-\frac{L_{mi}^2}{2\delta^2}\right), \tag{8}$$

$$\delta = \frac{\text{neighborhood window size} - 1}{4}. \tag{9}$$

Comparing (4) with (2) and (3), it can be seen that there are no pre-defined parameters a or λ in our model. Note that the prior probability π_{ij} in (4) represents the prior distribution of pixel y_i belonging to class j, which satisfies the constraint

$$0 \le \pi_{ij} \le 1 \text{ and } \sum_{j=1}^{K}\pi_{ij} = 1 . \tag{10}$$

Traditional estimation of fuzzy membership is in a FCM way. However, this derivation ignores the relationship between the neighborhoods of image pixels, thus lacks of spatial information. One possible solution of this problem is the well-known HMRF model. However, the HMRF model is too complex and time consuming. In this paper, we introduce another algorithm to apply a mean template on the membership function, given as follows

$$\pi_{ij} = \frac{\left(\sum_{m\in\mathcal{N}_i} w_m u_{mj}\right)^\beta}{\sum_{k=1}^{K}\left(\sum_{m\in\mathcal{N}_i} w_m u_{mk}\right)^\beta}, \tag{11}$$

where $\mathcal{N}_i$ is the neighborhood of the i-th pixel. β is the strength factor and can be set as 2, 3, 4…… for increasing the performance. Here, we set β=2. One possible choice of weighted parameter is $w_m = 1/\left(1 + L_{mi}^2\right)$.

For parameter learning, let us first consider the derivation of the fuzzy membership function values. This can be obtained by minimizing the objective function J over u under the constraints $\sum_{j=1}^{K} u_{ij} = 1$. Using the Lagrange multiplier, we have $J_u = J + \lambda\left(1 - \sum_{j=1}^{K} u_{ij}\right)$.

Taking the derivative of J_u with respect to u_{ij} and setting the result to zero, we have

$$u_{ij}^{(k+1)} = \frac{\pi_{ij}^{(k)}\exp\left\{-\frac{1}{2}\left(d_{ij}^{(k)} + \sum_{m\in\mathcal{N}_i}\frac{w_m}{R_i} d_{mj}^{(k)}\right)\right\}}{\sum_{h=1}^{K}\pi_{ih}^{(k)}\exp\left\{-\frac{1}{2}\left(d_{ih}^{(k)} + \sum_{m\in\mathcal{N}_i}\frac{w_m}{R_i} d_{mh}^{(k)}\right)\right\}}. \tag{12}$$

Taking the first derivatives of J_{mn} with respect to means μ_j and the covariance matrices Σ_j and then setting them to zero, yields

$$\mu_j^{(k+1)} = \frac{\sum_{i=1}^{N} u_{ij}^{(k)} \left\{ y_{ij} + \sum_{m \in \mathcal{N}_i} \frac{w_m}{R_i} y_m \right\}}{2\sum_{i=1}^{N} u_{ij}^{(k)}}. \tag{13}$$

$$\Sigma_j^{(k+1)} = \frac{\sum_{i=1}^{N} u_{ij}^{(k)}}{2\sum_{i=1}^{N} u_{ij}^{(k)}} \left\{ \left(y_i - \mu_j^{(k)}\right)\left(y_i - \mu_j^{(k)}\right)^T + \sum_{m \in \mathcal{N}_i} \frac{w_m}{R_i} \left(y_m - \mu_j^{(k)}\right)\left(y_m - \mu_j^{(k)}\right)^T \right\}. \tag{14}$$

For a deep understanding of our algorithm, we summarize the computation process of our algorithm as follows

Algorithm: Parameter learning in our model

Step 1. Use mean template for calculating the prior probability $\pi_{ij}^{(k)}$ given by (11).

Step 2. Compute the fuzzy membership function $u_{ij}^{(k+1)}$ using (12).

Step 3. Compute the quantities $\mu_j^{(k+1)}$ and $\Sigma_j^{(k+1)}$ by (13) and (14) respectively.

Step 4. Terminate the iterations if the object function converges; otherwise, increase the iteration (k=k+1) and repeat steps 1 through 4.

4. EXPERIMENTAL RESULTS

In this section, we experimentally evaluate our algorithm in a set of synthetic images and real images. We also evaluate FCM_S1 and FCM_S2 [5], FLICM [3], HMRF-FCM [19] and EGMM [20] for fair comparison. Our experiments have been developed in Matlab R2009b, and are executed on an Intel Pentium Dual-Core 2.2 GHZ CPU, 2G RAM.

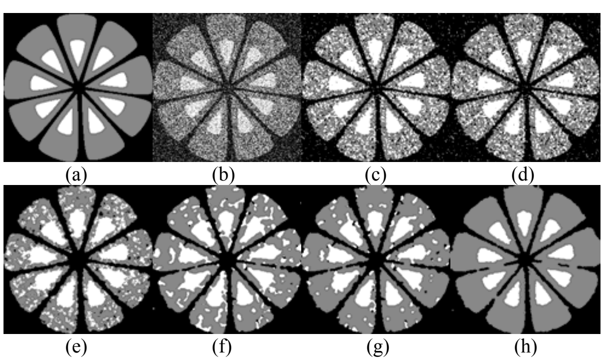

Fig. 1. (a) Original three-class image; (b) Corrupted by Gaussian noise (zero mean, 0.15 variance); (c) FCM_S1, MCR=17.50%; (d) FCM_S2, MCR=18.22%; (e) FLICM, MCR=15.46%; (f) HMRF-FCM, MCR=12.10%; (g) EGMM, MCR=8.87%; (h) Proposed method, MCR=2.92%.

In the first experiment, a three-class synthetic image (246×246, shown in Fig. 1(a)) is used to compare the performance of the proposed method with others. Fig. 1(b) shows the same image corrupted by Gaussian noise with zero mean and 0.15 variance. In order to evaluate the segmentation results, we employ the misclassification ratio (MCR) [15] in our experiments. The value of MCR is in the [0%-100%] range, where lower values indicate better segmentation performance. The segmentation results of the noised image (Fig. 1(b)) by FCM_S1, FCM_S2, FLICM, HMRF-FCM, EGMM and the proposed method are shown in Figs. 1(c)-(h). The class number is set to 3, based on previous experience. As we observe, FCM_S1 and FCM_S2 do not segment images well. Although FLICM, HMRF-FCM and EGMM can reduce the effect of noise to some extent, as [3, 19, 20] claim, they are still sensitive to heavy noise and misclassify some portions of pixels, as shown in Figs. 1(e)-(g). However, we observe that the proposed method yields outstanding segmentation results compared to the poor performance of their competitors, as seen in Fig. 1(h). The results obtained by different noise intensities are given in Table 1. As we observe, the proposed method obtains the best results compared to the other methods, and especially for heavy noised image segmentation.

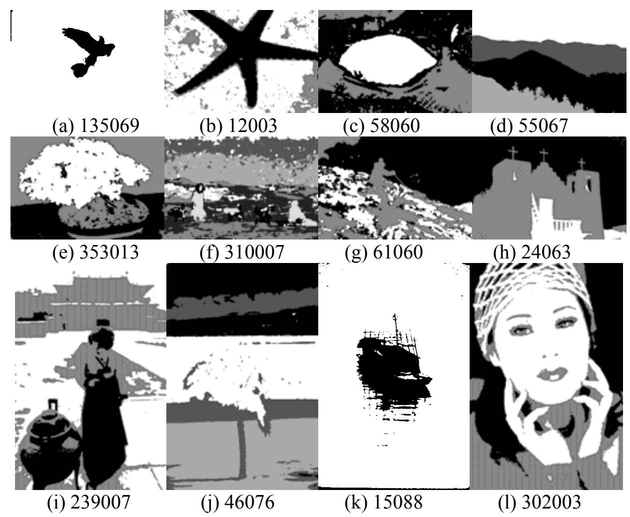

Fig. 2. Image segmentation results by proposed algorithm.

In the second experiment, we evaluate the performance of the proposed method based on a subset of the Berkeley image dataset [21], which is comprised of a set of real-world color images along with segmentation maps provided by different individuals. We employ the Probabilistic Rand (PR) index to evaluate the performance of the proposed method, with the multiple ground truths available for each image within the dataset [22]. The PR index takes values between 0 and 1, with values closer to 0 (indicating an inferior segmentation result) and values closer to 1 (indicating a better result).

Fig. 2 shows the segmentation results for original Berkeley images of various methods. For fair comparison, we also evaluate the performance of FCM_S1, FCM_S2, FLICM, HMRF-FCM and EGMM in addition to our methods. Table 2 presents the average PR values for all methods. Compared to other methods, the proposed algorithm yields the best segmentation results with the highest PR values.

In the last experiment, we try to segment the multidimensional RGB color image into three classes: the blue sky, the red roof and the white wall. The original image

(481×321) shown in Fig. 3(a) is corrupted by heavy Gaussian noise, with mean=0 and covariance=0.15. The noised image is shown in Fig. 3(b), and the segmentation results of FCM_S1, FCM_S2, FLICM, HMRF-FCM, EGMM and our proposed method are shown in Figs. 3(c)-(h), respectively. The accuracy of segmentation for FCM_S1 and FCM_S2 is quite poor. FLICM and HMRF-FCM obtain better results, but they are still sensitive to heavy noise. Although EGMM demonstrates better segmentation performance, it still misclassifies some portions of pixels at the edge region between the sky and the roof, as well as the edge region between the sky and the wall. The accuracy of the segmentation results from the proposed method, as shown in Fig. 3(h), is better than that of other methods, obtaining the highest PR values. We also evaluate the computation time for all methods in the previous experiment. The computation time t of the different methods is also presented in Fig. 3. It is noted that the computation of our methods is much faster than that of other methods except for FCM_S methods. Compared to other methods, our models can be calculated more quickly and achieve the best segmentation results.

Table 1. The misclassification ratio (MCR %) of synthetic image with additive Gaussian noise for different methods

Methods	var=0.06	var=0.09	var=0.12	var=0.15
FCM_S1	4.18	7.69	13.18	17.50
FCM_S2	3.51	8.39	13.38	18.22
FLICM	3.02	7.24	10.13	15.46
HMRF-FCM	2.96	4.69	10.44	12.10
EGMM	1.63	2.86	4.90	8.87
Proposed	1.46	1.98	2.41	2.92

5. CONCLUSION

In this paper, we propose a new effective fuzzy clustering approach for image segmentation. Gaussian distance function and spatial constraints are incorporated into the fuzzy objective function by dealing with FCM in a Bayesian way. Moreover, we add weighting for distant pixels in order to distinguish among the contributions of different pixels, as the weighted parameters decrease with increasing distance. In our algorithm, the prior probability of an image pixel is influenced by the fuzzy membership of pixels in its immediate neighborhood with the help of a mean template. Different from the HMRF model, our model is fully free of any empirically adjusted parameters and has less computation complexity. Compared with state-of-the-art technologies based on FCM, GMM, HMRF and their hybrid models, the experimental results demonstrate the improved robustness and effectiveness of our proposed algorithm.

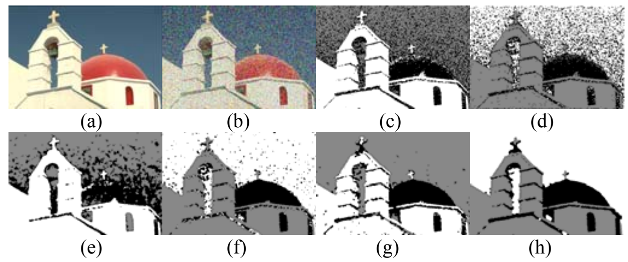

Fig. 3. RGB Image segmentation with image noise. (a) Original image; (b) Noised image; (c) FCM_S1, PR=0.0.7587, t=4.23s; (d) FCM_S2, PR=0.7523, t=4.18s; (e) FLICM, PR=0.7794, t=28.18s; (f) HMRF-FCM, PR=0.8426, t=167.53s; (g) EGMM, PR=0.8461, t=66.61s; (h) Proposed method, PR=0.8545, t=16.97s.

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7. RELATION TO PRIOR WORK

Our work has focused on the image segmentation by fuzzy clustering approach based on [3, 5, 19]. The major special characteristics of our model are summarized below:

1. Use mean template instead of HMRF to incorporate more local spatial information.
2. Use multivariate Gaussian distribution instead of traditional Euclidean distance (L_2 norm) in standard FCM algorithm.
3. Is free of parameter selection.
4. Decrease the influence of the neighborhood pixels with the increasing of their distance from the central pixel.

Table 2. Comparison of different methods for Berkeley image dataset, Probabilistic Rand (PR) Index.

Image #	Class	FCM_S1	FCM_S2	FLICM	HMRF-FCM	EGMM	Proposed
135069	2	0.981	0.981	0.983	0.984	0.985	0.972
124084	3	0.510	0.506	0.510	0.526	0.558	0.781
69020	3	0.535	0.534	0.552	0.559	0.605	0.676
12003	3	0.608	0.605	0.614	0.618	0.623	0.732
58060	3	0.573	0.563	0.584	0.615	0.622	0.645
239007	3	0.633	0.634	0.645	0.668	0.671	0.687
46076	4	0.715	0.715	0.725	0.826	0.828	0.845
55067	4	0.879	0.869	0.879	0.888	0.891	0.890
353013+0.01 noise	3	0.633	0.636	0.663	0.741	0.757	0.783
310007+0.01 noise	7	0.664	0.661	0.708	0.677	0.733	0.738
61060+0.01 noise	3	0.617	0.627	0.625	0.575	0.639	0.698
15088+0.02 noise	2	0.656	0.654	0.717	0.855	0.851	0.862
24063+0.02 noise	3	0.819	0.817	0.826	0.834	0.831	0.839
374067+0.02 noise	4	0.711	0.715	0.729	0.744	0.710	0.782
302003+0.02 noise	3	0.705	0.705	0.713	0.715	0.708	0.717
Mean		**0.683**	**0.681**	**0.698**	**0.722**	**0.734**	**0.776**

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