FAST FAN/PARALLEL BEAM CS-BASED LOW-DOSE CT RECONSTRUCTION

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ABSTRACT

Low dose X-ray Computed Tomography (CT) is clinically desired to reduce the risk of cancer caused by X-ray radiation. Compressed Sensing (CS), which allows images to be formed from incomplete data, enables large dose reduction to be achieved. Though this remains to be clinically unrealized due to excessive computation times. In this paper we demonstrate a fast, complete CS-based ℓ_2-TV minimizing CT reconstruction method applicable to both parallel and fan beam geometries to recover high quality images from highly undersampled (thus low-dose) data. We apply the fast pseudo-polar Fourier algorithm and the Central Slice Theorem to reduce the computation time of CS recovery. On a typical desktop computer, we are able to reconstruct a 512×512 CT image in approximately 30 seconds: a clinically-significant speedup compared to the many hours required by previous CS methods.

Index Terms— Computed Tomography, Central Slice Theorem, Pseudo-Polar Fourier Transform, Compressed Sensing

1. INTRODUCTION

Computed Tomography (CT) utilization has increased dramatically over the last two decades; principally due to the unsurpassed speed and detail with which cross-sectional views of soft tissues and organs can be obtained. However, CT scans deliver a relatively large radiation dose to patients, giving rise to concerns that this results in increased risk of developing cancer [1]. Using standard reconstruction methods such as Filtered Back Projection (FBP), low dose computed tomography (CT) images suffer from low contrast-to-noise ratios. A lower-dose CT scan protocol that nonetheless yields acceptable image quality is clinically desired and has been eagerly sought, especially in the last decade. Iterative reconstruction methods methods [2, 3] for obtaining high-quality images from low-dose CT scans are the subject of active research. While they outperform conventional FBP, which directly calculates the image in a single reconstruction step, they are computationally expensive enough to hinder their widespread clinical adoption. Compressed Sensing (CS) is a relatively recent innovation in signal processing that allows images to be recovered from fewer projections than required by the Nyquist sampling theorem. As CS permits reconstructions from fewer X-ray exposures, the radiation dose from a CS-based CT scan is lower than for a conventional scan. It works by combining data with valid priors about the image [4, 5]. CS-based reconstruction methods have been shown to reconstruct the exact image from approximately one tenth of the number of views needed in FBP [6], permitting a much lower-dose scanning protocol than that needed for conventional iterative reconstructions. However, most available CS-based CT reconstruction algorithms are either prohibitively computationally intensive for clinical use [7, 8] or make unphysical assumptions to accelerate the algorithm [9]. Fast solutions to CS-based CT imaging problems could enable practical lower-dose CT imaging. To find an image consistent with both data and prior assumptions about the image, CS requires solving an optimization problem similar to:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2^2 + \lambda \operatorname{TV}(\mathbf{x}), \quad (1)$$

where $\|\mathbf{x}\|_2^2 = \sum_i x_i^2$, $\hat{\mathbf{x}}$ is the vectorized reconstructed image, $\mathbf{Ax}$ is the expected data for an image $\mathbf{x}$, $\mathbf{y}$ is the observed data ($\mathbf{y} = \mathbf{Ax} + \mathbf{n}$), where $\mathbf{n}$ is noise. TVis the total variation norm (a measure of the departure of $\mathbf{x}$ from the prior assumption that images are partially smooth; $\operatorname{TV}(\mathbf{x}) = \sum_i \sqrt{(\triangle_i^h \mathbf{x})^2 + (\triangle_i^v \mathbf{x})^2}$, $\triangle_i^h$ and $\triangle_i^v$ are horizontal and vertical first order local difference operator) λ is a regularization parameter specifying the relative importance of the adherence of $\hat{\mathbf{x}}$ to the small TVnorm compared with the importance of fitting the data with a small $\|\mathbf{y}-\mathbf{Ax}\|_2^2$. Currently, most available CS-based reconstruction methods use a huge sampling matrix, $\mathbf{A}$, which models the rays going through the patient. $\mathbf{A}$ is a $(kn_\theta) \times n^2$ matrix whose entries are 1 if the k^{th} ray at angle θ_i passes through a pixel and 0 otherwise. Therefore, to reconstruct a 512×512 pixel image from 900 sensors and 1200 projection angles, $\mathbf{A}$ would be a 1080000×262144 matrix. As typical iterations each usually require two multiplications by $\mathbf{A}$ and $\mathbf{A}^\mathbf{T}$, it takes several hours of computation on typical desktop computers to reconstruct a 512×512 image with such methods [7, 8]. Here, we propose a fast CS-based CT reconstruction method which decouples information taken at different angles, making it able to recover high-quality images from few parallel projections in less than a minute using a regular desktop computer. The method is then modified for use with a fanbeam geometry. Section 2 of this paper describes the proposed method. Section 3 presents the simulation results. Section 4 contains the concluding remarks and finally in section 5 the relation between the proposed method to prior research are discussed.

2. PROPOSED METHOD

To construct a fast CT image from incomplete measurements, we perform the following steps:

1. CT scan data is obtained at the slightly nonuniformly spaced angles that correspond to those used for a pseudo-polar Fourier transform - see Fig. 1A [10].

2. The CT data thus obtained is rebinned from the fan beam to a parallel beam geometry [11], keeping track of the confidence level of each rebinned observation point.

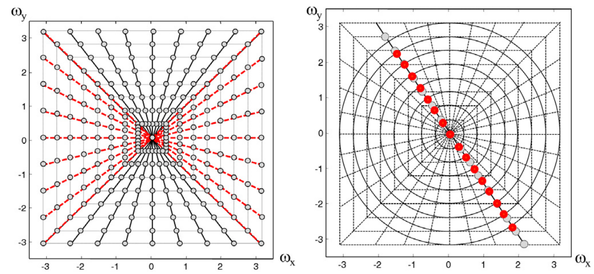
3. A 1-D Fourier transform is then taken of the rebinned data obtained for each projection angle.

4. Fourier-transformed data are interpolated from the polar measurement basis to the pseudo-polar basis using 1-D interpolations in radial direction.

5. TwIST (a CS solver) is applied on the pseudo-polar basis to find an image that simultaneously fits the data and has a low TVnorm, weighing the importance of each observation by our confidence in it, as described in step 2. The fast pseudo-polar Fourier transform allows TwIST to satisfy observations on a pseudo-polar basis while enforcing TVsparsity on a real space Cartesian basis in a computationally-efficient way [12].

Our key innovation is that it is possible to reduce the problem complexity presented to the CS solver by decomposing the image reconstruction problem into a collection of much less complex radial problems. This decomposition is made possible through the use of the Central Slice Theorem (CST) [13], which has been used in Direct Fourier Reconstruction (DFR) methods. Ignoring limitations due to finite sampling, the CST proves the following relationship between the 1D Fourier transform of the parallel projections in different angles ϕ, $P(\omega,\phi)$, and the 2D Fourier transform of the desired image, $F(\omega_x,\omega_y)$:

$$P(\omega,\phi) = F(\omega\cos\phi, \omega\sin\phi) \tag{2}$$



(A) (B)

Fig. 1: (A) Pseudo-polar grids, with basically-horizontal lines in dashed red and basically-vertical lines in solid black. (B) Polar grids (red dots) superimposed on pseudo-polar grids (gray dots). Interpolating data from a polar to a pseudo-polar coordinate system requires only a 1-D interpolation.

where ω is the radial frequency. Since CST is valid for parallel geometry, for fanbeam geometry the rays should be first rebinned to parallel beams. This additional step potentially adds extra error. We propose to use the confidence measure, computed in step 2, to modify the CS formulation to handle this extra error.

To apply TwIST to a polar-coordinate problem such as CT reconstruction it is necessary to have a rapid method to compute the real-space image given pseudo-polar Fourier transformed (PPFT - see Fig. 1) data. The PPFT can be computed quickly using 1D FFT and fractional Fourier transform [10]. As can be seen in Fig. 1, pseudo-polar grids contain basically-horizontal (BH) and basically-vertical (BV) lines of samples. The PPFT for BH samples is defined as follows; a similar expression is used for BV samples [14]:

$$\begin{aligned}\hat{x}^{BH}_{l,m} &= \frac{1}{n}\sum_{j_2=0}^{n-1}\sum_{j_1=o}^{n-1} x_{j_1,j_2} e^{-i\pi l(j_1 n + 2 j_2 m)/n^2} \\ &= \frac{1}{\sqrt{n}}\sum_{j_2=0}^{n-1}\left(\frac{1}{\sqrt{n}}\sum_{j_1=o}^{n-1} x_{j_1,j_2} e^{-i2\pi j_1 l/2n}\right).e^{-i2\pi\alpha j_2 m/n}\end{aligned} \tag{3}$$

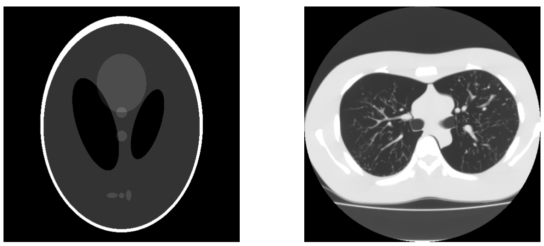
where $\alpha = \frac{l}{n}$. To reconstruct an $N \times N$ image, PPFT needs $4N^2$ samples ($2N$ projections and $2N$ samples in each projection), with a complexity of $O(N^2 \log N)$. Taking initial data along the angles of the pseudo-polar transform (which are not exactly evenly-spaced) eliminates the need for 2D interpolation to convert polar measurements to a pseudo-polar basis and reduces interpolation artifacts. A 1D interpolation suffices to transform this data into a pseudo-polar coordinate system. Since the Fourier domain could be oversampled by zeropadding the projections, this 1D interpolation is sufficiently accurate. In addition, the PPFT's adjoint has a closed form solution [10], which accelerates the reconstruction process in our algorithm.

As mentioned, we construct images by solving (1), in which we use the fast PPFT function as $\mathbf{A}$ [10]. If the true image has a sparse TVnorm, (1) enables $\mathbf{x}$ to be found given enough data.

To solve (1), TwIST applies the following equation to update the estimated image at the $(t+1)^{th}$ iteration [12]:

$$\mathbf{x}_{t+1} = (1-\alpha)\mathbf{x}_{t-1} + (\alpha-\beta)\mathbf{x}_t + \beta\Gamma_\lambda(\mathbf{x}_t). \tag{4}$$

Here, $\Gamma_\lambda(\mathbf{x}) = TV(\lambda, \mathbf{x} + \mathbf{A}^T(y - \mathbf{A}\mathbf{x}))$ where $TV(\lambda, \cdot)$ is the total variation denoising that uses λ as its threshold [15]. In the first iteration $\mathbf{x}_1 = \Gamma_\lambda(\mathbf{x}_0)$ is used. In fanbeam geometry,



(A) Shepp-Logan (B) Chest phantom

Fig. 2: Test images used here.

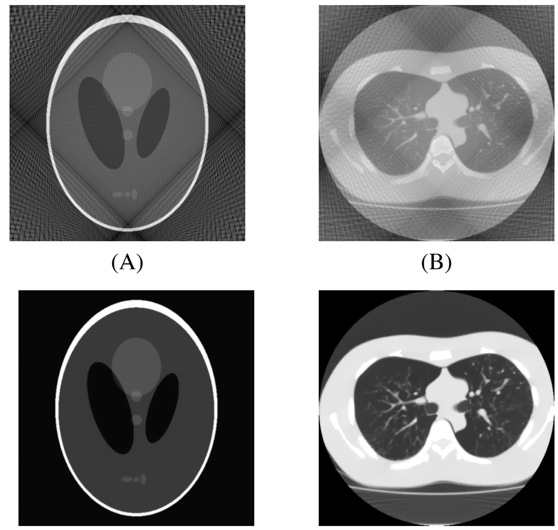
$\mathbf{y}$ is the Fourier transform of the rebinned data in the pseudo-polar coordinate system. Our confidence in each entry of $\mathbf{y}$ depends on how close the interpolated point is to a raw observation and on expected rebinning error. To improve reconstruction accuracy, we scale the ℓ_2 penalty term in (1) as follows to reflect our confidence $\mathbf{C}$ in the value of a particular observation:

$$\hat{\mathbf{x}} = \underset{\mathrm{x}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{C}\bullet\mathbf{y} - \mathbf{C}\bullet\mathbf{A}\mathbf{x}\|_2^2 + \lambda\mathrm{TV}(\mathbf{x}), \tag{5}$$

in which $\bullet$ is the element-wise multiplication and $\mathbf{C}_{i,m}$ is calculated from the distance between the closest samples in fanbeam geometry and the rebinned ray at an angle ϕ_i for the m^{th} sensor.

3. RESULTS

The proposed method was tested on the two ground truth images shown in Fig. 2. We first demonstrate that our method is capable of generating high-quality images from incomplete data Fig. 3 compares images recovered with inverse PPFT (iPPFT) to our method's output when 100 projections (only $1/10$th of the projections normally needed for FBP) and 512 samples (half of the number of samples needed for FBP) are available (5% of data needed for FBP). As can be seen in this figure, ℓ_2-TV recovery generates high-fidelity images from 5%-complete datasets. iPPFT pressrves some features of the image (highly correlated with original image) but changes the gray scale values (Hounsfield Units) and introduces artifacts (high error). The comparisons are based on a normalized error of $\|\hat{\mathbf{x}} - \mathbf{x}\|_2/\|\mathbf{x}\|_2$ and the correlation between the original image and the results, ρ. Fig. 4 shows the recovery error



(A) (B) (C) (D)

Fig. 3: Images recovered from 100 projections and 512 samples on each ray (5% of the data needed for FBP): (A) Shepp-Logan phantom recovered by iPPFT, error 98% and $\rho = 0.86$. (B) Chest phantom recovered by iPPFT, error 98% and $\rho = 0.92$. (C) Shepp-Logan phantom recovered by our proposed algorithm, error 0.9% and $\rho = 0.99$. (D) Chest phantom recovered by proposed algorithm, error 0.5% and $\rho = 0.99$.

using iPPFT and ℓ_2-TV for different numbers of projections. In each case, 512 samples per angle are taken: half the number required by FBP. Fig. 5 compares our ℓ_2-TVmethod,

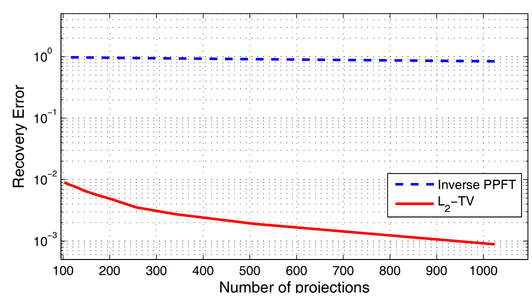


Fig. 4: Recovery error (MSE of 10 runs) of the undersampled data with different number of projections and 512 samples in each projection.

iPPFT, and weighted back projection for fanbeam geometry. 342 fanbeam projections are used to estimate the 342 closest

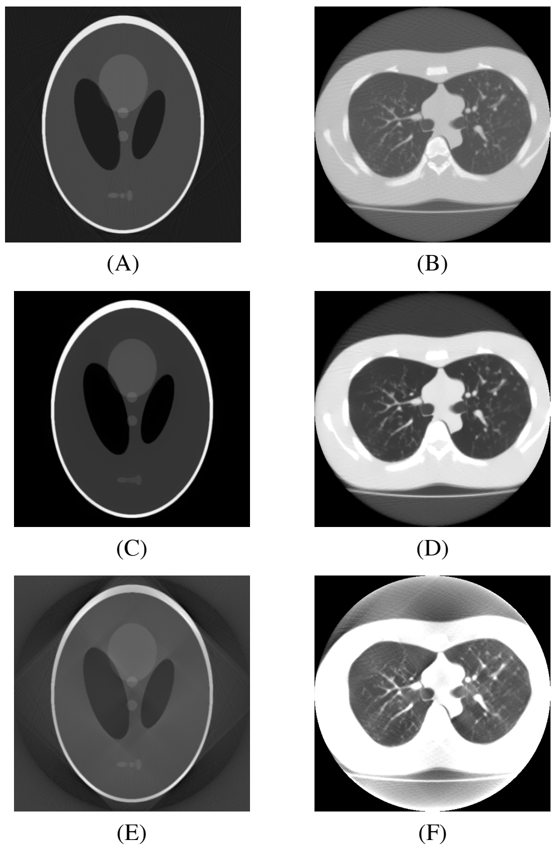


Fig. 5: Comparison of different reconstruction methods of undersampled fanbeam data (342 projections): (A), (B) inverse PPFT; (C), (D) ℓ_2-TV; (E), (F) weighted back projection. (A) and (B) have different window/level since the Hounsfield units are completely changed. Other figures have the same window/level (W=1500, L=-600 for Chest and W=1, L=0.5 for Shepp-Logan).

parallel projections on pseudo-polar grids; this is 1/3rd of the number of projections needed for FBP. As can be seen, our proposed method results in smaller error ($\approx$1%, $\rho \geq 0.99$) than iPPFT ($\approx$98%, $\rho \approx 0.90$) or weighted back projection ($\approx$56%, $\rho \approx 0.80$). Most suitable CS-based CT reconstruction methods use ART and TV smoothing [8] to iteratively solve (1). Using a standard desktop computer, Fig. 6 compares the computation time of an ART algorithm [16] as the core of such algorithms with our proposed method for parallel geometry. Typically, 100 ART iterations are required for a high-quality CT image [8]. Therefore, to get a rough estimate of the computation time of the method proposed in [8], one should multiply the computation time for ART in Fig. 6 by 100. Since our computer did not have sufficient RAM for a

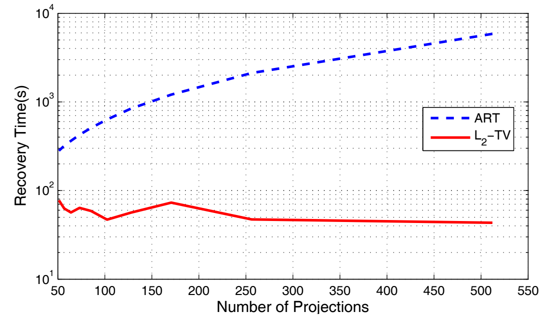


Fig. 6: Computation time of a single ART iteration and our proposed method. Images size is 256×256 in this example.

512×512 ART iteration, we have used 256×256 images in Fig. 6. In contrast, a full reconstruction of a 512×512 image with our method takes approximately 30 seconds.

4. CONCLUSION

CS has been used in CT recovery since 2006 [6] and has been used in clinical research since 2008 [7]. However, these methods are either computationally intensive or not directly applicable to CT scans, thereby hindering the clinical adoption. To our knowledge, the proposed method is the first capable of being used in a clinical settings with reasonable image computation time using a regular desktop computer. Our method uses a novel combination of the CST, CS and the pseudo-polar Fourier transform. The proposed method is able to recover 512×512 CT images with less than 1% error from undersampled parallel or fanbeam data in 30s (30000 times faster than the conventional methods) on a desktop computer.

5. RELATION WITH PRIOR WORKS

Pseudo-polar Fourier transform was proposed in 1996 [17, 18] which facilitates the direct fourier tomography. A fast and accurate PPFT calculation method was proposed by the same group in [10, 14] and has been used in different tomography based modalities such as electron microscopy [19, 20, 21], X-ray absorption and phase-contrast CT [22]. In [22, 19], TV smoothing is used iteratively with forward/backward application of PPFT to reduce the artifacts which are made by incompleteness of the available data. In [14], a 2D Discrete radon transform is proposed for fanbeam geometry for linear X-ray movement. Here we used a new combination of CST, CS and PPFT to improve our CS-based CT reconstructions.

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