

A SPARSE REPRESENTATION-BASED CLASSIFIER FOR IN-SET BIRD PHRASE VERIFICATION AND CLASSIFICATION WITH LIMITED TRAINING DATA

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ABSTRACT

The performance of a sparse representation-based (SR) classifier for in-set bird phrase verification and classification is studied. The database contains phrases segmented from songs of the Cassin's Vireo (*Vireo cassinii*). Each test phrase belongs to one of 33 phrase classes – 32 in-set categories, and 1 collective out-of-set category. Only in-set phrases are used for training. From each phrase segment, spectrographic features were extracted, followed by dimension reduction using PCA. A threshold is applied on the sparsity concentration index (SCI) computed by the SR classifier, for in-set bird phrase verification using a limited number of training tokens (3 - 7) per phrase class. When evaluated against the nearest subspace (NS) and support vector machine (SVM) classifiers using the same framework, the SR classifier has the highest classification accuracy, due to its good performances in both the verification and classification tasks.

Index Terms— Bird phrase classification, in-set verification, sparse representation, limited data, l_1 minimization.

1. INTRODUCTION

Machine-based bird song recognition facilitates animal behavior research of birds [1], and the measurement of biodiversity using bioacoustics recordings [2], which is especially useful in environments where visual identification is difficult. Applications include species identification [3, 4, 5, 6], individual bird recognition [7], and syllable or phrase classification of songs with complex lexicons [8, 9]. As “soundscape ecology” [10] receives greater attention, bird song applications will gain importance and popularity.

Classifying particular bird calls or song elements becomes especially challenging when the song repertoire is diverse, and comprises a large number of variant syllables or phrases; some species have thousands of distinct phrases in their lexicons [11]. The occurrence frequencies of individual bird song elements often resembles a “Zipf curve” [12], where a few phrases are observed frequently, but the majority of phrases are rare. The amount of training data can be further limited due to the opportunistic nature of bird song collection in geographical locations of interest, and the availability of human experts to identify and annotate the phrase types in these bird song recordings. Hence, besides the ability to perform accurate bird song phrase classification with limited data, the ability of an automated classifier to detect new phrase types that are unseen in the training set is also important. This could potentially reduce the amount of data that requires manual inspection in new audio recordings.

Since bird phrase classification is similar to automatic speech recognition (ASR), techniques that were proposed for ASR have also been applied to bird songs. For example, techniques employing Hidden Markov Models (HMMs) [8] and neural networks [13] have shown good acoustic unit recognition performance in bird songs. However, these models generally require a substantial amount of data for parameter estimation. In our previous study [14], we applied a sparse representation-based (SR) classifier on a closed-set bird phrase classification task using a few training samples per phrase class. The work is inspired by [15] which proposes this SR technique for face recognition and achieves high accuracies with just 7 images per subject. [14] is the first work to use the SR classifier for automated bird song recognition.

In this paper, we expand the classification task to distinguish between in-set (seen in training set) and out-of-set (unseen in training set) phrase classes. Out-of-set phrase classes are collectively grouped under the “Others” category. In-set bird phrase verification is performed by applying a threshold on a confidence measure of the classification output. The confidence measure computed from the SR classifier is the sparsity concentration index (SCI), which represents the maximum concentration of the computed sparse coefficients on a single in-set class. The SCI, first defined in [15], is shown to be a robust confidence measure in an outlier full/partial face image rejection application (i.e. detect images that do not belong to any of the subjects found in the training set), even in the presence of occlusions [16]. We compare the performance of the SR classifier to the support vector machine (SVM) [9, 17], and the nearest subspace (NS) [15, 18] classifiers, which have demonstrated good classification accuracies with limited training data.

2. DATA

The bioacoustic database is the same as that used in [14]. The song fragments (phrases) were segmented from audio recordings of Cassin's Vireo (*Vireo cassinii*). Fig. 1 shows the spectrogram of a song segment containing three different phrases. A Cassin's Vireo song is described as “... a jerky series of burry phrases, separated by pauses of 1 s. Each phrase is made up of 2 to 4 notes [syllables], with song often alternating between ascending and descending phrases ...” [19]. Songs from two males on two different territories in a conifer-oak forest in California were recorded, and the phrase repertoires of these two birds were similar, though not identical. Manual inspection was done using Praat (<http://www.fon.hum.uva.nl/praat>) to identify the phrase class, and mark the start and end times of each phrase in the song. The phrase classes were identified based on both auditory recognition and spectrogram inspection.

Recent changes to the annotations or textgrids of this data set

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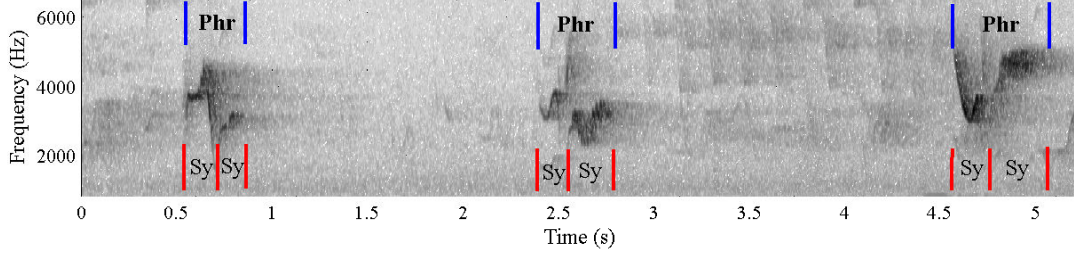


Fig. 1. A spectrogram of a Cassin's Vireo song segment. The phrase (Phr) boundaries are marked by blue lines, while the syllable (Sy) boundaries are marked by red lines.

include merging a few very similar classes to remove redundant phrase types, and corrections of wrong class labels in the annotations/textgrids. Each phrase segment or token is extracted from the original WAV file based on its start and end times in the manual annotations. There are 63 phrase classes, each with 1 to 69 tokens. In this study, the more frequently observed 32 phrase classes that have at least 10 tokens (these are the classes used in the closed-set classification task in [14]), consist of 1034 tokens, and they form the *in-set* training and test data; the 82 tokens of the remaining 31 classes form the *out-of-set* test data. For more information, please refer to [14]. The recordings and annotations for this study are available online at <http://taylor0.biology.ucla.edu/al/bioacoustics/>.

3. VERIFICATION AND CLASSIFICATION FRAMEWORK



Fig. 2. The verification and classification framework

The block diagram in Fig. 2 illustrates the common framework used with each classification algorithm (SR, SVM, or NS) to perform in-set bird phrase verification and classification. We also name this joint task as the 33-phrase class (32 in-set classes + 1 out-of-set class) bird phrase classification task. Note that no token from the *out-of-set* classes is used for training. A feature vector, b , is extracted from the input bird phrase audio segment, followed by the in-set classification. The classifier outputs an in-set class label – O , and a confidence or uncertainty measure – q , regarding the correctness of O . In-set bird phrase verification is performed by applying a fixed threshold on q . If q is a confidence measure, and it is larger than the threshold, O will be the final class label attributed to the input bird phrase, else it is classified to “Others”. The feature extraction procedure, and the implementation of the SR, SVM, and NS classifiers and the respective confidence or uncertainty measure used in our experiments are described in the following subsections.

3.1. Feature Extraction

Since spectrograms contain discriminative information that aids manual phrase annotation, we derived the features explicitly from the time-frequency spectrogram of each phrase segment. The feature extraction methodology follows that used in our previous study [14]. A file-duration-dependent frame shift is calculated, so that the spectrogram of each phrase token (of variable duration) always contains 64 frames in time. Only the 128 frequency bins between 1.5 and 6.5 kHz in the spectrogram are used, where most of the bird phrase acoustic energy falls within. For each spectrogram, the magnitudes are log-compressed and its dynamic range is normalized. The normalized spectrographic image is reshaped into a 1-D

feature vector, followed by a principal component analysis (PCA) to reduce its dimension, d to 32, 50, and 128 (corresponding to an image resizing factor of 1/16, 1/12, and 1/8, respectively). We vary d to investigate the classifiers’ performance dependency on feature dimension. Finally, the $d \times 1$ feature vector is normalized to unit length. The same feature extraction procedure is used in all the classification techniques evaluated for a fair comparison.

3.2. Classification Algorithms

3.2.1. Sparse Representation-based (SR) Classifier

The SR classification algorithm summarized in Eqs. (1) – (3), follows “Algorithm 1” described in [15] for a face recognition application. First, the SR classifier seeks a sparse linear combination of feature vectors or exemplars present in the training set that best represent the test feature vector, b . This sparse linear combination is found by solving for a sparse vector x via the l_1 minimization convex optimization problem defined in Eq. (1), where each column in matrix $\mathbf{A} \in \mathbf{R}^{d \times m}$, contains one exemplar (corresponding to one token) from the training set. Thus, $m = Kn$ in our study, where $K(=32)$ is the total number of in-set phrase classes, and n is the number of training tokens per class. The l_1 solver used to solve Eq. (1) is the l_1 -MAGIC MATLAB toolbox [20], with ϵ set to 0.05.

$$\min \|x\|_1 \text{ subject to } \|\mathbf{A}x - b\|_2 \leq \epsilon \quad (1)$$

$$r_i = b - \mathbf{A}\delta_i(x), \text{ for } i = 1, 2, \dots, 32. \quad (2)$$

$$O_{SR} = \arg \min_i \|r_i\|_2 \quad (3)$$

$$\text{SCI}(x) = \frac{K \max_i \|\delta_i(x)\|_1 / \|x\|_1 - 1}{K - 1} \in [0, 1] \quad (4)$$

After the sparse representation is found, the residual vector, r_i between b and $\mathbf{A}\delta_i(x)$ is computed in Eq. (2). The $\delta_i(x)$ function sets all x coefficients to zero, except those corresponding to class i ’s training exemplars. The class that yields the minimum residual norm is the classification decision O_{SR} , as shown in Eq. (3).

For the SR classifier, the sparsity concentration index ($\text{SCI}(x)$) in Eq. (4), is the confidence measure used for in-set bird phrase verification. The $\text{SCI}(x)$ has been successfully used for outlier face rejection in [15, 16]. This confidence measure indicates the maximum concentration of the computed sparse x coefficients are on a single in-set class. At one extreme, $\text{SCI}(x)=1$ when only the coefficients of one class are activated in x . At the other extreme, $\text{SCI}(x)=0$ when the coefficients of all classes are equally activated. In general, an in-set bird phrase should have a sparse representation where most of its non-zero coefficients corresponds to exemplars from its corresponding phrase class. On the other hand, an out-of-set bird phrase should have sparse coefficients distributed across multiple in-set classes, since it is usually not well-represented by any single class.

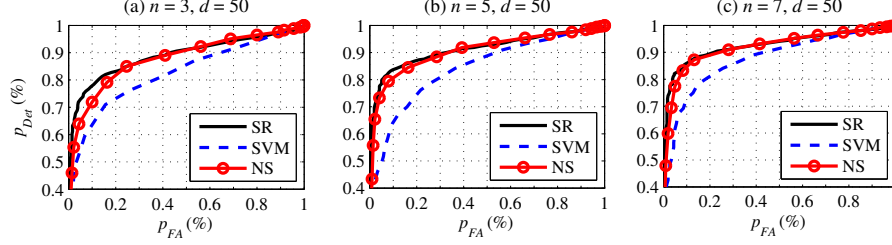


Fig. 3. ROC curves for in-set bird phrase verification with $d = 50$, for (a) $n = 3$, (b) $n = 5$, and (c) $n = 7$, where d is the dimension of the feature vector retained after PCA, and n is the number of training tokens per class.

3.2.2. Support Vector Machine (SVM) Classifier

The SVM classifier is implemented using the LIBSVM [21] – a software library for support vector machines. The confidence measure, q , used for in-set bird phrase verification is the probability estimate of the most-likely class computed by LIBSVM. The Gaussian radial basis function (RBF), $K(y, z) = \exp(-\gamma\|y - z\|^2)$ is the selected kernel, and a five-fold cross-validation (CV) training strategy is used to select the best penalty parameter, C from $\{2^{-1}, 2^0, \dots, 2^7\}$, and γ from $\{2^{-4}, 2^{-3}, \dots, 2^5\}$. This parameter tuning is done for all pairs of n and d , except when $n = 3$ and $d = 128$. For this pair, only C is tuned using CV, with a fixed $\gamma = 4$, because a significant decrease ($> 20\%$ in absolute difference) in the classification accuracy is observed with a CV-tuned γ value, due to over-fitting to the small set of training set when d is large. We have also experimented with a linear kernel function, and it gives better classification accuracies than the RBF only at $d = 128$ when $n = 3, 4$, and 5 . Even in these cases, the linear SVM still performs significantly worse than the comparative algorithms. Hence, only the RBF SVM results will be presented.

3.2.3. Nearest Subspace (NS) Classifier

The NS classifier [22] finds the class subspace that best represents the test vector, b . The class that yields the minimum residual norm between b and all class-subspace projected b , is the output of the NS classifier, O_{NS} , as shown in Eqs. (5) and (6), where P_i is the matrix containing the basis vectors of class-subspace i . Please refer to [14] for the computation of P_i . The uncertainty measure used for in-set bird phrase verification is the minimum residual norm, i.e. $q = \|r_{O_{NS}}\|_2$. If $\|r_{O_{NS}}\|_2$ is less than the threshold, O_{NS} will be the output class label.

$$r_i = b - P_i P_i^T b \quad (5)$$

$$O_{NS} = \arg \min_i \|r_i\|_2 \quad (6)$$

4. RESULTS

To evaluate the performance of the classifiers under limited data conditions, the amount of data for training is varied, with $n = 3, 4, \dots$ and 7 training tokens per class. For each value of n , five independent experiments were conducted such that a random set of in-set bird phrase tokens is used for training in each experiment. For each classifier, the averaged results over these five experiments (for each pair of n and d) are presented in this section.

4.1. ROC Curves for In-set Bird Phrase Verification

The performance of the classifiers for in-set bird phrase verification are evaluated based on the receiver operating characteristic (ROC) curve, which plots p_{Det} – the proportion of in-set phrases that is correctly identified as in-set (true positives), versus p_{FA} – the proportion of out-of-set phrases that is incorrectly identified as in-set

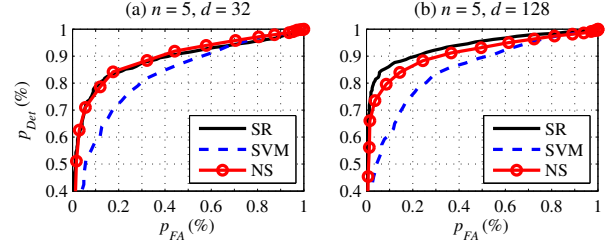


Fig. 4. ROC curves for in-set bird phrase verification with $n = 5$, for (a) $d = 32$ and (b) $d = 128$.

(false positives). To show the performance trends of the classifiers' with different values of n and d , Fig. 3 plots the ROC curves of each classifier when n is varied at a fixed $d = 50$, while Fig. 4 plots the ROC curves when d is varied at a fixed $n = 5$. In Fig. 3, it is observed that the performance of all classifiers generally improves as n increases. It is also observed in Fig. 3 that when $d = 50$, the SR classifier has the highest p_{Det} for $p_{FA} < 0.2$ among the other classifiers, and the NS classifier has the second-best performance, with an ROC curve that is very similar to the SR classifier's at higher p_{FA} . From Fig. 4, we found that the SR classifier's in-set bird phrase verification performance has a greater improvement over the NS classifier when $d = 128$ (Fig. 4(b)) compared to $d = 50$ (Fig. 3(b)). At $d = 32$, the SR classifier's performance is slightly worse than the NS's for $p_{FA} > 0.2\%$ (see Fig. 4(a)). The SVM classifier performs significantly worse than the SR and NS classifiers in this in-set bird phrase verification task for all cases of n and d investigated.

4.2. Classification Accuracy for the 33 Phrase Classes

The classification accuracy for the 33 phrase classes, Acc , of each experiment is calculated by taking an average of the percentage of in-set bird phrases that is correctly classified ($\frac{C_{in}}{N_{in}}$) and the percentage of out-of-set bird phrases that is correctly labeled as "Others" ($\frac{C_{out}}{N_{out}}$), as shown in Eq. (7). C_{in} and C_{out} is the number of in-set and out-of-set bird phrases correctly classified, respectively, while N_{in} and N_{out} is the total number of test tokens belonging to the in-set and out-of-set, respectively.

$$Acc = 0.5 \left(\frac{C_{in}}{N_{in}} + \frac{C_{out}}{N_{out}} \right) \% \quad (7)$$

Taking the average of these two components ensures that an equal importance is placed on the classification accuracies of both in-set and out-of-set test phrases, which is necessary since the number of in-set test tokens ($N_{in} = 1034 - 32n > 800$) is much greater than the number of out-of-set test tokens ($N_{out} = 82$). Otherwise, the results would be biased towards the classification accuracy of in-set phrase tokens, and the selected threshold would yield an undesirably high p_{FA} ($= 1 - \frac{C_{out}}{N_{out}}$) for the verification task. For each pair of n and d , the verification threshold is varied between 0 and 1, in steps of 0.005, and the value that yields the highest average Acc (over the

Table 1. Average *Acc* (%) for different values of n and d . The highest value for each case is boldfaced.

n	Classifier	$d = 32$	$d = 50$	$d = 128$
3	SR	81.8	83.6	N.A.
	SVM	73.5	74.8	72.4
	NS	79.8	79.6	82.3
5	SR	83.9	87.0	88.6
	SVM	75.3	76.8	77.3
	NS	82.0	84.7	84.7
7	SR	85.5	88.2	89.6
	SVM	78.7	80.8	81.6
	NS	84.5	86.4	86.4

five experiments) is the threshold used for each classifier. Table 1 shows the averaged *Acc* obtained with different values of n and d .

From Table 1, the SR classifier achieves the highest *Acc* for this 33-class bird phrase classification task in all cases, except when $n = 3$ and $d = 128$ for which the SR classifier is not able to generate a classification result. This is because when $n = 3$ and $d = 128$, there are more columns than rows in matrix \mathbf{A} such that Eq. (1) becomes an over-determined linear system, and the l_1 -solver used is usually unable to find a feasible x solution.

Note that the *Acc* values are generally lower than the closed-set classification accuracies reported in [14] for all classifiers, due to additional verification errors. This is especially true for the SVM classifier, thus its *Acc* is much lower than the SR and NS classifiers in all cases. In general, the improvement in *Acc* of the SR classifier over the second-best performing algorithm (NS classifier) increases with d at a fixed n , due to the SR classifier’s increasing verification performance gain over the other algorithms. This is also true in the case of $n = 7$, when the SR classifier’s 32-class, in-set classification accuracy (prior to in-set verification) is just slightly higher than the NS and SVM classifiers (these classification results are not shown here but reported in [14]). On the other hand, at $d = 32$, even though SR’s verification performance is slightly worse than NS’s (from the previous ROC curves), the SR classifier still has a higher *Acc* due to its higher in-set bird phrase classification accuracies [14].

5. DISCUSSION

We expect the SCI computed by the SR classifier to be a robust measure for the verification task because the sparse x vector is computed with full (global) information from all phrase classes in the training set [15]. In contrast, each class residual of the NS classifier is separately computed using information only from its respective phrase class; while LibSVM’s probability estimates [23] are obtained by combining pair-wise class probabilities derived from each one-against-one binary classifier in the multi-class SVM.

For our in-set bird phrase verification task, the SCI yields the best ROC curve at $d = 128$, and a comparable performance to the NS classifier’s residual measure at $d = 32$. The additional between-class differences that are retained in the training exemplars when a larger d is used, result in higher sparse weights concentration on the correct in-set class, which in turn improve the reliability of the SCI measure for this verification task. However, for the SR algorithm, the maximum d allowed is also upper-bounded to the total number of training tokens. One possible way to increase d is to generate additional training tokens from existing ones by perturbing the time boundaries of the token extracted. This might also improve classification accuracy when an automated bird phrase segmentation algorithm is used, whose segmented phrase boundaries could be less consistent within the same phrase class. This will be explored in future work.

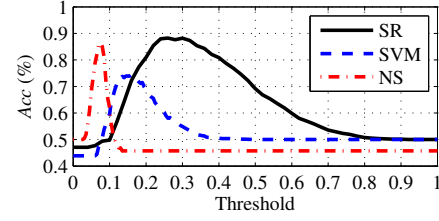


Fig. 5. *Acc* variation with verification threshold for an experiment with $n = 5$ and $d = 50$.

The 33-phrase class classification accuracy, *Acc* of the SR algorithm is also the least sensitive to verification threshold perturbations compared to NS and SVM. This is shown in Fig. 5, which plots the variation of *Acc* with the threshold used in each classifier, for a particular experiment with $n = 5$ and $d = 50$ (similar trends in *Acc* are observed at other values of n and d). The *Acc* of the SR classifier has the broadest peak lobe, while the NS classifier has the narrowest.

Since the completion of this study, new data from more Cassin’s Vireo individuals (5 different birds) were recorded. The annotators noted that the spectrograms of the 32 in-set phrase classes in the new data are similar to those used in this study. Hence, we expect similar performance trends to hold for the new data.

6. CONCLUSION

The performance of an SR classifier for an in-set bird phrase verification and classification task is studied. The data set consists of segmented Cassin’s Vireo phrases. The task is to classify these phrases into one of the 33 phrase classes – 32 in-set phrase classes, and 1 collective out-of-set class for the remaining phrase classes. The training set contains only bird phrases from the 32 in-set phrase classes. The sparsity concentration index (SCI) computed by the SR classifier, which was proposed for an outlier face rejection application [15, 16], is the measure used for in-set bird phrase verification. Compared to the NS classifier’s residual, and the SVM classifier’s probability estimate, the ROC curves show that when the feature dimension is large enough, the SR’s SCI is a more reliable measure for distinguishing between in-set and out-of-set bird phrases. The SR classifier also outperforms the NS and the SVM classifiers in the classification accuracy for these 33 phrase classes, due to good performances for both in-set verification and classification.

7. RELATION TO PRIOR WORK

In this work, an SR classifier is used to perform in-set bird phrase classification, as well as in-set bird phrase verification. This is an extension of our previous work in [14], which to our best knowledge, is the first paper to evaluate the exemplar-based SR classification technique on a bird sound recognition application. In [14], the SR classifier is used to classify bird phrases from the in-set (closed-set) classes only. The SR classification algorithm and the SCI measure were first proposed by Yang et. al. [15] for face recognition and outlier face rejection, respectively. In [15], the performances of the SR classifier for in-set face recognition and outlier face rejection are evaluated separately. New contributions of the present study include – (1) evaluating the SCI for in-set bird phrase verification, and investigating its performance dependency on feature dimension, (2) showing the overall classification accuracy of a complete system that performs both in-set bird phrase classification and verification, and (3) a sensitivity analysis of the overall classification accuracy to threshold variations during verification.

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