SHANNON ENTROPY BASED ON THE S-TRANSFORM SPECTROGRAM APPLIED ON THE CLASSIFICATION OF HEART SOUNDS

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ABSTRACT

The aim of this study is to present a complexity measure (Normalized Shannon Entropy) based on the S-Transform Spectrogram (ST-Spectrogram) plane, which can be considered as a variant of the Cohen's class. The ST-Spectrogram verifies the non-negativity condition which makes the application of the famous Shannon Entropy measure possible. A concrete application presented in this paper consists to detect pathologic heart sounds with systolic murmurs. A systolic period which contains murmurs is usually more complex than normal sound. The complexity measure applied on the ST-Spectrogram is used as feature to classify normal and pathologic heart sounds. A comparison with the classical spectrogram (STFT-Spectrogram) is performed by calculating the different Receiver Operating Curves (ROC) and the robustness against additive Gaussian noise is discussed.

Index Terms**—** Time-Frequency, Shannon Entropy, Stockwell Transform, heart sounds, classification.

1. INTRODUCTION

The Time-Frequency (TF) complexity measures aim to quantify the complexity of the signal via the Time-Frequency Representation (TFR) and not directly via the signal. In this case, the TFRs play an analogous role to a 2-D Probability Density Function (PDF) [1]. Although, not all TFRs can be considered strictly as PDF; for example, the Cohen's class which generalized all quadratic TFRs cannot satisfy the the time marginal property and the non-negativity (eq. 2) and (eq. 1) simultaneously [2]. However, it does not preclude applying complexity measures based on Cohen's TFRs planes especially when it could prove more appropriate for certain classes of signals [1].

$$\int_{-\infty}^{+\infty} C_x(t,f)df = |x(t)|^2 \ , \ \int_{-\infty}^{+\infty} C_x(t,f)dt = |X(f)|^2 \quad (1)$$

$$C_x(t,f) \geq 0 \quad (2)$$

The Stockwell Transform (ST) can be considered as a hybrid between the Short Time Frequency Transform (STFT) and the wavelet transform [3]. It can be viewed as a frequency dependent STFT or a phase corrected wavelet transform. The ST has gained popularity in the signal processing community because of its easy interpretation and fast computation [4]. A normal transition between the ST which is a linear transform and the corresponding time-frequency energy distribution is the square of the magnitude of the S-matrix named in this paper the ST-Spectrogram which is a quadratic transform. The ST-spectrogram can be considered as a variation of the Cohen's class distributions with a frequency dependent kernel function [4]. It verifies the non-negativity property which is desirable for physical interpretation and makes the famous Shannon Entropy complexity measure [5] possible which is not the case of Wigner-Ville distribution for example [1]. In addition, the ST has been shown high performance in classification and feature extraction problems applied on non-stationary signals, such as heart sounds [6, 7], power quality signals [8], etc.

The main contribution of this study is the presentation of a signal complexity measure based on Normalized Shannon Entropy (NSE) and ST-Spectrogram. The main studies which introduced the information content measure via the TFR plane [1, 9] focused on the Rényi Entropy measure applied on the Wigner-Ville distribution which precludes the using of Shannon Entropy due to the existence of negative coefficients in this distribution. In this paper, the ST-Spectrogram is used for the first time, as TFR plane, from which we estimate the information content with the NSE. A concrete application of this measure is the classification of normal and pathological heart sounds. The pathologic heart sounds are more complex than the normal sounds; hence, the possibility to use the NSE features to estimate the complexity of sounds via the ST-Spectrogram plane.

This paper is organized as follows: Section 2 presents the S-Transform and the link with the Cohen's class. It is followed by section 3 which presents a NSE complexity measure applied on the ST-Spectrogram coefficients. Section 4 describes the heart sounds and presents the results of the classification of normal and pathologic sounds. Finally, section 5 gives the conclusion and the future work.

2. S-TRANSFORM SPECTROGRAM AND COHEN'S CLASS

The S-Transform originates from two advanced signal processing tools, the Short Time Fourier Transform (STFT) and the Wavelet Transform (WT). It can be viewed as a frequency dependent STFT or a phase corrected WT. The generalized S-Transform of a time varying signal $x(t)$ is defined by:

$$S_x(t,f)=\int_{-\infty}^{+\infty}x(\tau)w(\tau-t,f)e^{-2\pi jf\tau}d\tau \qquad (3)$$

Where the window function $w(\tau-t,f)$ is chosen as:

$$w(t,f)=\frac{1}{\sigma(f)\sqrt{2\pi}}e^{\frac{-t}{2\sigma f^2}} \qquad (4)$$

And $\sigma(f)$ is a function of frequency as:

$$\sigma(f)=\frac{\alpha}{|f|} \qquad (5)$$

The window is normalized as:

$$\int_{-\infty}^{+\infty}w(t,f)dt=1 \qquad (6)$$

This gives the direct relation between the S-transform and the Fourier spectrum by averaging the local spectrum over time:

$$\int_{-\infty}^{+\infty}S_x(t,f)dt=X(f) \qquad (7)$$

Where $X(f)$ is the Fourier transform of $x(t)$.

If we consider the squared modulus of the S-Transform or the ST-Spectrogram, we obtain an energy distribution of the signal in time-frequency plane. The ST-Spectrogram is given as:

$$|S_x(t,f)|^2=\left|\int_{-\infty}^{+\infty}x(\tau)w(\tau-t)e^{-2\pi jf\tau}d\tau\right|^2 \qquad (8)$$

$$=S_x(t,f).S_x^*(t,f) \qquad (9)$$

$$=\iint x(\tau)w(\tau-t)x^*(\tau)w^*(\tau-t')e^{-j2\pi f(\tau-\tau')}d\tau d\tau' \qquad (10)$$

The relation with the Cohen Class can be given as [4]:

$$=\iiint x(u+\frac{1}{2}t)x^*(u-\frac{1}{2}t)\phi(\theta,t,f)e^{-j\theta t}e^{-jtf}e^{iu\theta}dudtd\theta \qquad (11)$$

With the frequency dependent Kernel function ∅ is given as:

$$\phi(\theta,t,f)=e^{-j\pi\theta}\int_{-\infty}^{+\infty}w(\frac{u}{f})w^*(\frac{u-\theta}{f})e^{j2\pi tu}du \qquad (12)$$

3. NORMALIZED SHANNON ENTROPY

The Shannon Entropy is a natural candidate for measuring the complexity of a signal through TFR. It is applicable on the ST-Spectrogram coefficients (C_x) since the ST-spectrogram verifies the non-negativity condition. The Shannon Entropy is defined as follows:

$$H(C_x)=-\iint C_x(t,f)\log_2 C_x(t,f)dtdf \qquad (13)$$

To normalize the Shannon entropy, we normalize first the coefficients of the ST-spectrogram as follows:

$$C_x^{norm}(t,f)=\frac{C_x(t,f)}{\iint C_x(u,v)dudv} \qquad (14)$$

The maximum of Shannon Entropy, which correspond to equiprobable events case, can be given as:

$$H_{\max}(C_x^{norm})=\log_2(n\times m) \qquad (15)$$

Where, n is the samples number of the signal $x(t)$, m is the number of frequency voices used to calculate the ST-spectrogram and $n\times m$ is the total number of coefficients in the $C_x^{norm}(t,f)$ distribution. Therefore, the normalized Shannon Entropy can be given as:

$$H_{norm}(C_x^{norm})=\frac{H(C_x^{norm})}{\log_2(n\times m)} \qquad (16)$$

4. APPLICATION ON THE CLASSFICATION OF HEART SOUNDS

4.1. Heart sounds

The analysis of the cardiac sounds solely based on the human ear is limited by the experience of the clinician for a reliable diagnosis of cardiac pathologies and to obtain all the qualitative and quantitative information about cardiac activity. Proposing an objective signal processing methods able to extract relevant information from heart sounds is a great challenge for specialists and auto-diagnosis fields. The electronic stethoscope is capable to register and optimize the quality of the acoustic heart signal, completed by the PhonoCardioGraphic (PCG) presentation of the auscultation signal.

The main application of this paper is the automatic detection of heart murmurs. Heart murmurs usually result from turbulence in blood flow or the vibration of heart tissues which can occur in a systolic or diastolic period. The presence of murmurs increases the heart sound complexity [10]. Several recent studies use methods for nonlinear and chaotic signals to estimate the signal complexity and detect murmurs [10-12]. These methods are generally based on the reconstructed state space which explores the non-linear behavior and the non-Gaussian components of the signal. However, even though it seems reasonable to expect the nonlinear and chaotic characteristics of turbulence in blood flow [13] through a vessel to be reflected in the murmurs, it is well accepted that recorded signals do not necessarily reflect the nonlinear and chaotic behavior of the underlying system [14, 15]. Moreover, application of such methods suited for nonlinear or chaotic signals might be an unnecessary increase in algorithm complexity compared to linear methods based on autocorrelation and power spectrum [15]. Therefore, in this study, we apply the complexity measure on the TFR plane (ST-Spectrogram) instead of the reconstructed state space, to detect murmurs in heart sounds.

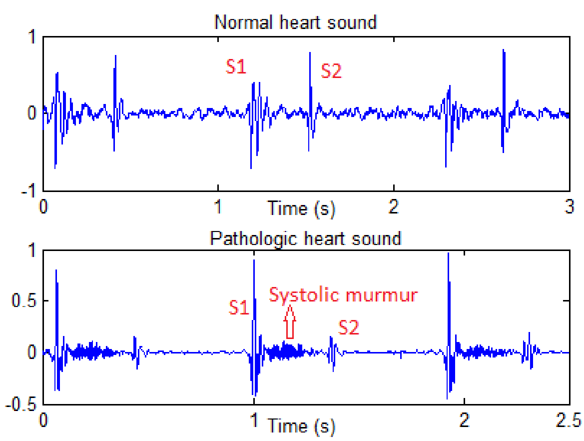

Fig.1. Example of a normal (top) and pathologic (bottom) heart sounds with systolic murmur.

4.2. Dataset

The heart sounds have been collected in the Hospital of Strasbourg (France) where Different cardiologists equipped with a prototype electronic stethoscope have contributed to a campaign of measurements. The sounds are recorded with 16 bits accuracy and 8000Hz sampling frequency in a wave format, using the software "Stetho" developed under Alcatel-Lucent license.

This study includes 30 heart sounds separated into two groups: 15 normal sounds and 15 pathologic sounds with systolic murmurs (see figure 1).

4.3. Results and discussion

The sounds are automatically segmented in order to extract systolic periods from heart sounds. Accurate segmentation of heart sounds is essential for the extraction of meaningful features, well oriented, from each part of heart cycles. The segmentation algorithm used in this paper is based on the ST (named OSSE algorithm) was proposed by the authors in [16].The results were visually inspected by a cardiologist and erroneously extracted sounds were excluded from the study. The feature extraction process extracts a NSE feature value (based on ST or STFT spectrogram) per extracted systolic signal and each of these features is averaged across available systolic signals from each subject. So from each subject in the database, we obtain one feature that quantifies the complexity of systolic periods in the heart sound. The NSE feature is evaluated by calculating the Area Under Curve (AUC) for ST and STFT for three levels of noise: first level correspond to the experimental sounds acquired in clinical conditions (the patient's position during auscultation, the surrounding noise, etc.) and two levels (Noise1 and Noise2) correspond to the two increasing levels of additive white Gaussian noise.

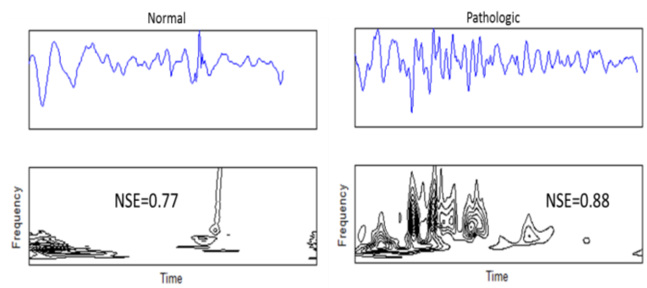

Fig.2. NSEs applied on the ST-Spectrogram plane for normal and pathologic segmented systolic sounds.

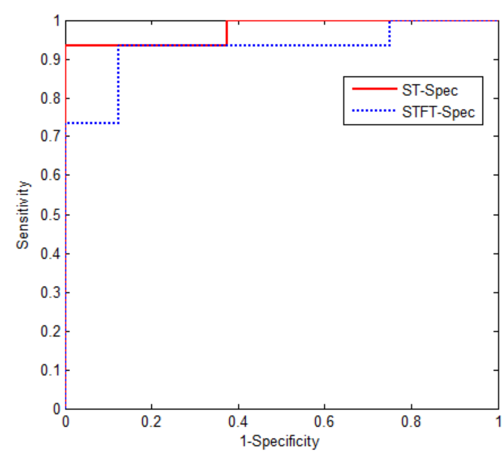

Fig.3. ROCs for the NSE applied on the ST-Spectrogram (solid line, AUC=0.98) and the STFT-Spectrogram (dashed line, AUC= 0.93).

The peaky TFRs of signals comprised of small numbers of elementary components would yield small entropy values, while the diffuse TFRs of more complicated signals would yield large entropy values [1]. Figure 2 shows an example of normal and pathologic systolic sounds and their NSEs based

on ST-Spectrogram. The number of component in pathologic sound with the presence of murmur is higher than the normal systole, which explains the higher NSE (0.88).
Figure 3 shows a comparison between the extracted NSEs based on the ST and the STFT spectrograms. The ST-Spectrogram showed the highest AUC (0.98).
The robustness of the ST-Spectrogram based features against additive noise is presented in figure 4. The AUC is 0.88 for the highest level of noise (Noise 3) which can be considered as good classification rate.

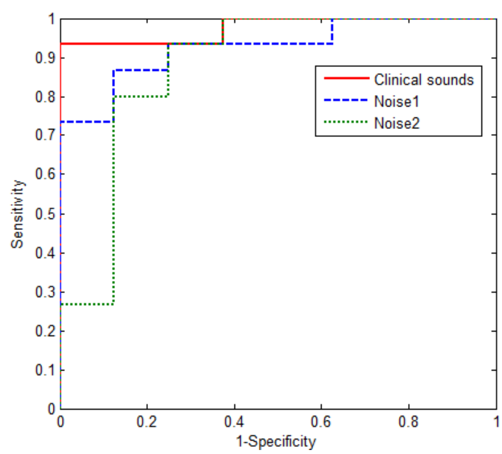

Fig.4. Variation of the ROC against white additive Gaussian noise for the ST-Spectrogram; clinical sounds without additive noise (AUC=0.98), noise1 (medium level noise, AUC=0.93) and noise2 (high level noise, AUC=0.88)

Figure 5 shows the robustness of the NSE measures based on the STFT-Spectrogram against additive noise. For the clinical sounds the extracted features give a good performance (AUC=0.93) but this is not the case in the presence of additive noise (0.83 and 0.7) where the performance decreases significantly.
The NSE based on ST-Spectrogram shows a high performance in the classification of normal and pathologic heart sounds. The results are summarized in Table 1.

Table 1: Shows the variation of AUC against white additive noise for the ST and the STFT spectrograms.

Spectrogram	AUC1	AUC2	AUC3
ST	**0.98**	**0.93**	**0.88**
STFT	0.93	0.84	0.7

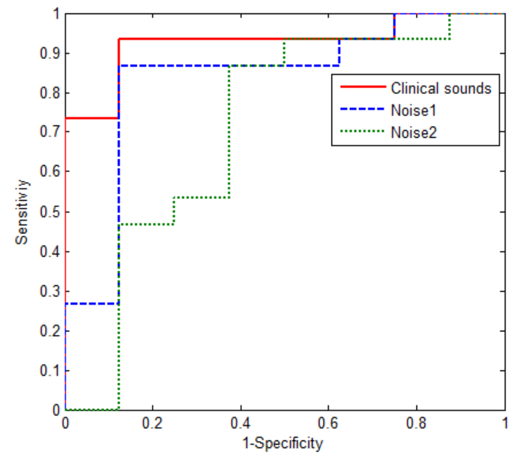

Fig.5. Variation of the ROC against white additive Gaussian noise for the STFT-Spectrogram; clinical sounds without additive noise (AUC=0.93), noise1 (medium level noise, AUC=0.84) and noise2 (high level noise, AUC=0.7)

5. CONCLUSION

We presented in this paper a Normalized Shannon Entropy measure based on the ST-Spectrogram coefficients in order to estimate the signal complexity via TFR plane. The ST-Spectrogram is a part of the Cohen's class and it verifies the non-negativity condition which makes the application of the Shannon Entropy measure possible. The NSE based on ST-Spectrogram showed a very good performance in the classification of normal and pathologic heart sounds which can be characterized by their increased complexity. A comparison with the classic STFT-Spectrogram was performed and the results showed a large preference for the ST-Spectrogram, mainly in presence of additive noise. However, to distinguish between the different origin of the detected murmurs (stenos aortic, mitral insufficiency, etc.) several features are needed and it calls to design more advanced classification approaches. Other positive TFRs [17] can be tested and compared and detailed study of mathematical foundations of ST-Spectrogram still needed.
Finally, the proposed NSE applied on ST-Spectrogram can be considered as a very promising measure to estimate signal complexity and to be applied to several feature extraction and classification tasks.

6. ACKNOWLEDGMENT

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