ON THE EIGENSPACE ESTIMATION FOR SUPERVISED MULTICHANNEL SYSTEM IDENTIFICATION

Karim Helwani[1] *and Herbert Buchner*[2]

[1]Quality and Usability Lab, Telekom Innovation Laboratories,[2] Machine Learning Group
Technische Universität Berlin, 10587 Berlin, Germany

ABSTRACT

Recently developed multichannel adaptive filtering algorithms aim at spatio-temporal decoupling of the signals by suitably chosen transformations. In this paper we establish the relation between the techniques of transform-domain adaptive filtering with application to multichannel acoustic cancellation. The link between the recently introduced source-domain and eigenspace adaptive filtering algorithms is shown by means of a generic spatial transform-domain adaptive filtering algorithm. We discuss the difference between regularizing the identification problem in the source domain and in the system eigenspace. Further, we study the estimation of the multiple-input multiple-output (MIMO) system eigenspace without modifying the highly cross correlated input signals or requiring prior knowledge of the system and highlight the validity of the estimated eigenspace due to system changes. Finally, we give simulation results proving our concept.

Index Terms— Acoustic echo, adaptive filtering, eigenspace filtering.

1. INTRODUCTION

Whenever hands-free and full-duplex communication is desired, acoustic echo cancellation (AEC) is required in order to prevent coupling the sound emitted from the P loudspeakers in the receiving room (near-end) into the outgoing Q microphone signals (which are sent back to the far-end listener or some multimedia terminal), AEC attempts to cancel out any contributions of the incoming loudspeaker signals from the microphone signal by subtracting filtered versions of the loudspeaker signals from the microphone signal. By modeling each echo path by an FIR filter of length L, multichannel AEC is a MIMO identification problem that can be solved by adaptive realizations of Wiener filtering. Wiener filters are the optimal solution in a linear least squares error (LSE) sense. In practice, adaptive filters are used to cope with time-varying systems. The solution of the adaptive filters converges asymptotically (in the mean) to the Wiener solution [1]. It has been shown that cross-correlations between the loudspeaker signals let the adaptive filter converge to a solution that depends on the characteristics of the loudspeaker signals. Any movement of the sound source in the transmission room results in a breakdown of the echo cancellation performance and requires a new adaptation of the cancellation filters [2]. Therefore, a preprocessing stage to decorrelate the transmitted signals for a unique identifiability of the echo paths is required to ensure robustness to sound source movements [2, 3]. However, for high-quality applications using music signals and massive multichannel reproduction techniques it is often desired to avoid introducing any distortion products to the desired loudspeaker signals.

A recently proposed approach for MIMO adaptive filtering in an optimally adjusted transform domains in [4] was shown to offer high convergence rates even in the absence of a preprocessing stage and prior knowledge about the system. The main idea of this approach is to restrict the estimation to the source domain which typically has lower dimensionality compared to the system eigenspace. Moreover, it can be shown that the estimated system is decoupled in the source domain. Theoretically, the system can be ideally decoupled if the eigenspace of the system to be estimated is a priori known. The eigenspace of the MIMO system can be obtained either by the singular value decomposition (SVD) of the system [5], which cannot be achieved in practice since the system is in general unknown, or by incorporating knowledge that can be obtained analytically by considering the underlying physical problem [6]. A major problem of this approach is that the analytical eigenfunctions are only available for rather simple geometries (e.g. sphere, box) and boundary conditions.

In this paper we give a study of the relation between the adaptation in the source domain and in the eigenspace of the system and highlight how an estimation of the eigenspace of a poorly excited MIMO system can be performed. As will be shown, the typical structure of a MIMO system is a matrix whose number of rows $P \cdot L$ is much greater than its number columns Q. Therefore, the left singular vectors corresponding to the non vanishing singular values span a subspace in $\mathbb{R}^{PL}$ of the dimension $R \leq Q$. The set of subspaces with particular dimension lie on a Grassmann manifold [7]. Hence, to efficiently estimate the eigenspace space of the MIMO system we can constraint our search space to a Grassmann manifold with the dimension R.

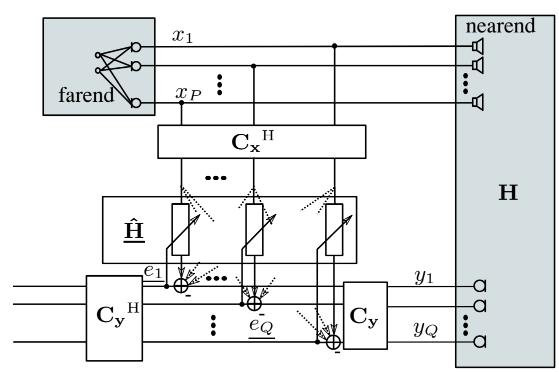

Fig. 1. Illustration of echo cancellation in transformed domain

2. PROBLEM DEFINITION AND SYSTEM MODEL

To cancel the echoes arising due to the acoustic path in the near-end, the reproduction signals $x_p(t)$ are filtered with the adaptively estimated $Q \times P \cdot L$ coefficients of the FIR filter, i.e., a replica of the actual acoustic MIMO system. The resulting signal $y(t)$ is subtracted from the near-end microphone signals, where t denotes the time instant. If the estimated echo paths $\widehat{\mathbf{H}}$ are equal to the true transfer paths $\mathbf{H}$, all disturbing echoes will be canceled from the microphone signal. Formally, the error that has to be minimized reads $\mathbf{e}(n) = \mathbf{y}(n) - \widehat{\mathbf{H}}^{\mathrm{T}}(n-1)\mathbf{x}(n)$, where $\mathbf{y}(n) = [y_1(n), y_2(n), \cdots, y_Q(n)]^{\mathrm{T}}$, $\widehat{\mathbf{H}}(n)$ denotes the $PL \times Q$ MIMO coefficient matrix composed by $P \cdot Q$ sub-filters, $\hat{\mathbf{h}}_{pq} = [\hat{h}_{pq,0}, \hat{h}_{pq,1}, \cdots, \hat{h}_{pq,L-1}]^{\mathrm{T}}$ and n the time instant. $\mathbf{x}(n)$ is the length-PL input signal vector (loudspeaker signals in the near-end) $\mathbf{x}(n) = [\mathbf{x}_1^{\mathrm{T}}(n), \mathbf{x}_2^{\mathrm{T}}(n), \cdots, \mathbf{x}_P^{\mathrm{T}}(n)]^{\mathrm{T}}$, $\mathbf{x}_p(n) = [x_p(n), x_p(n-1), \cdots, x_p(n-L+1)]^{\mathrm{T}}$.

Minimization of the error signal leads to the well known normal equation, which reads

$$\widehat{\mathbf{H}}(n) = \mathbf{R}_{\mathbf{xx}}^{-1}(n)\mathbf{R}_{\mathbf{xy}}(n), \tag{1}$$

the $PL \times PL$ correlation matrix $\mathbf{R}_{\mathbf{xx}} := \hat{\mathcal{E}}\{\mathbf{x}\,\mathbf{x}^{\mathrm{H}}\}$ contains all inter- and intrachannel correlations and is usually estimated by the recursive formula with the forgetting factor α,

$$\mathbf{R}_{\mathbf{xx}}(n) = \alpha\mathbf{R}_{\mathbf{xx}}(n) + (1-\alpha)\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n), \tag{2}$$

and $\mathbf{R}_{\mathbf{xy}} := \hat{\mathcal{E}}\{\mathbf{x}\,\mathbf{y}^{\mathrm{H}}\}$ is estimated analogously.

3. GENERIC SPATIALLY TRANSFORMED ADAPTIVE FILTERING FOR ILL-CONDITIONED PROBLEMS

To cope with the mentioned ill-conditioning problem due to the correlation of the loudspeakers signals supplementary prior solution knowledge has to be taken into account by regularizing the problem to determine an approximate solution that is stable under small changes in the initial data. A very popular regularization scheme is the energy-based regularization in the spirit of Tikhonov which can be understood as adding a constraint on the ℓ_2-norm of $\mathrm{vec}(\widehat{\mathbf{H}}(n))$. Recently developed algorithms for multichannel adaptive filtering aim at estimating the MIMO coefficients in a transform domain in which the system to be estimated has a sparse representation [4, 5, 6, 8]. So far, the choice of the transformation domain depends on the available system or signal statistics. The transformation is done by introducing the matrices $\mathbf{C}_{\mathbf{y}}$ and $\mathbf{C}_{\mathbf{x}}$ (see Fig. 1). The adaptation in the system eigenspace will be notationally highlighted by setting $\mathbf{C}_{\mathbf{x}} = \mathbf{C}_{\mathbf{x}}^{\mathrm{EAF}}$ and $\mathbf{C}_{\mathbf{y}} = \mathbf{C}_{\mathbf{y}}^{\mathrm{EAF}}$. For source-domain adaptive filtering we will set $\mathbf{C}_{\mathbf{x}} = \mathbf{C}_{\mathbf{x}}^{\mathrm{SDAF}}$ and $\mathbf{C}_{\mathbf{y}} = \mathbf{C}_{\mathbf{y}}^{\mathrm{SDAF}}$. The SDAF algorithm as presented in [4] exploits decoupling the estimate of the MIMO in the source domain. In general, the real MIMO system could not be diagonalized in the source domain. The SDAF algorithm aims at decoupling the projection of the real MIMO system onto the excitation signal subspace either by only decorrelating the sources in the far-end by considering the principal component analysis of the loudspeaker signals or ideally, by separating the sources of the far-end by employing a probabilistic models, which is connected to higher computational complexity. In this study, we concentrate on the efficient approach by decorrelating the sources as presented in [4]. In contrast to the adaptation in the source domain, in the eigenspace of the real system the MIMO system is decoupled and not only its restriction on the signal subspace.

In general, the transformed input and output signals are obtained by

$$\underline{\mathbf{y}} := \mathbf{C}_{\mathbf{y}}{}^{\mathrm{H}}\mathbf{y}, \qquad \underline{\mathbf{x}} := \mathbf{C}_{\mathbf{x}}{}^{\mathrm{H}}\mathbf{x}. \tag{3}$$

The transformed estimate of the MIMO system is

$$\underline{\widehat{\mathbf{H}}} := \mathbf{C}_{\mathbf{x}}{}^{\mathrm{H}}\widehat{\mathbf{H}}\mathbf{C}_{\mathbf{y}}. \tag{4}$$

Assuming a very special form of sparsity, namely a diagonalization of the estimated MIMO system in the transform domain, the ℓ_2-regularized cost function reads

$$\begin{aligned} J\left(\underline{\widehat{\mathbf{H}}}(n)\right) :=& \hat{\mathcal{E}}\left\{\left(\underline{\mathbf{y}}(n) - \underline{\widehat{\mathbf{H}}}^{\mathrm{T}}(n)\underline{\mathbf{x}}(n)\right)^2\right\} \\ &+ \lambda\left\|\mathrm{vec}\left(\mathrm{diag}\left(\underline{\widehat{\mathbf{H}}}(n)\right)\right)\right\|_2^2, \end{aligned} \tag{5}$$

where λ denotes the Lagrange multiplier. From a probabilistic point of view, regularization is strongly related to the maximum a posteriori criterion (MAP) which reads

$$\underline{\widehat{\mathbf{H}}}_{\mathrm{opt}} = \arg\max_{\underline{\widehat{\mathbf{H}}}} \quad p(\underline{\widehat{\mathbf{H}}}|\underline{\mathbf{x}}, \underline{\mathbf{y}}), \tag{6}$$

where $p(\cdot)$ denotes a probability density function. Note that we discarded the time dependency for clarity of presentation. $p(\underline{\widehat{\mathbf{H}}}|\underline{\mathbf{x}}, \underline{\mathbf{y}})$ denotes the a posteriori probability distribution and is given by the Bayesian rule [9],

$$p(\underline{\widehat{\mathbf{H}}}|\underline{\mathbf{x}}, \underline{\mathbf{y}}) \propto p(\underline{\mathbf{y}}|\underline{\mathbf{x}}, \underline{\widehat{\mathbf{H}}}) \cdot p(\underline{\widehat{\mathbf{H}}}). \tag{7}$$

The constraint in Eq. (5) corresponds to a prior multivariate normal distribution with zero mean and variance $\mathbf{\Sigma}_{\underline{\widehat{\mathbf{H}}}} = \sigma_{\underline{\widehat{\mathbf{H}}}}^2\mathbf{I}$,

$$p(\underline{\widehat{\mathbf{H}}}) = \frac{1}{\sqrt{(2\pi)^{PL}|\mathbf{\Sigma}_{\underline{\widehat{\mathbf{H}}}}|}} e^{-\frac{1}{2}\left(\mathrm{vec}(\underline{\widehat{\mathbf{H}}})\right)^{\mathrm{T}}\mathbf{\Sigma}_{\underline{\widehat{\mathbf{H}}}}{}^{-1}\mathrm{vec}\left(\mathrm{diag}(\underline{\widehat{\mathbf{H}}})\right)}, \tag{8}$$

where $\left|\mathbf{\Sigma}_{\underline{\widehat{\mathbf{H}}}}\right|$ denotes the determinant of $\mathbf{\Sigma}_{\underline{\widehat{\mathbf{H}}}}$. It is easy to see that maximizing the a posteriori log-likelihood is equivalent to minimizing the cost function in Eq. (5). ℓ_2 regularization aims at adding the same value to all eigenvalues of an ill-conditioned system. This has the positive effect that all eigenvalues are prevented from becoming zero.The transformed normal equation

$$\underline{\widehat{\mathbf{H}}} = \underline{\mathbf{T}}_{\mathbf{xx}}^{-1}\underline{\mathbf{T}}_{\mathbf{xy}}, \tag{9}$$

with $\underline{\mathbf{T}}_{\mathbf{xx}}$ and $\underline{\mathbf{T}}_{\mathbf{xy}}$ denoting the diagonal transformed auto- and crosscorrelation matrices $\mathbf{R}_{\mathbf{xx}}$, $\mathbf{R}_{\mathbf{xy}}$, respectively. Estimating the MIMO coefficients in the transform domain using a Newton-based algorithm can be summarized by the following steps:

1. Calculating the Hessian matrix of the constrained cost function as given in Eq. (5). This yields a summation of the transformed autocorrelation matrix with a weighted unity matrix $\mathbf{I}$. The transformed autocorrelation matrix can be estimated similarly to Eq. (2) using

$$\underline{\mathbf{T}}_{\mathbf{xx}}(n) = \alpha\underline{\mathbf{T}}_{\mathbf{xx}}(n-1) + (1-\alpha)\underline{\mathbf{x}}(n)\,\underline{\mathbf{x}}^H(n). \tag{10}$$

2. Computing the regularized inverse of the diagonal Hessian matrix

$$\mathbf{Hess}^{-1}(n) = \left(\underline{\mathbf{T}}_{\mathbf{xx}}(n) + \lambda\mathbf{I}\right)^{-1}, \tag{11}$$

3. A basis update step is required to ensure an estimation in the actual transform domain [8]

$$\mathbf{G}_{\mathbf{C}_\mathbf{x}} = {\mathbf{C}_\mathbf{x}}^{\mathrm{H}}(n)\mathbf{C}_\mathbf{x}(n-1), \tag{12}$$

$$\mathbf{G}_{\mathbf{C}_\mathbf{y}} = {\mathbf{C}_\mathbf{y}}^{\mathrm{H}}(n-1)\mathbf{C}_\mathbf{y}(n). \tag{13}$$

Note that the estimation in the system eigenspace does not require updating the transformation domain due to changes in the source domain (e.g., due to changes in the far-end).

4. Finally, computing the error and updating the filter coefficients:

$$\underline{\mathbf{e}}(n) = \underline{\mathbf{y}}(n) - \left(\mathbf{G}_{\mathbf{C}_\mathbf{x}}\underline{\widehat{\mathbf{H}}}(n-1)\mathbf{G}_{\mathbf{C}_\mathbf{y}}\right)^{\mathrm{T}}\underline{\mathbf{x}}(n), \tag{14}$$

$$\underline{\widehat{\mathbf{H}}}(n) = \mathbf{G}_{\mathbf{C}_\mathbf{x}}\underline{\widehat{\mathbf{H}}}(n-1)\mathbf{G}_{\mathbf{C}_\mathbf{y}} + \mathbf{Hess}^{-1}\underline{\mathbf{x}}(n)\underline{\mathbf{e}}^{\mathrm{T}}(n). \tag{15}$$

4. SYSTEM EIGENSPACE ESTIMATION

In the following, we outline how an estimation of the eigenspace ($\mathbf{C}_\mathbf{x}^{\mathrm{EAF}}$, $\mathbf{C}_\mathbf{y}^{\mathrm{EAF}}$) of an unknown and poorly excited MIMO system can be performed without preprocessing the input signals or incorporating prior knowledge. As an input of the estimation process, we assume having only the excitation signal $\mathbf{x}$ and an estimation of the MIMO system in the source domain. The advantages of the estimation in the system eigenspace are twofold. First, the system is decoupled in its eigenspace. Therefore, in the estimation process we do not have to consider the off-diagonals of the transformed autocorrelation matrix $\underline{\mathbf{T}}_{\mathbf{xx}}$. The second advantage becomes clear by assuming the system $\mathbf{H}$ to be non degenerate, such that their singular values are unique, a robust estimation can be performed simply by detecting the poorly excited modes corresponding to particular singular vectors of the system such that a mode-selective regularization can be done. The estimation in the source domain, i.e., when $\mathbf{C}_\mathbf{x}$ and $\mathbf{C}_\mathbf{y}$ depend on the signal in the far-end, leads to a different regularization strategy. Especially, if $\mathbf{C}_\mathbf{x}$ is chosen to contains the eigenvectors of $\mathbf{R}_{\mathbf{xx}}$ it can easily be verified that the ℓ_2-norm regularization leads to an estimate which is equivalent to the one obtained by the pseudoinverse ${\mathbf{R}_{\mathbf{xx}}}^{\dagger} := \lim_{\delta\to 0}\left({\mathbf{R}_{\mathbf{xx}}}^{\mathrm{H}}\mathbf{R}_{\mathbf{xx}} + \delta\mathbf{I}\right){\mathbf{R}_{\mathbf{xx}}}^{\mathrm{H}}$ [10].

Let us choose the following symmetric matrix for the projection on the signal subspace

$$\mathbf{P} := \mathbf{R}_{\mathbf{xx}}{\mathbf{R}_{\mathbf{xx}}}^{\dagger}. \tag{16}$$

Any ℓ_2-norm regularized solution of the ill-conditioned normal equation solution can be seen as a projection of an optimal estimation of the actual system on the signal subspace. This can be verified using the identities

$$\widehat{\mathbf{H}}^{\mathrm{T}}\mathbf{P} = {\mathbf{R}_{\mathbf{xy}}}^{\mathrm{T}}{\mathbf{R}_{\mathbf{xx}}}^{\dagger}\mathbf{P} = {\mathbf{R}_{\mathbf{xy}}}^{\mathrm{T}}{\mathbf{R}_{\mathbf{xx}}}^{\dagger}, \tag{17}$$

since $\mathbf{R}_{\mathbf{xx}}$ is symmetric, using Eq.(16) and the properties of the pseudoinverse. Hence,

$$\widehat{\mathbf{H}}^{\mathrm{T}}\mathbf{P} = \widehat{\mathbf{H}}^{\mathrm{T}}. \tag{18}$$

In the following we show how to estimate the right singular vectors of $\mathbf{H}$ from projections on low rank signal spaces. Once the right singular vectors are estimated, the left singular vectors can simply be calculated as we will show later on. The left singular vectors correspond to the column range of $\mathbf{H}$. Let us assume that a projection of the optimal estimated MIMO system on the rank deficient excitation data are given,

$$\widehat{\mathbf{H}}^{\mathrm{T}} = \widehat{\mathbf{H}}'^{\mathrm{T}}\mathbf{P}, \tag{19}$$

where $\widehat{\mathbf{H}}'$ denotes the optimal estimation of the MIMO system assuming full rank excitation. For an optimal embedding of the column vectors in an orthogonal subspace we have to minimize the following cost function

$$J(\mathbf{C_1}) := \left\|\mathbf{P}^{\mathrm{T}}\left(\widehat{\widehat{\mathbf{h}}}' - \mathbf{C_1}\underline{\widehat{\widehat{\mathbf{h}}}}'\right)\right\|_2^2, \tag{20}$$

where $\widehat{\widehat{\mathbf{h}}}'$ denotes a column in the matrix $\widehat{\mathbf{H}}$ representing the subfilters from all loudspeakers to a microphone. With $\mathbf{C_{1_P}} := \mathbf{P}^{\mathrm{T}}\mathbf{C_1}$ we can calculate $\underline{\widehat{\widehat{\mathbf{h}}}}'$ using

$$\underline{\widehat{\widehat{\mathbf{h}}}}' = \left({\mathbf{C_{1_P}}}^{\mathrm{T}}\mathbf{C_{1_P}}\right)^{-1}{\mathbf{C_{1_P}}}^{\mathrm{T}}\widehat{\widehat{\mathbf{h}}}', \tag{21}$$

where $\mathbf{C_1}$ is a unitary matrix with the dimensions $PL \times R$, and R denotes the rank of the MIMO system to be estimated. Note that in Eq. (21) $\widehat{\widehat{\mathbf{h}}}'$ is not explicitly required since it is multiplied with ${\mathbf{C_{1_P}}}^{\mathrm{T}}$. Minimizing the cost function in (20) can be done efficiently by incorporating suitable prior knowledge about the matrix $\mathbf{C_1}$ [11]. It has to represent an orthogonal basis for the subspace of the MIMO system column vectors. Hence, we can constrain our search space on a Grassmann manifold. In our special setup the Grassmannian is a compact Riemannian manifold in $\mathbb{R}^{PL}$. Each point in the Grassmann manifold of dimension R represents a subspace which is in turn represented by a unitary matrix $\mathbf{C_1}$ with the dimensions $PL \times R$. The geometry of algorithms with such constraints was studied in [7] and it has recently found a variety of applications in signal subspace estimation, e.g., in [12]. Here, we are interested in estimating the eigenspace of a poorly excited MIMO system. The derivation of gradient descent algorithm for the estimation of the subspace spanned by the left singular vectors of the MIMO system can be summarized as follows [7, 12]: The gradient of the cost function (20) on the Grassmannian reads

$$\nabla J = \left(\mathbf{I} - \mathbf{C_1}{\mathbf{C_1}}^{\mathrm{T}}\right)\frac{\partial J}{\partial \mathbf{C_1}}, \tag{22}$$

with

$$\frac{\partial J}{\partial \mathbf{C_1}} = -2\left(\mathbf{P}^{\mathrm{T}}\left(\widehat{\widehat{\mathbf{h}}}' - \mathbf{C_1}\underline{\widehat{\widehat{\mathbf{h}}}}'\right)\right)\underline{\widehat{\widehat{\mathbf{h}}}}'^{\mathrm{T}}. \tag{23}$$

Using the definition

$$\mathbf{r} := \mathbf{P}^{\mathrm{T}}\left(\widehat{\widehat{\mathbf{h}}}' - \mathbf{C_1}\underline{\widehat{\widehat{\mathbf{h}}}}'\right), \tag{24}$$

we substitute in (23) and obtain

$$\nabla J = -2\left(\mathbf{I} - \mathbf{C_1}{\mathbf{C_1}}^{\mathrm{T}}\right)\mathbf{r}\underline{\widehat{\widehat{\mathbf{h}}}}'^{\mathrm{T}}. \tag{25}$$

To derive a gradient descent algorithm we approximate the Hessian of the cost function by the unity matrix. The geodesic update rule in the direction ∇J for a step size η is then given according to [7] as

$$\mathbf{C_1} = {\mathbf{C_1}}' + \left[\begin{array}{cc} {\mathbf{C_1}}'\mathbf{V} & \mathbf{U} \end{array}\right] \cdot \left[\begin{array}{c} \cos(\mu\eta) - 1 \\ \sin(\mu\eta) \end{array}\right]\mathbf{V}^{\mathrm{T}}. \tag{26}$$

Here, ${\mathbf{C_1}}'$ denotes the old estimation, and μ is the single singular value of the rank-one matrix $\Delta := -\nabla J$ as obtained from its compact singular value decomposition. $\mathbf{U}$ denotes the matrix with the left singular vectors of Δ, and $\mathbf{V}$ contains the right singular vectors.

An orthogonal basis for the nullspace of the column vectors of $\widehat{\mathbf{H}}$ can be obtained using the Gram-Schmidt process on the matrix $\mathbf{I} - \mathbf{C_1}\mathbf{C_1}^{\dagger}$ or its QR-decomposition with pivoting and choosing the last $P \cdot L - R$ vectors [10]. We denote the resulting matrix containing the orthogonal basis vectors of the nullspace $\mathbf{N_c}$. Hence, the matrix $\mathbf{C}_{\mathbf{x}}^{\mathrm{EAF}} := [\mathbf{C_1}|\mathbf{N_c}]$ decomposes the system into two orthogonal parts: one in the system subspace as well as a part in the nullspace of the system.

Finally, applying the QR-decomposition with pivoting on the matrix $\widehat{\mathbf{H}}^{\mathrm{T}}\mathbf{C}_{\mathbf{x}}^{\mathrm{EAF}}$ offers the corresponding left singular vectors $\mathbf{C}_{\mathbf{y}}^{\mathrm{EAF}}$ of the estimated MIMO system. The reader is referred to the SVD algorithm in [10] for verification.

4.1. Validity of the Estimated Eigenspace

The obtained matrices $\mathbf{C_x}$ and $\mathbf{C_y}$ diagonalize simultaneously all MIMO systems $\{\mathbf{H}_i\}$ fulfilling the condition

$$\widehat{\mathbf{H}}^{\mathrm{T}}\mathbf{H}_i \stackrel{!}{=} \mathbf{H}_i^{\mathrm{T}}\widehat{\mathbf{H}}, \qquad \text{and} \qquad \widehat{\mathbf{H}}\mathbf{H}_i^{\mathrm{T}} \stackrel{!}{=} \mathbf{H}_i\widehat{\mathbf{H}}^{\mathrm{T}}. \qquad (27)$$

Hence, system changes within the set $\{\mathbf{H}_i\}$ do not require updating the transformation matrices in the process of the filter coefficient adaptation.

4.2. Adaptation Control

So far, the estimation of the filter coefficients in the transform domain and the estimation of the system eigenspace are done in a separate manner. Obviously, an automatism should be introduced to detect changes in the eigenspace. This automatism has to ensure an optimal embedding of the system under estimation in the chosen transform-domain. Equation (15) shows that a change in the system eigenspace will result in a non-diagonal update matrix. Hence, an adequate measure is the Frobenius norm of the off-diagonal matrix:

$$J_2 := \left\| \mathrm{off}(\underline{\mathbf{x}}\,\underline{\mathbf{e}}^{\mathrm{T}}) \right\|_{\mathrm{F}}^2. \qquad (28)$$

In practical implementations when J_2 exceeds a predefined threshold a basis update has to be considered.

5. EXPERIMENTAL RESULTS

5.1. Performance Measures

Since we are interested in estimating the eigenspace of a MIMO system, the most important quantity is the achieved diagonalization of the real MIMO system after a transformation into the estimated domain up to a permutation matrix. Therefore we introduce a measure for compactness and define:

$$\mathrm{compactness}(\underline{\mathbf{H}}) = 10\log_{10}\frac{\mathrm{vec}(\underline{\mathbf{H}})}{\max\mathrm{vec}(\underline{\mathbf{H}})}\,\mathrm{dB}. \qquad (29)$$

5.2. Simulation

To illustrate the properties of the developed algorithms, an AEC application scenario is considered. The simulation aims at a proof of our concept. More efficient implementations for complex scenarios can be obtained by considering a block formulation for the presented algorithm in a similar manner to the approach in [13]. Every FIR filter of the MIMO system has a length $L = 256$. The MIMO system has $P = 8$ inputs, and $Q = 12$ outputs. The filter coefficients

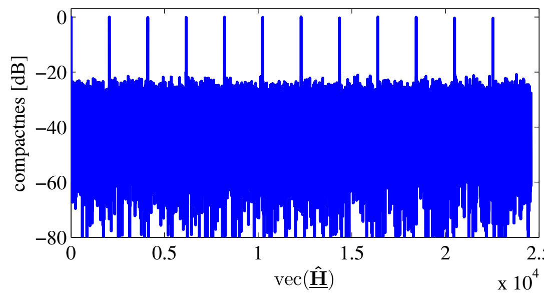

Fig. 2. Achieved compactness of the transformed real MIMO system.

were chosen to be normal distributed with zero mean. For generating the input signals, the same white noise signal is fed into all inputs with different delays. The chosen delay values lie between 0 and 3 samples such that the signals are spatially highly correlated. The estimation of the filter coefficients is performed in the source domain. The estimation of the eigenspace is performed as described in Sect. 4. Figure 2 depicts the achieved compactness of $\mathbf{C_x}^{\mathrm{H}}\mathbf{H}\mathbf{C_y}$. The simulation shows that an attenuation of more than 20 dB of the off-diagonal elements can be reached.

6. CONCLUSION

In this paper we studied the relation between the adaptation in the source domain and in the system eigenspace. We derived an algorithm for an iterative estimation of matrices for the domain transformation into the system eigenspace based on an estimation with an orthogonality constraints and taking into account the poor excitation of the MIMO system due to the highly cross correlated input signals. To prove our concept we innervate our approach with simulation results.

7. RELATION TO PRIOR WORK

Recently developed multichannel adaptive filtering algorithms aim at spatio-temporal decoupling of the signals by suitably chosen transformations [4, 5, 6]. The spatial decoupling is performed by introducing transformation matrices which allow the estimation of the MIMO system in transformed domain. So far the determination of the transformation matrices depends either on the system or on the signal statistics. In this paper we establish the relation between the signal and system dependent transformations. Our study is based on the estimation of the system eigenspace given projections of the system on a particular signal subspace. It differs from the estimation of the signal subspace estimation as presented in [14] since we explicitly consider projections of the system to be estimated. Our approach aims at an iterative estimation of the system eigenspace by incorporating prior knowledge on the transformation matrix. The problem returns to an estimation problem on a Grassmaniann manifold. We apply insights from the adaptation on Grassmannian manifolds as shown in [7] on our special setup while taking into account highly intra- and intercorrelated input signals.

8. REFERENCES

[1] S. Haykin, "Adaptive filter theory," *Prentice-Hall, Inc*, 1991.

[2] J. Benesty, D.R. Morgan, and M.M. Sondhi, "A better understanding and an improved solution to the specific problems of stereophonic acoustic echo cancellation," *IEEE Transactions on Speech and Audio Processing*, vol. 6, no. 2, pp. 156–165, 1998.

[3] J. Herre, H. Buchner, and W. Kellermann, "Acoustic echo cancellation for surround sound using perceptually motivated convergence enhancement," in *Proc. IEEE ICASSP*, 2007, vol. 1, pp. I–17I–20.

[4] K. Helwani, H. Buchner, and S. Spors, "Source-domain adaptive filtering for MIMO systems with application to acoustic echo cancellation," in *Proc. IEEE ICASSP*, 2010, pp. 321–324.

[5] S. Spors, H. Buchner, and R. Rabenstein, "Eigenspace adaptive filtering for efficient pre-equalization of acoustic MIMO systems," in *Proc. EUSIPCO, 2006*, pp. 1–5.

[6] H. Buchner, S. Spors, and W. Kellermann, "Wave-domain adaptive filtering: Acoustic echo cancellation for full-duplex systems based on wave-field synthesis," in *Proc. IEEE ICASSP*, 2004, vol. 4, pp. 117–120.

[7] A. Edelman, T. A. Arias, and S.T. Smith, "The geometry of algorithms with orthogonality constraints," *SIAM Journal on Matrix Analysis and Applications*, vol. 20, no. 2, pp. 303–353, Jan. 1998.

[8] S. Spors, H. Buchner, and K. Helwani, "Block-based multichannel transform-domain adaptive filtering," in *Proc. EUSIPCO, 2009*, pp. 1735–1739.

[9] C. Bishop, *Pattern recognition and machine learning*, Springer, New York, 2006.

[10] G.H. Golub and C.F. Van Loan, *Matrix computations*, Johns Hopkins University Press, 1996.

[11] H. Buchner, "A systematic approach to incorporate deterministic prior knowledge in broadband adaptive MIMO systems," in *Proc. 44-th Asilomar Conference on Signals, Systems, and Computers*, 2010, pp. 461–468.

[12] L. Balzano, R. Nowak, and B. Recht, "Online identification and tracking of subspaces from highly incomplete information," in *Proc. 48th Annual Allerton Conference on Communication, Control, and Computing*, 2010, pp. 704 –711.

[13] H. Buchner, J. Benesty, and W. Kellermann, "Generalized multichannel frequency-domain adaptive filtering: efficient realization and application to hands-free speech communication," *Signal Processing*, vol. 85, no. 3, pp. 549–570, 2005.

[14] B. Yang, "Projection approximation subspace tracking," *IEEE Transactions on Signal Processing*, vol. 43, no. 1, pp. 95–107, 1995.