ANALYSIS OF CLOSED-LOOP ACOUSTIC FEEDBACK CANCELLATION SYSTEMS

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ABSTRACT

In a previous study, the performance of an acoustic feedback/echo cancellation system was analyzed using a power transfer function method. Whereas the analysis result provides very accurate performance predictions in open-loop acoustic echo cancellation systems, it is less accurate in closed-loop acoustic feedback cancellation systems if there is a *strong* correlation between the loudspeaker signal and the signals entering the microphones. This work extends the performance analysis to include the effects of the nonzero correlation on the adaptive filters. Simulation results verify that this extension provides much more accurate performance predictions in closed-loop acoustic feedback cancellation systems.

Index Terms— Adaptive filters, acoustic feedback cancellation, closed-loop systems, estimation bias, steady-state behavior.

1. INTRODUCTION

Acoustic feedback problems arise when a microphone of an audio system picks up part of its acoustic output signal from the loudspeaker. Acoustic feedback cancellation using adaptive filters [1–3] in a system identification setup [4, 5] has evolved to be a state-of-the-art solution [6–12]. Much work has been done to analyze/characterize [13–19] and improve [20–25] these adaptive algorithms in terms of robustness, stability bounds, convergence rate, steady-state behavior, complexity, etc.

In [26], an analysis is performed to describe the frequency domain performance characteristics for acoustic feedback cancellation (AFC) and/or acoustic echo cancellation (AEC) in a multiple-microphone and single-loudspeaker (MMSL) system, illustrated in Fig. 1, in terms of the concept of power transfer function (PTF). The AFC/AEC is carried out by adaptive filters $\hat{\mathbf{h}}_i(n)$, where n is the time index, and $i = 1, ..., P$, where P is the number of microphones, and the beamformer filters $\mathbf{g}_i$ are performing a spatial filtering on the feedback/echo compensated signals $e_i(n)$. The PTF analysis in [26] determined a simple and accurate approximation $\hat{\xi}(\omega, n)$ of the expected magnitude-squared transfer function from point A to B in Fig. 1, where ω is the discrete frequency index. This approximation allowed prediction of the convergence rate, steady-state behavior, and the tracking ability of AFC/AEC systems without knowing the true acoustic feedback/echo paths $\mathbf{h}_i(n)$.

For simplicity, the analysis in [26] was performed in an open-loop system by omitting the forward path $\mathbf{f}(n)$ in Fig. 1, and the loudspeaker signal $u(n)$ was assumed to be uncorrelated with the incoming signals $x_i(n)$. Hence, whereas the results from [26] are very accurate for open-loop AEC systems, these results have certain limitations in closed-loop AFC systems.

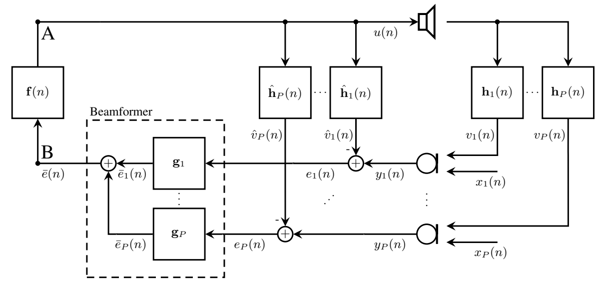

Fig. 1. A multiple-microphone and single-loudspeaker system.

Specifically, the most significant limitation occurs when the incoming signals $x_i(n)$ have long tails in their autocorrelation functions (compared to the system latency from microphone to loudspeaker), such as in most music and alarm signals. The loudspeaker signal $u(n)$ is then correlated with $x_i(n)$. This leads to a biased estimation of $\hat{\mathbf{h}}_i(n)$ [7], and it violates the assumption of uncorrelated $u(n)$ and $x_i(n)$ for the PTF prediction. Thus, for strongly correlated incoming signals, the derived PTF expressions in [26] provide poor predictions, although the expressions are relatively accurate when the incoming signals were speech signals as demonstrated in [26].

Another important application of the PTF approximation $\hat{\xi}(\omega, n)$ in AFC systems is to ensure system stability. The true PTF $\xi(\omega, n)$ is the unknown part of the expected magnitude-squared open-loop transfer function $E[|\text{OLTF}(\omega, n)|^2]$ of the MMSL system expressed by $E\left[|\text{OLTF}(\omega, n)|^2\right] = |F(\omega, n)|^2 \xi(\omega, n)$, where $F(\omega, n)$ is the, generally known, frequency response of $\mathbf{f}(n)$. If $|\text{OLTF}(\omega, n)| < 1$, system stability is guaranteed [27]. However, when the estimation of $\hat{\mathbf{h}}(n)$ is biased due to the correlation between $u(n)$ and $x_i(n)$, $\hat{\xi}(\omega, n)$ determined in [26] would generally be too small. Even if the forward path gain $|F(\omega, n)|$ was chosen as $|F(\omega, n)| < 1/\sqrt{\hat{\xi}(\omega, n)}$, stability could not be guaranteed.

In this work, we derive an extended PTF approximation that includes the influence of potentially biased estimation of $\hat{\mathbf{h}}_i(n)$. In particular, this is done by allowing the correlation function between the loudspeaker signal $u(n)$ and the incoming signals $x_i(n)$ to be nonzero.

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2. REVIEW OF POWER TRANSFER FUNCTION

The PTF describes the expected magnitude-squared transfer function from point A to B in Fig. 1, where the frequency responses $H_i(\omega, n)$ of the true feedback paths $\mathbf{h}_i(n)$ are unknown and considered stochastic. Hence, as in [26], we define the exact PTF of the MMSL system as $\xi(\omega, n) = E[|\sum_{i=1}^{P} G_i(\omega)\tilde{H}_i(\omega, n)|^2]$, where $G_i(\omega)$ is the frequency response of $\mathbf{g}_i$, and $\tilde{H}_i(\omega, n) = \hat{H}_i(\omega, n) - H_i(\omega, n)$ is the frequency response of $\tilde{\mathbf{h}}_i(n) = \hat{\mathbf{h}}_i(n) - \mathbf{h}_i(n)$. Clearly, $\xi(\omega, n) = \sum_{i=1}^{P}\sum_{j=1}^{P} G_i(\omega)G_j^*(\omega)\xi_{ij}(\omega, n)$, where * denotes complex conjugation and $\xi_{ij}(\omega, n) = E[\tilde{H}_i(\omega, n)\tilde{H}_j^*(\omega, n)]$.

In general, however, we can not calculate the PTF $\xi(\omega, n)$ directly because $H_i(\omega, n)$ is unknown. In [26], an approximation $\hat{\xi}_{ij}(\omega, n) \approx E[\tilde{H}_i(\omega, n)\tilde{H}_j^*(\omega, n)]$ was introduced, where $\hat{\xi}_{ij}(\omega, n)$ is expressed by a relatively simple function, leading to an approximate PTF $\hat{\xi}(\omega, n) = \sum_{i=1}^{P}\sum_{j=1}^{P} G_i(\omega)G_j^*(\omega)\hat{\xi}_{ij}(\omega, n)$.

In [26], we derived PTF approximations $\hat{\xi}(\omega, n)$ for several adaptive algorithms for estimating the feedback/echo paths $\mathbf{h}_i(n)$. In this work, to limit our scope, we focus on $\hat{\xi}(\omega, n)$ for the least mean square (LMS) adaptive algorithm. Under the assumptions of uncorrelated $u(n)$ and $x_i(n)$, the LMS step size $\mu(n) \to 0$, the length of the adaptive filter $L \to \infty$, and $r_{x_{ij}}(k) = E[x_i(n)x_j(n+k)] = 0\ \forall\ |k| > k_0 \in \mathbb{N}$, the PTF could be approximated as [26],

$$\hat{\xi}(\omega, n) = (1 - 2\mu(n)S_u(\omega))\,\hat{\xi}(\omega, n-1) + L\mu^2(n)S_u(\omega) \cdot \sum_{i=1}^{P}\sum_{j=1}^{P} G_{ij}(\omega)S_{x_{ij}}(\omega) + \sum_{i=1}^{P}\sum_{j=1}^{P} G_{ij}(\omega)S_{\bar{h}_{ij}}(\omega), \quad (1)$$

where $S_u(\omega)$ denotes the power spectrum density (PSD) of the loudspeaker signal $u(n)$, $S_{x_{ij}}(\omega)$ denotes the cross(auto) PSDs of the incoming signals $x_i(n)$ and $x_j(n)$, $G_{ij}(\omega) = G_i(\omega)G_j^*(\omega)$, and $S_{\bar{h}_{ij}}(\omega)$ is the PSD of the feedback/echo path variations over time.

Furthermore, system behavior in terms of the convergence rate, steady-state error, and the tracking error can be determined using Eq. (1), we refer to [26] for details.

3. EXTENDED PTF IN CLOSED-LOOP SYSTEMS

In this section, we derive an extended PTF approximation for the case where $u(n)$ and $x_i(n)$ may be correlated. We do this based on the bias of the Wiener solution of the adaptive filter estimation for AFC systems.

3.1. Definition of Extended PTF

We consider the adaptive filter estimate $\hat{\mathbf{h}}_i(n)$ as $\hat{\mathbf{h}}_i(n) = \bar{\mathbf{h}}_i(n) + \breve{\mathbf{h}}_i(n)$, where $\bar{\mathbf{h}}_i(n)$ denotes the unbiased estimate from the adaptive algorithms if $u(n)$ was uncorrelated with $x_i(n)$, i.e. $E[\bar{\mathbf{h}}_i(n)] = \mathbf{h}_i(n)$, and $\breve{\mathbf{h}}_i(n)$ is the additional bias vector due to the correlation between $u(n)$ and $x_i(n)$.

The expected value of the adaptive filter estimate $\hat{\mathbf{h}}_i(n)$ which minimizes $E[e_i^2(n)]$ in MMSL systems as in Fig. 1 can be shown to be $E[\hat{\mathbf{h}}_i(n)] = \mathbf{h}_i(n) + E[\breve{\mathbf{h}}_i(n)]$, and $E[\breve{\mathbf{h}}_i(n)]$ is the Wiener solution of the bias vector $\breve{\mathbf{h}}_i(n)$ given by

$$E\left[\breve{\mathbf{h}}_i(n)\right] = E\left[\mathbf{u}(n)\mathbf{u}^T(n)\right]^{-1} \cdot E\left[\mathbf{u}(n)x_i(n)\right], \quad (2)$$

where $\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-L+1)]^T$. An unbiased solution $E[\hat{\mathbf{h}}_i(n)] = \mathbf{h}_i(n)$ is obtained if $E[\mathbf{u}(n)x_i(n)] = \mathbf{0}$. However, this is usually not the case for AFC systems.

We now study the impact of $E[\breve{\mathbf{h}}_i(n)]$ on the PTF. The frequency response of $\hat{\mathbf{h}}_i(n)$ is denoted by $\hat{H}_i(\omega, n) = \bar{H}_i(\omega, n) + \breve{H}_i(\omega, n)$. We express the exact extended PTF $\breve{\xi}_{\text{exact}}(\omega, n)$ as

$$\breve{\xi}_{\text{exact}}(\omega, n) = E\left[\left|\sum_{i=1}^{P} G_i(\omega)\left(\hat{H}_i(\omega, n) - H_i(\omega, n)\right)\right|^2\right]. \quad (3)$$

Eq. (3) can be simplified. For a small step size $\mu(n)$, in principle $\mu(n) \to 0$, the fluctuation of the unbiased adaptive estimate $\bar{H}_i(\omega, n)$ tends to 0, i.e. $\bar{H}_i(\omega, n) - H_i(\omega, n) \to 0$. We can thereby neglect the cross-term $E[\breve{H}_i(\omega, n)(\bar{H}_i^*(\omega, n) - H_i^*(\omega, n))]$ compared to the auto-term $E[\breve{H}_i(\omega, n)\breve{H}_i^*(\omega, n)]$ when evaluating Eq. (3), which becomes

$$\begin{aligned}
&\breve{\xi}_{\text{exact}}(\omega, n) \\
=&E\left[\left|\sum_{i=1}^{P} G_i(\omega)\left(\bar{H}_i(\omega, n) - H_i(\omega, n)\right)\right|^2\right] \\
&+ E\left[\left|\sum_{i=1}^{P} G_i(\omega)\breve{H}_i(\omega, n)\right|^2\right] \\
=&\xi(\omega, n) + \sum_{i=1}^{P}\sum_{j=1}^{P} G_i(\omega)G_j^*(\omega)E\left[\breve{H}_i(\omega, n)\breve{H}_j^*(\omega, n)\right]. \quad (4)
\end{aligned}$$

Thus, the correlation between $x_i(n)$ and $u(n)$ leads to a nonzero bias term (the last term) in Eq. (4), which in turn leads to an increase in the extended PTF $\breve{\xi}_{\text{exact}}(\omega, n)$, over the PTF $\xi(\omega, n)$ which would have been achieved, had $x_i(n)$ and $u(n)$ been uncorrelated. To further simplify Eq. (4), we replace $\breve{H}_i(\omega, n)$ with its expected value $E[\breve{H}_i(\omega, n)]$, since we are studying the steady-state effects of $E[\breve{\mathbf{h}}_i(n)]$ in Eq. (2) on the PTF, and $\breve{H}_i(\omega, n) \to E[\breve{H}_i(\omega, n)]$ in steady-state for the LMS step size $\mu(n) \to 0$ [28]. Furthermore, by replacing the PTF $\xi(\omega, n)$ in Eq. (4) with its approximation $\hat{\xi}(\omega, n)$, we get the extended PTF approximation $\breve{\xi}(\omega, n)$, as

$$\begin{aligned}
\breve{\xi}(\omega, n) &= \hat{\xi}(\omega, n) \\
&+ \sum_{i=1}^{P}\sum_{j=1}^{P} G_i(\omega)G_j^*(\omega)E\left[\breve{H}_i(\omega, n)\right] \cdot E\left[\breve{H}_j^*(\omega, n)\right]. \quad (5)
\end{aligned}$$

The last term in Eq. (5) is independent of the step size parameter in applied adaptive algorithms, since the expected value of adaptive filter bias $E[\breve{\mathbf{h}}_i(n)]$ only depends on the incoming signals x_i(n) and the loudspeaker signal $u(n)$ as given in Eq. (2). Furthermore, this bias term can be considered as an additional error contribution to the steady-state error given by $\hat{\xi}(\omega, n)$. Therefore, while the convergence of $\breve{\xi}(\omega, n)$ is only determined by $\hat{\xi}(\omega, n)$, the steady-state behavior of $\breve{\xi}(\omega, n)$ is determined by both $\hat{\xi}(\omega, n)$ and this bias term. In the following sections, we study the influences of this bias term.

3.2. Extended PTF Analysis

We model the forward path $\mathbf{f}(n)$ as $\mathbf{f}(n) = \mathbf{f}_0(n) * \delta(n-d)$, where $*$ denotes convolution, i.e. a filtering part $\mathbf{f}_0(n)$ and a delay of $d > 0$ samples. Let $\mathbf{F}_0(n) \in \mathbb{R}^{L\times L}$ and $\mathbf{G}_i \in \mathbb{R}^{L\times L}$ be the Toeplitz structured convolution matrices of the forward path filter $\mathbf{f}_0(n)$ and the beamformer filter $\mathbf{g}_i$, respectively. We define the incoming signal vector as $\mathbf{x}_i(n) = [x_i(n), x_i(n-1), \ldots, x_i(n-L+1)]^T$, and by considering the case that $H_i(\omega,n) - \hat{H}_i(\omega,n)$ is relatively small in a steady-state situation, i.e. the adaptive filter provides a relatively precise estimate $\hat{H}_i(\omega,n)$ despite an eventual bias term $\breve{H}_i(\omega,n) \neq 0$, we can neglect the closed-loop effect, given by the transfer function $1/(1 - F_0(\omega,n)G_i(\omega)(H_i(\omega,n) - \hat{H}_i(\omega,n)))$, on the loudspeaker signal vector $\mathbf{u}(n)$, which is now simply expressed by

$$\mathbf{u}(n) = \sum_{i=1}^{P} \mathbf{F}_0(n)\mathbf{G}_i\mathbf{x}_i(n-d). \tag{6}$$

The correspondence of the resulting theory and the simulation results presented later showed that Eq. (6) is reasonable for even relatively large values of the bias term $\breve{H}_i(\omega,n)$ and thereby $H_i(\omega,n) - \hat{H}_i(\omega,n)$. Inserting Eq. (6) in Eq. (2), $E[\breve{\mathbf{h}}_i(n)]$ can be written as

$$E\left[\breve{\mathbf{h}}_i(n)\right] = \left(\sum_{p=1}^{P}\sum_{q=1}^{P} \mathbf{F}_0(n)\mathbf{G}_p\mathbf{R}_{x_{pq}}(0)\mathbf{G}_q^T\mathbf{F}_0^T(n)\right)^{-1} \cdot \sum_{p=1}^{P} \mathbf{F}_0(n)\mathbf{G}_p\mathbf{r}_{x_{pi}}(d), \tag{7}$$

where $\mathbf{R}_{x_{ij}}(k) = E[\mathbf{x}_i(n)\mathbf{x}_j^T(n+k)]$ and $\mathbf{r}_{x_{ij}}(k) = E[\mathbf{x}_i(n)x_j(n+k)]$.

We compute the frequency response $E[\breve{H}_i(\omega,n)]$ of $E[\breve{\mathbf{h}}_i(n)]$ using the discrete Fourier transform (DFT) matrix $\mathbf{D} \in \mathbb{C}^{L\times L}$. Since the DFT matrix $\mathbf{D}$ diagonalizes any Toeplitz matrix asymptotically, as $L \to \infty$ [29], and the matrices $\mathbf{F}_0(n)$, $\mathbf{G}_i$, and $\mathbf{R}_{x_{ij}}(0)$ are all asymptotically Toeplitz matrices, each element $E[\breve{H}_i(\omega,n)]$ of the frequency response vector $\mathbf{D}E[\breve{\mathbf{h}}_i(n)]$ can be shown to be

$$E\left[\breve{H}_i(\omega,n)\right] = \frac{\sum_{p=1}^{P} F_0(\omega,n)G_p(\omega)\Gamma_{x_{pi}}(\omega)}{\sum_{p=1}^{P}\sum_{q=1}^{P} F_0(\omega,n)G_p(\omega)S_{x_{pq}}(\omega)G_q^*(\omega)F_0^*(\omega,n)}. \tag{8}$$

$\Gamma_{x_{pi}}(\omega)$ are elements of the vector $\mathbf{D}\mathbf{r}_{x_{pi}}(d)$, which is the DFT of the autocorrelation tail sequence $r_{x_{pi}}(d), r_{x_{pi}}(d+1), \ldots, r_{x_{pi}}(d+L-1)$. Furthermore, $S_{x_{pq}}(\omega)$ are the diagonal entries of the matrix $\frac{1}{L}\mathbf{D}\mathbf{R}_{x_{pq}}(0)\mathbf{D}^H$, these approach the DFT of the autocorrelation sequence $r_{x_{pq}}(0), r_{x_{pq}}(1), \ldots, r_{x_{pq}}(L-1)$, as $L \to \infty$. Finally, inserting Eq. (8) in Eq. (5), we get the expression for the extended PTF approximation $\breve{\xi}(\omega,n)$ as

$$\begin{aligned}\breve{\xi}(\omega,n) = \hat{\xi}(\omega,n) + \sum_{i=1}^{P}\sum_{j=1}^{P} G_i(\omega)G_j^*(\omega) \\ \cdot \left(\frac{\sum_{p=1}^{P} F_0(\omega,n)G_p(\omega)\Gamma_{x_{pi}}(\omega)}{\sum_{p=1}^{P}\sum_{q=1}^{P} F_0(\omega,n)G_p(\omega)S_{x_{pq}}(\omega)G_q^*(\omega)F_0^*(\omega,n)}\right) \\ \cdot \left(\frac{\sum_{p=1}^{P} F_0(\omega,n)G_p(\omega)\Gamma_{x_{pj}}(\omega)}{\sum_{p=1}^{P}\sum_{q=1}^{P} F_0(\omega,n)G_p(\omega)S_{x_{pq}}(\omega)G_q^*(\omega)F_0^*(\omega,n)}\right)^*.\end{aligned} \tag{9}$$

4. DISCUSSIONS

Generally, Eq. (9) is not easily interpreted. However, for a single-microphone and single-loudspeaker (SMSL) system ($P = 1$), the extended PTF approximation $\breve{\xi}(\omega,n)$ given by Eq. (9) simply reduces to

$$\breve{\xi}(\omega,n) = \hat{\xi}(\omega,n) + \frac{|\Gamma_x(\omega)|^2}{|F_0(\omega,n)|^2 S_x^2(\omega)}. \tag{10}$$

In general, $\breve{\xi}(\omega,n) > \hat{\xi}(\omega,n)$ since $|\Gamma_x(\omega)| > 0$ in Eq. (10). However, for incoming signals $x(n)$ fulfilling $r_x(k) = 0 \;\forall\; |k| > k_0 \in \mathbb{N}$, increasing the forward path delay d would generally decorrelate $u(n)$ from $x(n)$, see e.g. [30–32], and for a large value of $d > k_0$, we get $\Gamma_x(\omega) = 0$ leading to $\breve{\xi}(\omega,n) = \hat{\xi}(\omega,n)$ in Eq. (10).

Furthermore, Eq. (10) reveals that increasing the forward path gain $|F_0(\omega,n)|$ leads to a smaller bias term in $\breve{\xi}(\omega,n)$. Intuitively this can be explained by the fact when the forward path gain $|F_0(\omega,n)|$ gets larger, the larger is $u(n)$ compared to $x(n)$, see e.g. Eq. (6), the amplitude of each element in the expected bias vector $E[\breve{\mathbf{h}}_i(n)]$ would thereby be smaller as also seen from Eq. (2). However, although a large forward path gain $|F_0(\omega,n)|$ leads to a small $\breve{\xi}(\omega,n)$ in Eq. (10), $|F_0(\omega,n)|$ is still a compromise between being large enough to ensure a reasonably small bias, and being small enough to maintain system stability. In particular, $|F_0(\omega,n)|$ should be chosen according to $|F_0(\omega,n)| < 1/\sqrt{\breve{\xi}(\omega,n)}$ to ensure stability. Furthermore, using Eq. (10), a lower bound for the magnitude-squared open-loop transfer function $|\mathrm{OLTF}(\omega,n)|^2 = |F_0(\omega,n)|^2\breve{\xi}(\omega,n)$ is obtained as $|\Gamma_x(\omega)|^2/S_x^2(\omega)$ for $\hat{\xi}(\omega,n) \to 0$, and it is interesting to note that it is actually independent of $|F_0(\omega,n)|$.

Table 1. Common parameters for simulation experiments.

Symbol	Value	Description
D_s	80000	Duration of simulation.
R	100	Number of sim. runs.
μ	2^{-11}	LMS step size.
L	32	Length of $\hat{\mathbf{h}}(n)$.
$\mathbf{g}$	$0.1 \cdot [10, 3, -2.5, 1, 0.5]^T$	Beamformer filter.
$\mathbf{h}(0)$	$0.01 \cdot [6, 0.84, -1.38]^T$	Initial values of $\mathbf{h}(n)$.
$N(\mu_h, \sigma_h^2)$	$N(0, 0.0019^2)$	Feedback path variation.
$\mathbf{h}_x$	$0.01 \cdot [10, -3, 4, -1, 0.5]^T$	Shaping filter for $x(n)$.
$\mathbf{f}_0$	$[10]^T$	Forward path filter.
d	1	Forward path delay.

5. SIMULATION VERIFICATIONS

In this section, we perform simulation experiments to verify the extended PTF approximation $\breve{\xi}(\omega,n)$ in Eq. (9) and show the improvements by comparing to the PTF approximation $\hat{\xi}(\omega,n)$ in Eq. (1). We consider an SMSL system ($P = 1$) with a known feedback path $\mathbf{h}(n)$, which remains fixed during the first half of the simulation experiment, but undergoes variations in the second half. We use the same procedure as described in Sec. VI-A of [26] for these simulation experiments. We refer to [26] for details. The only difference is that $u(n)$ is no longer independently generated, but it is rather computed in the closed-loop system as the error signal $\bar{e}(n)$ filtered through a time invariant forward path $\mathbf{f}$. Table 1 shows the general simulation parameters; the beamformer filter $\mathbf{g}$, initial feedback path $\mathbf{h}(0)$, and the forward path filter $\mathbf{f}_0$ are chosen to be lower order filters for reproducibility in these experiments.

In the first simulation experiment, we verify that the extended PTF expression in Eq. (9) can accurately predict biased steady-state values for an SMSL AFC system using an LMS algorithm. A biased

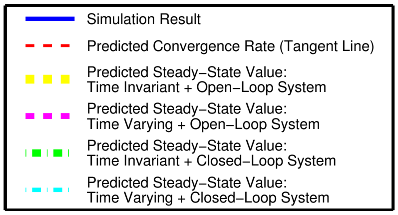

Fig. 2. Legends for Figs. 3-5.

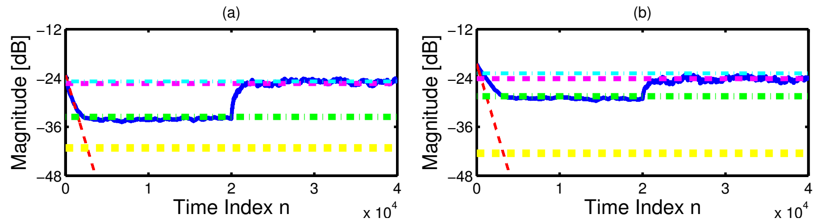

Fig. 3. PTF values for frequency bins (a) $l = 3$. (b) $l = 7$. See Fig. 2 for legend.

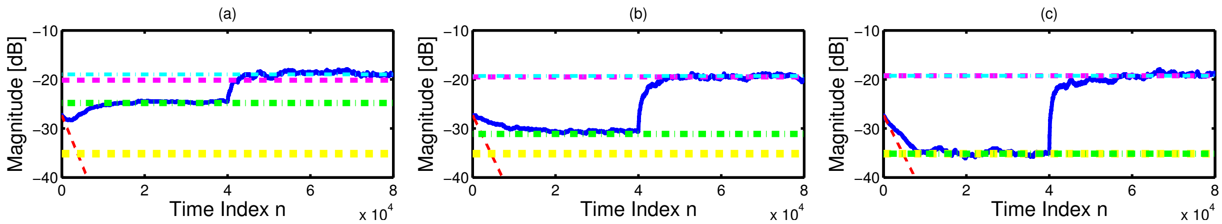

Fig. 4. PTF values for frequency bin 3. The forward path delay is (a) $d = 1$ sample. (b) $d = 3$ samples. (c) $d = 5$ samples. See Fig. 2 for legend.

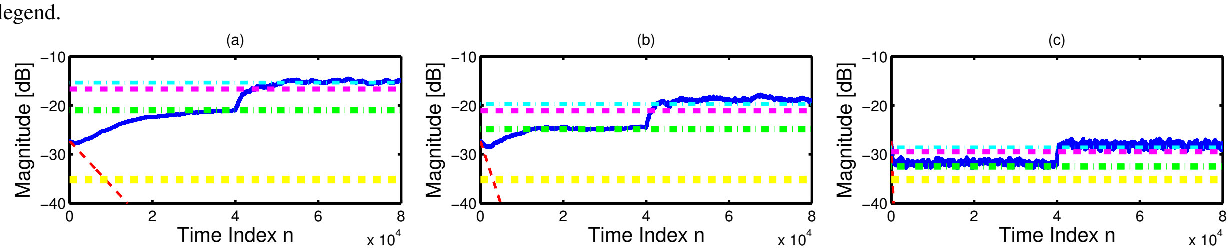

Fig. 5. PTF values for frequency bin 3. The forward path gain is (a) $|F_0(\omega)| = 6$ dB. (b) $|F_0(\omega)| = 10$ dB. (c) $|F_0(\omega)| = 20$ dB. See Fig. 2 for legend.

estimation of $\mathbf{h}(n)$ is expected due to the choices of the forward path delay d and the shaping filter $\mathbf{h}_x$, which is used to generate the incoming signals $x(n)$ by convolving $\mathbf{h}_x$ with a white noise sequence.

Fig. 3 shows the results at two representative example frequency bins $l = 3, 7$. The true PTF values can be calculated in simulations because $\mathbf{h}(n)$ is known. These true PTF values confirm the PTF prediction values using Eqs. (1) and (10) for computing the convergence rate and steady-state behaviors for the open-loop (see details in [26]) and closed-loop systems, respectively. Furthermore, the closed-loop PTF values for biased estimation of $\hat{\mathbf{h}}(n)$ are generally found at higher levels than the open-loop PTF values without biased estimation.

In the second simulation experiment, we show the dependence between forward path delay d and the PTF values. All simulation parameters are the same as given in Table 1, except the forward path delay, which is $d = 1, 3, 5$ in three different simulations. Fig. 4 shows the simulation results. The shaping filter $\mathbf{h}_x$ is a fourth-order filter, it means that $r_x(k) = 0 \; \forall \; |k| > 4$. Clearly, with a forward path delay $d = 1, 3$, $\breve{\xi}(\omega) > \hat{\xi}(\omega)$ due to biased estimation of $\hat{\mathbf{h}}(n)$. For a longer enough delay $d = 5$, $\breve{\xi}(\omega) = \hat{\xi}(\omega)$.

In the last simulation experiment, we show the dependence between the forward path gain $|F_0(\omega)|$ and the PTF values. We again use the general simulation parameters given in Table 1, only the forward path filter $\mathbf{f}_0$ varies so that the $|F_0(\omega)| = 6, 10, 20$ dB, respectively. Fig. 5 shows the simulation results for different forward path gains $|F_0(\omega)|$. As expected, a higher forward gain $|F_0(\omega)|$ gives lower steady-state PTF values, as expressed in Eq. (10).

6. CONCLUSION

In previous work, PTF approximations have been derived for different adaptive algorithms in open-loop MMSL systems. In this work, we derived extensions to these PTF expressions for closed-loop MMSL systems. We showed that this extension provides more accurate and useful performance predictions of closed-loop AFC systems, especially if there are strong correlations between the loudspeaker and incoming signals, and the adaptive filter estimates therefore become heavily biased. The results also showed the relations between the forward path delay/gain in closed-loop systems and the biased adaptive filter estimates and thereby the PTF prediction values. This knowledge is important in designing AFC systems and provides a very useful upper-limit for the forward path gain to guarantee system stability in closed-loop AFC systems.

7. RELATIONS TO PRIOR WORK

This work is an extension of the power transfer function analysis of a multiple-microphone and single-loudspeaker cancellation system introduced in [26]. The extended analysis provides accurate cancellation performance predictions in closed-loop systems even if there is a *strong* correlation between the loudspeaker and incoming signals, see details in the Introduction. The power transfer function analysis is inspired by the studies in [33, 34] of tracking characteristics for an open-loop single-microphone and single-loudspeaker cancellation system, which is a special case of the presented framework.

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