STUDY OF THE OPTIMAL AND SIMPLIFIED KALMAN FILTERS FOR ECHO CANCELLATION

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ABSTRACT

In this paper, we study the time-domain Kalman filter in the context of echo cancellation. We explain the fundamental differences between the Kalman filter and the recursive least-squares (RLS) algorithm. Also, we show that the normalized least-mean-square (NLMS) algorithm has a clear relationship with the Kalman filter. Furthermore, a simplified Kalman filter is derived and by a judicious choice of its parameters, this algorithm behaves like a variable step-size adaptive filter. Simulation results indicate the good performance of the optimal and simplified Kalman filtering algorithms.

Index Terms— Echo cancellation, Kalman filter, recursive least-squares (RLS) algorithm, normalized least-mean-square (NLMS) algorithm, variable step-size (VSS) adaptive filter.

1. INTRODUCTION

The Kalman filter is a very interesting signal processing tool, which is widely used in many practical applications [1]. This algorithm recursively estimates a set of unknown variables from a set of (noisy) observations acquired over time. Due to its optimal performance, the Kalman filter can be used in many system identification problems [2].

Similarly, the main goal in echo cancellation is to estimate an unknown system, i.e., the echo path, from the microphone signal that contains the echo signal corrupted by different types of "noise" (e.g., the background noise and the near-end speech) [3]. However, despite its performance, the Kalman filter has been avoided in this context.

The main objective of this paper is to promote the work by G. Enzner and his co-authors [4]–[9], and motivate the use of the Kalman filter (and its variants) in the echo cancellation problem. We believe that this optimal filter has the potential to become one of the most interesting choices for this important problem. Therefore, we give a brief overview of the Kalman filtering for cancelling echoes.

Our contribution is organized as follows. The state variable model for echo cancellation is described in Section 2. Based on this model, the Kalman filter is derived in Section 3; here, we also explain its relationship with the recursive least-squares (RLS) algorithm. In order to reduce the computational complexity, a simplified Kalman filter is developed in Section 4; also, it is shown that the normalized least-mean-square (NLMS) algorithm has a clear relationship with the Kalman filter. Using a judicious choice of its parameters as shown in Section 5, the simplified Kalman filter behaves like a variable step-size (VSS) algorithm. Simulation results presented in Section 6 support the theoretical findings and outline the appealing performance of the optimal and simplified Kalman filtering algorithms.

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2. STATE VARIABLE MODEL FOR ECHO CANCELLATION

Let us consider the echo cancellation scenario, where the microphone (or desired) signal at the discrete-time index n is defined as

$$d(n) = \mathbf{x}^T(n)\mathbf{h} + v(n) = y(n) + v(n), \tag{1}$$

where $\mathbf{x}(n) = \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-L+1) \end{bmatrix}^T$ is a vector containing the L most recent time samples of the input (or loudspeaker) signal $x(n)$, superscript ^{T} denotes transpose of a vector or a matrix, $\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} \end{bmatrix}^T$ is the impulse response (of length L) of the system (from the loudspeaker to the microphone) that we need to identify, and $v(n)$ is a zero-mean stationary white Gaussian noise signal. The variance of this additive noise is $\sigma_v^2 = E\left[v^2(n)\right]$, where $E[\cdot]$ denotes mathematical expectation. In the context of echo cancellation, the signal $y(n)$ is called the echo that we want to cancel with an adaptive filter [3], [10].

In the mentioned application, our objective is to estimate or identify $\mathbf{h}$ with an adaptive filter:

$$\widehat{\mathbf{h}}(n) = \begin{bmatrix} \widehat{h}_0(n) & \widehat{h}_1(n) & \cdots & \widehat{h}_{L-1}(n) \end{bmatrix}^T, \tag{2}$$

in such a way that for a reasonable value of n, we have for the (normalized) misalignment:

$$\frac{\left\|\widehat{\mathbf{h}}(n) - \mathbf{h}\right\|_2^2}{\|\mathbf{h}\|_2^2} \leq \iota, \tag{3}$$

where ι is a predetermined small positive number and $\|\cdot\|_2$ is the ℓ_2 norm.

The system impulse response can be modeled as a state equation. In our context, $\mathbf{x}(n)$ is the measurement vector and $x(n)$ is considered as deterministic. Expression (1) is called the observation equation. We assume that $\mathbf{h}(n)$ is a zero-mean random vector, which follows a simplified first-order Markov model, i.e.,

$$\mathbf{h}(n) = \mathbf{h}(n-1) + \mathbf{w}(n), \tag{4}$$

where $\mathbf{w}(n)$ is a zero-mean white Gaussian noise signal vector, which is uncorrelated with $\mathbf{h}(n-1)$ and $v(n)$. The correlation matrix of $\mathbf{w}(n)$ is assumed to be $\mathbf{R}_\mathbf{w}(n) = \sigma_w^2(n)\mathbf{I}_L$, where $\mathbf{I}_L$ is the $L \times L$ identity matrix. The variance, $\sigma_w^2(n)$, captures the uncertainties in $\mathbf{h}(n)$. Expression (4) is called the state equation.

Therefore, the echo cancellation problem may be restated as follows. Given the two fundamental equations:

$$\begin{aligned} \mathbf{h}(n) &= \mathbf{h}(n-1) + \mathbf{w}(n), & (5)\\ d(n) &= \mathbf{x}^T(n)\mathbf{h}(n) + v(n), & (6) \end{aligned}$$

our objective is to find the optimal recursive estimator of $\mathbf{h}(n)$ denoted by $\widehat{\mathbf{h}}(n)$. In the context of echo cancellation, the values of $\sigma_w^2(n)$ play a major role in the performance of the estimator. Indeed, small values of $\sigma_w^2(n)$ imply a good misalignment but a poor tracking; while large values of $\sigma_w^2(n)$ (meaning that the uncertainties in the echo path are high) imply a good tracking but a high misalignment. In other words, the values of $\sigma_w^2(n)$ highly determine the tracking abilities and the convergence of the Kalman filter to be derived. Therefore, there is always a compromise between good tracking and low misalignment. As simulations will show, this simplified model is very satisfactory for the echo cancellation problem.

3. KALMAN FILTER VERSUS RLS ALGORITHM

The Kalman filter can be derived based on the simplified (with respect to the state equation) model presented in the previous section. In the context of the linear sequential Bayesian approach, the optimal estimate of the state vector, $\mathbf{h}(n)$, has the form [11]:

$$\begin{aligned}\widehat{\mathbf{h}}(n) &= \widehat{\mathbf{h}}(n-1) + \mathbf{k}(n)\left[d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n-1)\right] \\ &= \widehat{\mathbf{h}}(n-1) + \mathbf{k}(n)e(n), \qquad (7)\end{aligned}$$

where $\mathbf{k}(n)$ is the Kalman gain vector and

$$e(n) = d(n) - \widehat{y}(n) = d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n-1) \qquad (8)$$

is the a priori error signal between the microphone signal and the estimate of the echo signal. Also, the a posteriori error signal is

$$\epsilon(n) = d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n) = \mathbf{x}^T(n)\boldsymbol{\mu}(n) + v(n), \qquad (9)$$

where

$$\boldsymbol{\mu}(n) = \mathbf{h}(n) - \widehat{\mathbf{h}}(n) \qquad (10)$$

is the state estimation error or a posteriori misalignment. The correlation matrix of $\boldsymbol{\mu}(n)$ is

$$\mathbf{R}_{\boldsymbol{\mu}}(n) = E\left[\boldsymbol{\mu}(n)\boldsymbol{\mu}^T(n)\right]. \qquad (11)$$

We can also define the a priori misalignment as

$$\mathbf{m}(n) = \mathbf{h}(n) - \widehat{\mathbf{h}}(n-1) = \boldsymbol{\mu}(n-1) + \mathbf{w}(n), \qquad (12)$$

for which its correlation matrix is

$$\mathbf{R}_{\mathbf{m}}(n) = E\left[\mathbf{m}(n)\mathbf{m}^T(n)\right] = \mathbf{R}_{\boldsymbol{\mu}}(n-1) + \sigma_w^2(n)\mathbf{I}_L. \qquad (13)$$

Thus, the a priori misalignment appears in the a priori error signal as

$$e(n) = \mathbf{x}^T(n)\mathbf{m}(n) + v(n). \qquad (14)$$

The Kalman gain vector is obtained by minimizing the criterion:

$$J(n) = \frac{1}{L}\mathrm{tr}\left[\mathbf{R}_{\boldsymbol{\mu}}(n)\right] \qquad (15)$$

with respect to $\mathbf{k}(n)$. From this minimization, we find that

$$\mathbf{k}(n) = \frac{\mathbf{R}_{\mathbf{m}}(n)\mathbf{x}(n)}{\mathbf{x}^T(n)\mathbf{R}_{\mathbf{m}}(n)\mathbf{x}(n) + \sigma_v^2} \qquad (16)$$

and

$$\mathbf{R}_{\boldsymbol{\mu}}(n) = \left[\mathbf{I}_L - \mathbf{k}(n)\mathbf{x}^T(n)\right]\mathbf{R}_{\mathbf{m}}(n). \qquad (17)$$

Summarizing, the following equations define the well-known Kalman filter [12]:

$$\begin{aligned}\mathbf{R}_{\mathbf{m}}(n) &= \mathbf{R}_{\boldsymbol{\mu}}(n-1) + \sigma_w^2(n)\mathbf{I}_L, &(18)\\ \mathbf{k}(n) &= \frac{\mathbf{R}_{\mathbf{m}}(n)\mathbf{x}(n)}{\mathbf{x}^T(n)\mathbf{R}_{\mathbf{m}}(n)\mathbf{x}(n) + \sigma_v^2}, &(19)\\ e(n) &= d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n-1), &(20)\\ \widehat{\mathbf{h}}(n) &= \widehat{\mathbf{h}}(n-1) + \mathbf{k}(n)e(n), &(21)\\ \mathbf{R}_{\boldsymbol{\mu}}(n) &= \left[\mathbf{I}_L - \mathbf{k}(n)\mathbf{x}^T(n)\right]\mathbf{R}_{\mathbf{m}}(n). &(22)\end{aligned}$$

This algorithm has striking resemblances with the classical RLS algorithm [12]. However, contrary to what it may be believed, the two algorithms are very much different and do not behave the same way in practice; this is perhaps the reason why the Kalman filter was not really so deeply studied in echo cancellation except by G. Enzner [4]. Nevertheless, there are at least four fundamental differences between these two filters. First, the Kalman filter does not require any matrix inversion, which is not the case for the RLS (whose inverse input signal correlation matrix is implicitly calculated at each iteration). Second, the Kalman filter depends explicitly on the correlation matrix of the misalignment while the RLS adaptive filter depends on the (inverse) correlation matrix of the input signal. Third, the RLS does not depend on the variance of the additive noise. Finally, the RLS does not depend on the uncertainties in $\mathbf{h}(n)$ since it is considered as deterministic in its derivation. The two parameters $\sigma_w^2(n)$ and σ_v^2 in the Kalman filter (for which the RLS does not depend on) allow us to better control it.

At infinity, we have [13]

$$\begin{aligned}\lim_{n\to\infty}\mathrm{tr}\left[\mathbf{R}_{\boldsymbol{\mu}}(n)\right] &= L\sigma_{\boldsymbol{\mu}}^2(n), &(23)\\ \lim_{n\to\infty}\mathrm{tr}\left[\mathbf{R}_{\mathbf{m}}(n)\right] &= L\left[\sigma_{\boldsymbol{\mu}}^2(n) + \sigma_w^2(n)\right], &(24)\end{aligned}$$

where $\sigma_{\boldsymbol{\mu}}^2(n)$ is a small positive number to which all diagonal elements of $\mathbf{R}_{\boldsymbol{\mu}}(n)$ converge. Therefore, the normalized misalignment as defined in Section 2 should be after convergence:

$$\frac{\left\|\widehat{\mathbf{h}}(n) - \mathbf{h}\right\|_2^2}{\|\mathbf{h}\|_2^2} \approx \frac{L\left[\sigma_{\boldsymbol{\mu}}^2(n) + \sigma_w^2(n)\right]}{\|\mathbf{h}\|_2^2}, \qquad (25)$$

where $\mathbf{h}$ is the acoustic impulse response that we try to identify. It is instructive to observe how the final misalignment is determined by the values of $\sigma_w^2(n)$. Also, when $\sigma_w^2(n) = 0$, we have

$$\lim_{n\to\infty}\mathbf{R}_{\boldsymbol{\mu}}(n) = \mathbf{0}, \quad \lim_{n\to\infty}\mathbf{k}(n) = \mathbf{0}, \qquad (26)$$

and, obviously, the Kalman filter will never be able to track the changes in $\mathbf{h}(n)$. On the other hand, for large values of $\sigma_w^2(n)$, the Kalman gain vector never goes to zero, which allows the update equation (7) to stay "alert" to any possible random changes of the echo path.

4. SIMPLIFIED KALMAN FILTER

Let us assume that the Kalman filter converged to its steady-state. In this case, $\mathbf{R}_{\mathbf{m}}(n)$ tends to become a diagonal matrix with all its elements equal to a small positive number, $\sigma_{\mathbf{m}}^2(n)$; so we can make the approximation:

$$\mathbf{R}_{\mathbf{m}}(n) \approx \sigma_{\mathbf{m}}^2(n)\mathbf{I}_L. \qquad (27)$$

As a result, the Kalman gain vector simplifies to

$$\mathbf{k}(n) \approx \mathbf{k}_{\mathrm{NLMS}}(n) = \frac{\mathbf{x}(n)}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta(n)}, \tag{28}$$

where

$$\delta(n) = \frac{\sigma_v^2}{\sigma_{\mathbf{m}}^2(n)} \tag{29}$$

can be seen as a variable regularization parameter. We deduce that the Kalman filter simplifies to the regularized NLMS algorithm:

$$e(n) = d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n-1), \tag{30}$$

$$\widehat{\mathbf{h}}(n) = \widehat{\mathbf{h}}(n-1) + \frac{\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta(n)}. \tag{31}$$

Simulations confirm that with these parameters, the NLMS algorithm behaves the same way as the Kalman filter at the convergence. Of course, some a priori information on $\delta(n)$ is required.

A better possibility is the following. Assume that the update matrix that appears explicitly in $\mathbf{R}_{\boldsymbol{\mu}}(n)$ can be approximated as

$$\mathbf{I}_L - \mathbf{k}(n)\mathbf{x}^T(n) \approx \left[1 - \frac{\mathbf{k}^T(n)\mathbf{x}(n)}{L}\right]\mathbf{I}_L. \tag{32}$$

The previous approximation is reasonable. Indeed, when the filter starts to converge, the matrix $\mathbf{R}_{\boldsymbol{\mu}}(n)$ tends to become a diagonal one since the misalignment of the individual coefficients tend to become uncorrelated. As a consequence of (32), the two matrices $\mathbf{R}_{\mathbf{m}}(n)$ and $\mathbf{R}_{\boldsymbol{\mu}}(n)$ become diagonal, i.e., $\mathbf{R}_{\mathbf{m}}(n) \approx \sigma_{\mathbf{m}}^2(n)\mathbf{I}_L = r_{\mathbf{m}}(n)\mathbf{I}_L$ and $\mathbf{R}_{\boldsymbol{\mu}}(n) \approx \sigma_{\boldsymbol{\mu}}^2(n)\mathbf{I}_L = r_{\boldsymbol{\mu}}(n)\mathbf{I}_L$. Then, it is not hard to deduce that the Kalman filter simplifies to

$$r_{\mathbf{m}}(n) = r_{\boldsymbol{\mu}}(n-1) + \sigma_w^2(n), \tag{33}$$

$$\delta(n) = \frac{\sigma_v^2}{r_{\mathbf{m}}(n)}, \tag{34}$$

$$e(n) = d(n) - \mathbf{x}^T(n)\widehat{\mathbf{h}}(n-1), \tag{35}$$

$$\widehat{\mathbf{h}}(n) = \widehat{\mathbf{h}}(n-1) + \frac{\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta(n)}, \tag{36}$$

$$r_{\boldsymbol{\mu}}(n) = \left\{1 - \frac{\mathbf{x}^T(n)\mathbf{x}(n)}{L\left[\mathbf{x}^T(n)\mathbf{x}(n) + \delta(n)\right]}\right\} r_{\mathbf{m}}(n). \tag{37}$$

This simplified Kalman filter, first proposed in [8] and named broadband Kalman filter (BKF), behaves like a VSS-type NLMS algorithm. Simulations prove this statement. This algorithm may look very similar, at first glance, to the one proposed in [14] but the two are very much different. Indeed, in [14], the echo path is considered as time invariant so that no state equation is involved; this is equivalent to taking $\sigma_w^2(n) = 0$, which may be a problem in tracking.

5. PRACTICAL CONSIDERATIONS

There are two parameters that need to be set or estimated within the Kalman filter (and its simplified version). The first (and perhaps the most important) one is $\sigma_w^2(n)$, which plays a major role in the overall performance of the algorithms, as explained in Sections 2 and 3. In order to evaluate this parameter, let us rewrite (4) as

$$\mathbf{w}(n) = \mathbf{h}(n) - \mathbf{h}(n-1). \tag{38}$$

Using the ℓ_2 norm in both sides of (38), together with the approximation $\|\mathbf{w}(n)\|_2^2 \approx L\sigma_w^2(n)$ (which is valid when $L \gg 1$), and replacing $\mathbf{h}(n)$ by its estimate $\widehat{\mathbf{h}}(n)$, we can evaluate

$$\widehat{\sigma}_w^2(n) = \frac{1}{L}\left\|\widehat{\mathbf{h}}(n) - \widehat{\mathbf{h}}(n-1)\right\|_2^2. \tag{39}$$

The estimation from (39) is designed to achieve a proper compromise between good tracking and low misalignment. When the algorithm starts to converge or when there is an abrupt change of the system (e.g., when the echo path changes), the difference between $\widehat{\mathbf{h}}(n)$ and $\widehat{\mathbf{h}}(n-1)$ is significant, so that the parameter $\widehat{\sigma}_w^2(n)$ takes large values, thus providing fast convergence and tracking. On the other hand, when the algorithm starts to converge to its steady-state, the difference between $\widehat{\mathbf{h}}(n)$ and $\widehat{\mathbf{h}}(n-1)$ reduces, thus leading to small values of $\widehat{\sigma}_w^2(n)$ and, consequently, to a low misalignment.

In the case of the BKF [8], the state equation is slightly different as compared to (4), i.e.,

$$\mathbf{h}(n) = a\mathbf{h}(n-1) + \mathbf{w}(n), \tag{40}$$

where $0 \leq a \leq 1$ is the transition coefficient. This implies that the estimation of $\sigma_w^2(n)$ becomes [8]

$$\widehat{\sigma}_w^2(n) = \frac{1-a^2}{L} E\left[\widehat{\mathbf{h}}^T(n)\widehat{\mathbf{h}}(n)\right]. \tag{41}$$

In general, the value of the parameter a should be chosen very close to one [8], which could be problematic for the estimator given in (41). From this point of view, the evaluation of $\widehat{\sigma}_w^2(n)$ from (39) should be more reliable in practice.

The second parameter to be found is the noise power, σ_v^2. Usually, it can be estimated during silences of the near-end talker, i.e., in the single-talk scenario [15]. However, this is not always an easy task. The most critical situation in echo cancellation is the double-talk case, when the near-end signal is a combination of the background noise and the near-end speech. In this scenario, the parameter $\sigma_v^2(n)$ can be estimated as proposed in [16] or [17].

6. EXPERIMENTAL STUDY

Experiments are performed in the context of both network and acoustic echo cancellation. Two echo paths are used: the first one is the fourth impulse response from G168 Recommendation [18] (with 128 taps); the second one is a measured acoustic impulse response (with 512 taps). The sampling rate is 8 kHz. All adaptive filters used in the experiments have the same length as the echo paths. The far-end signal (i.e., the input signal) is either a white Gaussian signal or a speech sequence. The output of the echo path is corrupted by an independent white Gaussian noise with 20 dB signal-to-noise ratio (SNR). We assume that the variance of the noise, σ_v^2, is available in all the simulations. In order to evaluate the tracking capabilities of the algorithms, an echo path change scenario is simulated in all the experiments, by shifting the impulse responses to the right by 12 samples. The performance measure used in simulations is the normalized misalignment (in dB) evaluated based on (3).

It was shown in Section 4 that the Kalman filter and the NLMS algorithm behave the same way at the convergence [with a proper selection of their parameters, see (27) and (29)]. In Fig. 1, the input signal is a white Gaussian noise and the network echo path is used. The regularization parameter of the NLMS algorithm is evaluated according to (29). Constant values of the parameter $\sigma_w^2(n)$ are considered in this experiment. As we can see, both algorithms converge to the same steady-state misalignment. In terms of the initial convergence rate, the Kalman filter outperforms the NLMS algorithm; however, they behave the same way in steady-state and their tracking reaction is similar. Also, it can be noticed that a smaller value of $\sigma_w^2(n)$ leads to a lower misalignment but also to a slower tracking. This aspect motivates the use of a variable parameter $\widehat{\sigma}_w^2(n)$ [see (39)], which will be involved in all the following experiments.

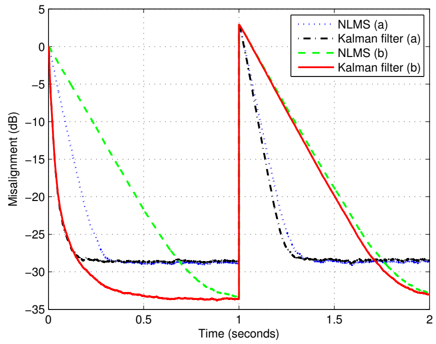

Fig. 1. Misalignment of the Kalman filter and the NLMS algorithm. (a) $\sigma_w^2(n) = 10^{-9}$. (b) $\sigma_w^2(n) = 10^{-10}$. The input signal is white Gaussian, $L = 128$, and SNR = 20 dB.

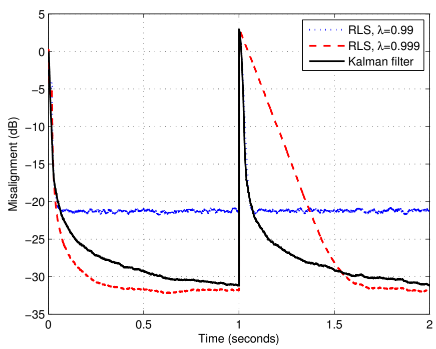

Fig. 2. Misalignment of the Kalman filter and the RLS algorithm using different values of the forgetting factor λ. Other conditions as in Fig. 1.

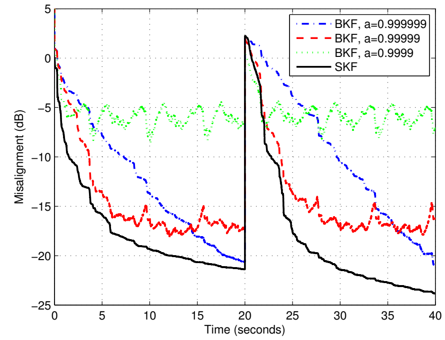

Fig. 3. Misalignment of the SKF and the BKF using different values of the parameter a. The input signal is speech, $L = 512$, and SNR = 20 dB.

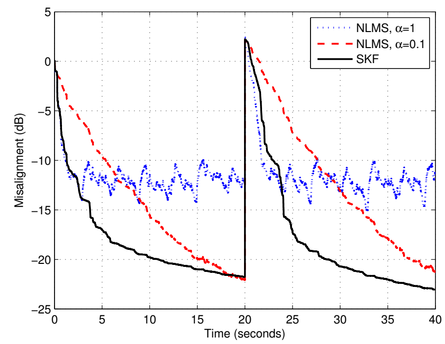

Fig. 4. Misalignment of the SKF and the NLMS algorithm using different values of the normalized step-size α. Other conditions as in Fig. 3.

In Fig. 2 (using the network impulse response), the Kalman filter is compared to the RLS algorithm using different values of the forgetting factor ($0 < \lambda \leq 1$). This specific parameter of the RLS algorithm addresses the compromise between convergence rate/tracking capabilities on the one hand and misadjustment/stability on the other hand. It can be noticed that the Kalman filter compromises better between the tracking capability and steady-state misalignment level, as compared to the RLS algorithm. To improve the tracking reaction of the RLS algorithm, the value of the λ should be decreased, but this increases the misalignment and could affect the stability [19].

In Fig. 3, the simplified Kalman filter (SKF) developed in Section 4 is compared to the BKF [8] in the context of acoustic echo cancellation. The parameter $\widehat{\sigma}_w^2(n)$ is estimated in different ways within these two algorithms [see (39) and (41)]. We can see that the BKF gives a lower misalignment level when the value of the parameter a increases; but its tracking capability is reduced in this case. On the other hand, the SKF achieves a good compromise between these performance criteria. This algorithm tracks faster than the BKF, while it also shows a reasonable low steady-state misalignment.

As explained in Section 4, the SKF behaves like a VSS adaptive filter. In Fig. 4 (using the acoustic impulse response), the SKF is compared to the NLMS algorithm using two different values of the normalized step-size. This positive constant α (usually $\alpha \leq 1$) multiplies the update term of the NLMS algorithm [i.e., the second term in the right-hand side of (31)], in order to achieve a good compromise between the convergence rate and misadjustement. It can be noticed that the convergence rate of the SKF (and its tracking reaction) is similar to the NLMS algorithm using the largest normalized step-size, while the SKF obtains a lower steady-state misalignment specific to the NLMS algorithm with a smaller normalized step-size.

7. CONCLUSIONS

In this paper, we have studied the time-domain Kalman filter in the context of echo cancellation. We have explained the fundamental differences between the Kalman filter and the RLS algorithm. Also, it was proved that the NLMS algorithm has a clear relationship with the Kalman filter. Finally, we have developed a simplified Kalman filter, which behaves like a VSS adaptive filter. Simulation results support the theoretical findings, recommending the proposed solutions for real-world echo cancellation scenarios.

8. RELATION TO PRIOR WORK

The work presented here has focused on the study of the time-domain Kalman filter for echo cancellation with a simplified first-order Markov model, which seems to work very well in this context. Most of the work by G. Enzner and his co-authors consider the frequency-domain approach, e.g., [5], [6], [9], except for the BKF algorithm presented in [8] (and also some treatments in [7]). The SKF algorithm given in this paper (which has the same structure as the BKF) uses a different way to estimate the key parameter $\sigma_w^2(n)$.

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