A STATISTICAL APPROACH TO REVERBERATION IN NON-DIFFUSIVE RECTANGULAR ROOMS BASED ON THE IMAGE SOURCE MODEL

Albino Nogueiras Rodríguez and Jordi Colom Olivares

Universitat Politècnica de Catalunya (UPC). Barcelona, Spain.
albino.nogueiras@upc.edu

ABSTRACT

In this paper, a novel procedure for the estimation of the energy decay curve of the reverberation on rectangular non-diffusive rooms is presented. It is based on the calculation of the expected sound intensity using a room characteristic factor, the specific attenuation factor, also introduced in the paper. Complete knowledge of the probability density function of this factor leads to exact estimation of the energy decay curve of reverberation, even in the case of heavily irregular rooms and/or un-homogeneous walls.

Index Terms— Reverberation, Energy Decay Curve, Image Source Model

1. INTRODUCTION

Reverberation plays a central role in room acoustics. In the seminal work by Sabine [1], he proposed the reverberation decay time as a measure of the acoustic quality of a room, and empirically deduced a formula that is still used. Later work by Eyring [2] tried to provide a more theoretically founded analysis, but it was based on the assumption that the room was perfectly diffusive, what is only true under very strict conditions of geometry of the room and absorption of its walls, that are difficultly met. Other works, such as those from Fitzroy [3], Arau [4] or Neubauer [5], try to overcome the problems associated with standard rooms where the perfect diffusive assumption cannot be made, but still are focused on the estimation of the reverberation decay time.

More recently, several authors have tried to characterize not only the decay time of reverberation, but its whole evolution along time as well. Most of these works are based on geometric considerations, either using wave propagation ideas [6], or the image source model [7].

In this work, we propose a novel estimation of the energy decay curve of reverberation for non diffusive rectangular rooms based on the image source model. Unlike previous works, it does not rely on geometric considerations, but on the probability density function (**PDF**) of a room dependent factor. A complete and exact knowledge of this PDF leads to error-free estimation of the energy decay both for low frequency and broad band signals.

Our approach is similar to that of Lehmann and Johansson [7] but, while they rely on complex geometric considerations, our method is based on more tractable statistic properties of the room.

2. THE IMAGE SOURCE MODEL IN NON-DIFFUSIVE RECTANGULAR ROOMS

The image source model is based on the fact that the effects of the reflection of sound on a non-diffusive limiting surface can be modeled by substituting this surface with a specular virtual source. By considering all possible reflections on the different walls of an enclosure, we get a constellation of virtual sources. The total reverberation can be thus estimated as the sum of all these individual contributions.

In the case of rectangular rooms, the shape of the constellation of virtual sources is regular. It can be readily seen that the whole space is divided in non-overlapping cells of exactly the same shape of the room, each holding one single virtual source. This regularity in the partition of space enables us to estimate the density of sources with an arrival time to the listener equal to t [8]:

$$\mathbb{m}(t) \approx 4\pi \frac{c^3}{V} t^2 \tag{1}$$

Where the approximation becomes exact for times growing up to infinity.

3. THE SPECIFIC ATTENUATION FACTOR

3.1. The specific attenuation

The intensity of sound at a certain point due to a single source of power W depends on three independent factors: the distance between them, and the attenuation due to air absorption and to the reflections on the walls of the room. Discarding the effects of air absorption, we can express the intensity as:

$$I(t) = W \cdot \frac{1}{4\pi(ct)^2} \cdot \prod_i (1-\alpha_i)^{n_i} \tag{2}$$

Where r is the distance between the source and the point, α_i is the absorption coefficient of wall i, and n_i is the number of reflections on that wall.

The third term of this equation only depends on the reflections suffered by the sound in its travel from the source to the listener. In rectangular rooms, we get six walls. We call S_{+xy} the wall placed on the xy plane in the positive direction of axis z and, similarly, S_{-xy} the one opposite to that. The number of reflections on each of these walls, that we call n_{+xy} and n_{-xy}, respectively, will differ at most in one, so we will suppose that they are equal and call it $n_z \approx n_{+xy} \approx n_{-xy}$. The attenuation due to reflections on this couple of walls becomes:

$$\mathrm{A_{xy}} \approx \left(1-\alpha_{+xy}\right)^{-n_{+xy}} \left(1-\alpha_{-xy}\right)^{-n_{-xy}} = \beta_z^{-2n_z} \tag{3}$$

Where $\beta_z = \sqrt{\left(1-\alpha_{+xy}\right)\left(1-\alpha_{-xy}\right)}$ is the mean reflection coefficient for walls parallel to plane xy, i.e. normal to z. The total attenuation due to reflections on the walls of the room becomes:

$$\mathrm{A_{ref}} = \prod_{\mathrm{i=x,y,z}} \beta_\mathrm{i}^{-2\mathrm{n_i}} \tag{4}$$

In the image source model this attenuation only depends on the location of the virtual source. In general, the longest the distance, more reflections will be involved. Being L_i the dimension of the room on the i^{th} axis, we have:

$$t = \frac{1}{c}\sqrt{(n_x L_x)^2 + (n_y L_y)^2 + (n_z L_z)^2} \tag{5}$$

And the attenuation due to reflections can be expressed as

$$\mathrm{A_{ref}} = \prod_{\mathrm{i=x,y,z}} \beta_\mathrm{i}^{-2ct\frac{n_i}{\sqrt{(n_x L_x)^2+(n_y L_y)^2+(n_z L_z)^2}}} = \mathrm{S}(t,\theta)^{ct} \tag{6}$$

$$\mathrm{S}(t,\theta) = \mathrm{S}(\theta) = \prod_{\mathrm{i=x,y,z}} \beta_\mathrm{i}^{\frac{-2n_i}{\sqrt{(n_x L_x)^2+(n_y L_y)^2+(n_z L_z)^2}}} \tag{7}$$

Where we call $\mathrm{S}(t,\theta)$ the *specific attenuation* of the room for a virtual source at a distance $r = ct$ in the θ arrival direction. This specific attenuation always depends on the arrival direction but notice that, in the case of rectangular rooms, it is independent of time because it does not depend on the particular values of n_i but just on the ratios n_j/n_i.

3.2. The specific attenuation factor

In non-diffusive rectangular rooms, the specific attenuation is bounded. The lower limit corresponds to the direction in space with the minimum attenuation. Let's consider first tangential sources, i.e. those where just two confronted walls are involved. For this kind of source just one of n_i is different from zero, so the logarithm of the specific attenuation in its direction is $\ln \mathrm{S}(\theta) = -2\ln\beta_\mathrm{i}/L_i$. Thus, the minimum occurs for direction:

$$\begin{gathered} i_{min} = \operatorname{argmin}\{-2\ln\beta_\mathrm{i}/L_i\} \\ \beta_{max}^L = \ln\beta_{i_{\min}}/L_{i_{min}} \\ \mathrm{S_{min}} = e^{-2\beta_{max}^L} \end{gathered} \tag{8}$$

Any other direction of space, either tangential or not, can be formed by substituting a sub-path on this minimum $\mathrm{S}(\theta)$ direction with sub-paths on the rest of directions. But each of these sub-paths will have, at least, as much attenuation as the former, so we conclude that $\mathrm{S_{min}}$ is the absolute minimum of the specific attenuation.

The specific attenuation is also upper-bounded. Its maximum can be obtained by constrained maximization of equation (6) subject to the restriction in equation (5). Using Lagrange multipliers we get:

$$\begin{gathered} \beta_{min}^L = \sum \frac{\ln^2\beta_i}{L_i^2\sqrt{(\ln\beta_x/L_x)^2 + (\ln\beta_y/L_y)^2 + (\ln\beta_z/L_z)^2}} \\ \mathrm{S_{max}} = e^{-2\beta_{min}^L} \end{gathered} \tag{9}$$

Using these upper and lower limits, a convenient way of representing the specific attenuation is by means of the specific attenuation factor, $\mathrm{O}(\theta)$, defined as:

$$\mathrm{O}(\theta) = \frac{\ln(\mathrm{S}(\theta)/\mathrm{S_{min}})}{\ln(\mathrm{S_{max}}/\mathrm{S_{min}})} \tag{10}$$

Using $\mathrm{O}(\theta)$, the specific attenuation becomes:

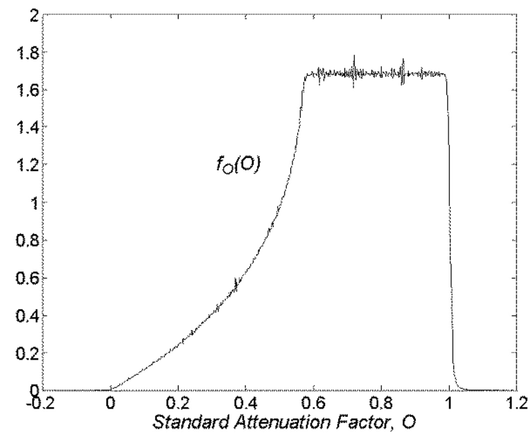

Figure 1: *Probability density function of the specific attenuation factor for cubic rooms with homogeneous absorption.*

$$\mathrm{S}(\theta) = \mathrm{S_{min}}\left(\frac{\mathrm{S_{max}}}{\mathrm{S_{min}}}\right)^{\mathrm{O}(\theta)} = \mathrm{S_{min}}\Upsilon^{\mathrm{O}(\theta)} \tag{11}$$

Where $\Upsilon=\mathrm{S_{max}}/\mathrm{S_{min}}$ is a room dependent figure that quantifies the dynamic range of the reflections arriving at a same time.

The specific attenuation factor is a variable of the geometry of the room and the arrival direction, whose value ranges from 0, for the direction of minimum attenuation, to 1, for the direction of maximum attenuation. Knowing its value for all the virtual sources in the room is enough to have a complete knowledge of the evolution of reverberation over time.

$\mathrm{O}(\theta)$ has several interesting properties, particularly its probability density function (**PDF**), $f_O(O)$. In first place, it is invariant to uniform scaling of $\ln\beta_\mathrm{i}/L_i$. This includes both scaling of the dimensions of the room, and raising the reflection coefficients β_i to a same exponent. For instance, Figure 1 shows the PDF of the specific attenuation coefficient for cubic rooms with the same absorption coefficient α in all its walls. This function is independent of the dimensions of the room or the value of α, as long as the room is cubic and homogeneous.

We have empirically found another interesting property of $f_O(O)$: in general, it shows a non-descent trend. We believe that this is due to the fact that, given a certain path from a virtual source to the listener, it is possible to find other sources at the same distance by substituting a portion of this path with another. In most situations, there will be many more possibilities of finding a substituting sub-path with a greater number of reflections than the original, than of finding ones with fewer reflections. In general, more reflections means more attenuation, and this would justify the non-descent nature of $f_O(O)$. Nevertheless, this situation is not always accomplished. For instance, if the room has two pairs of confronted walls with very low attenuation and small distance, while the other pair has bigger attenuation and distance, low attenuation paths can be more frequent than high attenuation ones. Yet, in our experiments, we have only found this situation in extremely irregular and un-homogeneous rooms.

The approximately crescent trend of $f_O(O)$ implies that the paths with the minimum attenuation will also be the less frequent. Yet, as the total reverberation will be the combination of all the contributions, the asymptotical behavior will be dominated by them. On the other hand, for small arrival times there will be little attenuation, independently of the value of O. So, in this case, the global behavior will be dominated by the most frequent values of O. This implies that the whole PDF carries valuable information about the evolution of reverberation, with the more frequent values ruling the initial shape of its energy decay curve, and the rarer values ruling the asymptotical behavior.

4. EXPECTED VALUE OF THE REVERBERATION INTENSITY

Using the specific attenuation coefficient, the sound intensity contribution due to a single virtual source located at a distance $r = ct$ in the θ direction is:

$$I_i(t) = W \cdot \frac{1}{4\pi(ct)^2} \cdot \mathrm{S}_{\min}^{-\mathrm{ct}} (\Upsilon^{-ct})^{\mathrm{O}(\theta)} \tag{12}$$

In order to estimate the total reverberation, we must account for all the virtual sources arriving at time t. This can be done combining them for all possible arrival directions, as in the standard geometry-based procedures, or taking all possible values of $\mathrm{O}(\theta)$ along with their frequency.

If we use this second approach, we need to know how many sources we get for each value of $\mathrm{O}(\theta)$, i.e. its PDF $f_O(O)$, and how we must combine the individual contribution to get the global intensity.

The question of how to determine $f_O(O)$ on a specific room will be subject of further research in the future. By now, on the scope of this paper, we will use histograms evaluated using image source model-based simulations. This means that the PDF will be in the form:

$$f_O(O) = \sum_k f_i \Pi\left(\frac{O - O_k}{\Delta O}\right) \tag{13}$$

Where $\Pi(\mathrm{O})$ is the rectangular pulse function, so $\Pi\left(\frac{O-O_k}{\Delta O}\right)$ is a pulse of amplitude one, width ΔO, and centered in the value O_k, and f_k is the frequency of values of O inside this margin.

The way individual sources are to be combined to get the global intensity depends on the correlation between them which, in turn, depends on the spectrum of the sound. In the case of virtual sources coming from the reflections of a real source, all the sources produce the same signal so, in principle, they combine coherently. The problem with this coherence assumption is that virtual sources do not conform a continuum, but a discrete grid with different arrival times that break the coherence. On the other hand, out of perfectly impulsive sounds, sound coming from sources at different distances will overlap in time.

In this paper we will consider two extreme opposite different conditions: complete incoherence and maximum coherence of the virtual sources.

4.1. Source combination assuming complete incoherence of the virtual sources

Complete incoherence means that the correlation of the signals is zero at the origin. As all the sources produce the same signal with different amplitudes and arrival times, this only can happen if the signal has a flat spectrum on an infinite bandwidth.

Assuming complete incoherence, the global intensity due to a number of different sources the sum of the individual intensities:

$$I_T(t) = \sum I_k(t) \tag{14}$$

The expected value of $I_T(t)$ for $\mathbb{m}(t)$ sources is:

$$E\{I_T(t)\} = \sum E\{I_k(t)\} = \mathbb{m}(t) E\{I_k(t)\} \tag{15}$$

Where we can determine the expected value of one virtual source using the PDF of the specific attenuation factor:

$$\begin{aligned} E\{I_k(t)\} &= \int_{-\infty}^{\infty} \frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} (\Upsilon^{-ct})^O f_O(O) dO = \\ &= \frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} \int_{-\infty}^{\infty} \Upsilon^{-ctO} f_O(O) dO \end{aligned} \tag{16}$$

Expression which, assuming the histogram defined in (13), becomes:

$$\begin{aligned} E\{I_k(t)\} &= \\ &= \frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} \int_{-\infty}^{\infty} \Upsilon^{-ctO} \sum f_k \Pi\left(\frac{O - O_k}{\Delta O}\right) dO = \\ &= \frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} \sum f_k \int_{O_k - \Delta O/2}^{O_k + \Delta O/2} \Upsilon^{-ctO} dO = \\ &= \frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} \sum f_k 2 \frac{\Upsilon^{-ctO_k}}{ct \ln \Upsilon} \sinh\left(ct \frac{\Delta O}{2} \ln \Upsilon\right) \end{aligned} \tag{17}$$

And the expected value of $I_T(t)$ for $\mathbb{m}(t)$ sources is:

$$E\{I_T(t)\} = 2 \frac{W}{V \ln \Upsilon} \frac{\mathrm{S}_{\min}^{-\mathrm{ct}} \sinh\left(ct \frac{\Delta O}{2} \ln \gamma\right)}{t} \sum f_k \gamma^{-ctO_k} \tag{18}$$

4.2. Source combination assuming maximum coherence of the virtual sources

In the case of low frequency signals, the incoherence assumption is not applicable because they will present little or none phase lag. When sound coming from all the sources is in the same phase, we must combine them coherently.

Coherent signals combine together adding their amplitudes instead of their intensities:

$$\sqrt{I_T(t)} = \sum \sqrt{I_k(t)} \tag{19}$$

$$E\left\{\sqrt{I_T(t)}\right\} = \sum E\left\{\sqrt{I_k(t)}\right\} = \mathbb{m}(t) E\left\{\sqrt{I_k(t)}\right\} \tag{20}$$

$$\begin{aligned} E\left\{\sqrt{I_k(t)}\right\} &= \int_{-\infty}^{\infty} \sqrt{\frac{W}{4\pi(ct)^2} \mathrm{S}_{\min}^{-\mathrm{ct}} (\Upsilon^{-ct})^O} f_O(O) dO = \\ &= \sqrt{\frac{W}{4\pi(ct)^2}} \mathrm{S}_{\min}^{-\mathrm{ct}/2} \int_{-\infty}^{\infty} \Upsilon^{\frac{-ctO}{2}} f_O(O) dO \end{aligned} \tag{21}$$

Which, using the histogram defined in (13):

$$\begin{aligned} E\left\{\sqrt{I_k(t)}\right\} &= \sqrt{\frac{W}{4\pi(ct)^2}} \mathrm{S}_{\min}^{-\mathrm{ct}/2} \sum f_k \int_{O_k - \Delta O/2}^{O_k + \Delta O/2} \Upsilon^{\frac{-ctO}{2}} dO = \\ &= \sqrt{\frac{W}{4\pi(ct)^2}} \mathrm{S}_{\min}^{-\mathrm{ct}/2} \sum f_k 4 \frac{\Upsilon^{\frac{-ctO_k}{2}}}{ct \ln \Upsilon} \sinh\left(ct \frac{\Delta O}{4} \ln \Upsilon\right) \end{aligned} \tag{22}$$

And the expected value of $\sqrt{I_T(t)}$ for $\mathbb{m}(t)$ sources is:

$$E\left\{\sqrt{I_T(t)}\right\} = \\ = \frac{c\sqrt{64\pi W}}{V \ln \Upsilon} \mathrm{S}_{\min}^{-\mathrm{ct}/2} \sinh\left(ct\frac{\Delta O}{4}\ln \Upsilon\right) \sum f_k \Upsilon^{-ctO_k/2} \tag{23}$$

5. SIMULATION RESULTS

In order to assess the validity of equations (18) and (23), we compare the results given by them with those got with a MATLAB implementation of the image source model proposed by Allen and Berkley [9].

Table 1 shows the results obtained when using our formulae in the estimation of the three fundamental parameters of rooms: the reverberation time, the intensity level due to reverberation, and the energy decay curve. Two kinds of signal are used in the comparisons: in the broad band signal experiments, the source produces a perfectly impulsive sound (a *Dirac impulse*) which is not further filtered, so the different sources will add un-coherently; in the low-pass signal experiments, this impulsive sound is filtered with a low-pass filter with cut-off frequency equal to one half of the speed of sound divided by the largest dimension in the room. This choice of band ensures that virtual sources at similar distance add coherently, while keeping the shape of the energy decay. Room dimensions and walls attenuation coefficients are chosen so that they range all possible situations: Room 1 is cubic (regular) and all the walls share the same attenuation coefficient (homogeneous); Room 2 is regular, but highly un-homogeneous; Room 3 is homogeneous, but highly irregular; and Rooms 4-9 are both highly irregular and un-homogeneous.

In the reverberation time results we can see the very different values obtained using the classical Sabine's formula and the image source model (in particular, see Rooms 7 and 9), this is probably due to the fact that Sabine's formula assumes perfectly diffusive rooms. On the contrary, our formulae adhere to the values of the image source model with an error below 10% in all cases.

It can also be seen that the image source model and the theoretical classical formula for the total intensity of the reverberation field, $I = 4W/R$ [11], coincide to a great extent for broad-band signals, but differ by more than 20dB for low-pass signals. Our formulae coincide with the simulations with an error less than 2dB. They also coincide, with less than 3dB root mean square error, when the whole energy decay curve is compared.

6. CONCLUSIONS AND FUTURE WORK

As it can be seen in the previous section, the herein proposed method provides very good estimation of the energy decay curve even in the case of highly irregular and un-homogeneous rooms. Yet, this good result is based on the previous knowledge of the PDF of the specific attenuation factor of the room. One of our close future efforts will be how to infer this PDF out of the characteristics of the room.

In our opinion, extension to non-rectangular rooms or to take into account diffusion only require a redefinition of the density of sources defined in equation (1), and the evaluation of the corresponding PDF of the specific attenuation factor, which will surely become time-dependent. Anyway, equations (16) and (21), should still apply to these situations.

Finally, we propose two different approximations for low frequency and broad band signals. Future work will also address the more interesting case of band-pass signals, as those generally considered in the evaluation of audition rooms.

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Room	Dimensions [m]			Attenuation Coefficients			Reverberation Time (RT60) [s]			%Error in RT60		Total Intensity Level (Lp) [dB]			Error in Lp [dB]		RMS Error in EDC [dB]	
	Lx	Ly	Lz	Ax	Ay	Az	Sabine	RT_∞	RT_0	ERT_∞	ERT_0	$4W/R$	Lp_∞	Lp_0	Ep_∞	Ep_0	RMS_∞	RMS_0
1	10	10	10	0.50	0.50	0.50	0.54	0.43	0.51	-0.14%	0.47%	-10.77	-8.54	-30.37	-0.60	-0.45	1.86	1.41
2	10	10	10	0.90	0.50	0.10	0.54	1.97	2.15	-0.69%	0.20%	-10.77	-8.36	-31.92	-1.58	-1.17	1.55	0.98
3	2.5	10	40	0.50	0.50	0.50	0.31	1.34	0.87	-6.34%	-0.94%	-13.20	-12.34	-37.26	1.71	-0.60	1.18	0.45
4	2.5	10	40	0.10	0.50	0.90	0.72	0.63	0.38	-3.39%	-1.93%	-7.56	-8.19	-29.02	-0.02	-1.34	2.11	1.22
5	2.5	10	40	0.10	0.90	0.50	0.57	1.35	0.84	-3.02%	-1.09%	-8.91	-9.60	-32.52	0.05	-1.30	1.68	1.13
6	2.5	10	40	0.50	0.10	0.90	0.35	2.09	1.88	-5.78%	-0.29%	-12.20	-10.87	-34.50	0.71	-1.23	1.54	1.06
7	2.5	10	40	0.50	0.90	0.10	0.28	6.47	7.21	-5.37%	0.76%	-14.20	-13.76	-42.04	1.56	-1.24	1.42	1.02
8	2.5	10	40	0.90	0.10	0.50	0.21	2.20	1.88	-2.68%	-0.32%	-17.49	-13.46	-39.17	0.82	-1.12	1.56	1.01
9	2.5	10	40	0.90	0.50	0.10	0.20	6.75	7.57	-1.05%	1.84%	-18.84	-15.09	-43.87	-0.62	-1.10	2.68	1.94

Table 1: *Simulation and theoretical results for 9 different rooms. Reverberation Time: Sabine stands for RT60 using Sabine's formula,* $RT_{Sabine} = 0.161V/\bar{\alpha}S$, RT_∞ *and* RT_0 *are the values of* $RT60$ *estimated using the image source model for broadband and low-pass signals, respectively;* ERT_∞ *and* ERT_0 *are the percent error committed when estimating* $RT60$ *with the formulae proposed in this paper for the energy decay curve; Total Intensity Level:* Lp *is the energy level in dB,* $4W/R$ *stands for the classical value of the total intensity of the reverberant field,* Lp_∞ *and* Lp_0 *are the values of* Lp *estimated using the image source model for both kinds of signal, and ,*Ep_∞ *and* Ep_0 *the error in dB committed using our formulae;* RMS_∞ *and* RMS_0 *are the root mean square error committed when approximating the energy decay curve (EDC) obtained with the image source model using our formulae.*

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