SOME INSIGHTS INTO FARFIELD WIDEBAND BEAMFORMERS IN THE PRESENCE OF MICROPHONE MISMATCHES

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ABSTRACT

In this paper, the performance analysis is carried out for farfield wideband beamformers with microphone mismatches, from the perspective of variance analysis of array response. Some insightful properties on farfield wideband beamformers have been revealed, which are helpful to better understand the robustness of farfield wideband beamformers. The comparative study is performed, regarding the effects of microphone position errors, and of microphone gain and phase errors on farfield wideband beamformers. It is found that the effects of microphone position errors depend on source angle of arrival as well as the temperature in homogeneous environments, while these factors have little impact on the effects of microphone gain and phase errors. Several numerical examples are presented to further confirm the theoretical results.

***Index Terms*—** Microphone array, wideband beamformer, robustness, microphone mismatches.

1. INTRODUCTION

Beamforming, i.e., spatial filtering, has long been the important technique in array signal processing field, and has been found wide applications in radar, sonar, wireless communication, and microphone arrays, etc. [1, 2, 3]. In general, beamformers can be classified into narrowband and wideband beamformers [1]. Due to wideband nature of audio and speech signals, wideband beamformers are usually required in microphone array processing [4, 5, 6, 7, 8].

In practical applications, there usually exist microphone mismatches, i.e., gain and phase errors, microphone position errors [9, 10, 11, 12]. It is known that wideband beamformers are highly sensitive to microphone mismatches, especially for small-size arrays. Although one can perform microphone calibration before use, unfortunately, the fact that microphone characteristics are usually not exactly available to the designers and can even change over time makes the microphone calibration a very challenging task [11]. Therefore, wideband beamformers robust against microphone mismatches are practically required. In recent years, several robust design methods of wideband beamformers have been proposed. Generally speaking, the existing robust design approaches can be classified into two categories. One is based on the probability density function (pdf) weighted criteria, which takes the pdf of microphone characteristics into consideration [11, 12, 13, 14, 15]. Another is based on the worst-case performance (WCP) optimization, which does not require the pdf knowledge of microphone characteristics, and just uses the uncertainties in microphone mismatches [16]. To overcome the over-constraint problem with the WCP optimization based design approach, recently we have proposed a new design approach based on the worst-case mean performance optimization with uncertain prior statistical knowledge of microphone characteristics [17]. Besides, some theoretical results on array response variance (ARV) of nearfield wideband beamformers with microphone gain and phase errors are also presented therein, which are helpful to understand the robustness characteristics of nearfield wideband beamformers.

The work presented here has focused on the study of the effects of microphone position errors, as well as microphone gain and phase errors on farfield wideband beamformers. The work in [17] considers only the microphone gain and phase errors for nearfield wideband beamformers. Furthermore, the present work makes an interesting comparative study on the effects of microphone position errors, and of microphone gain and phase errors on farfield wideband beamformers through both the theoretical analysis and numerical evaluation, which was not considered in the earlier studies.

2. MATHEMATICAL DATA MODEL

Consider a farfield linear array consisting of M microphones that are placed along the x-axis with the positions x_0, x_1, $\cdots$, x_{M-1}, respectively. Let an L-tap finite impulse response (FIR) filter $\mathbf{w}_m \in \mathbb{R}^{L\times 1}$ $(m = 0, 1, \cdots, M-1)$ be used behind each microphone.

The array response of the filter-and-sum wideband beamformers to a source from the angle of arrival (AOA) θ defined with respect to the x-axis ($0^\circ \leq \theta \leq 180^\circ$, with $\theta = 90^\circ$ as

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the broadside direction) at frequency f can be expressed as

$$P(\theta, f) = \mathbf{w}^T \mathbf{g}(\theta, f) \quad (1)$$

where $\mathbf{w} = \left[\mathbf{w}_0^T, \mathbf{w}_1^T, \cdots, \mathbf{w}_{M-1}^T\right] \in \mathbb{R}^{ML\times 1}$ is the beamformer weight vector, $(\cdot)^T$ represents the transpose, $\mathbf{g}(\theta, f) = \mathbf{h}(\theta, f) \otimes \mathbf{e}(f)$ is the array steering vector, $\otimes$ stands for the Kronecker product, and

$$\mathbf{h}(\theta, f) = [h_0(\theta, f), h_1(\theta, f), \cdots, h_{M-1}(\theta, f)]^T \quad (2)$$

$$\mathbf{e}(f) = \left[1, e^{-j2\pi f/f_s}, \cdots, e^{-j2\pi f(L-1)/f_s}\right]^T \quad (3)$$

where $h_m(\theta, f) = \exp(-j2\pi f x_m \cos\theta/c)$ is the microphone transfer function from the reference point to the mth microphone, c denotes the sound speed in air, and f_s represents the sampling frequency. Without loss of generality, the origin has been chosen as the reference point.

In practice, there usually exist microphone mismatches, i.e., microphone position errors, and microphone gain and phase errors. Like [12, 18], consider the microphone position perturbations are along the x-axis. The microphone transfer function can be represented by [18]

$$\tilde{h}_m^{(p)}(\theta, f) = \exp\left[-j2\pi f(x_m + \delta_m)\cos\theta/c\right] \quad (4)$$

In the presence of microphone gain and phase errors, the characteristics of the m-th microphone is given by [11]

$$A_m(\theta, f) = [1 + g_m(\theta, f)]e^{-j\varphi_m(\theta, f)} \quad (5)$$

where $g_m(\theta, f)$ and $\varphi_m(\theta, f)$ are the gain and phase errors of the m-th microphone, respectively. Consequently, the actual microphone transfer function is given by

$$\tilde{h}_m^{(gp)}(\theta, f) = A_m(\theta, f)\exp(-j2\pi f x_m \cos\theta/c) \quad (6)$$

3. MAIN RESULTS

In the following, we make the common assumptions [12, 13, 14, 15]: all microphone mismatch errors are uncorrelated, and all microphones have the same characteristics.

3.1. Microphone Position Errors

For small microphone position errors, applying Taylor series expansion with first-order approximation to (4), we obtain

$$\begin{aligned}\tilde{h}_m^{(p)}(\theta, f) &\simeq \exp(-j2\pi f x_m \cos\theta/c) - j2\pi f \delta_m \cos\theta/c \\ &\quad \times \exp(-j2\pi f x_m \cos\theta/c) \\ &= h_m(\theta, f) + \Delta h_m^{(p)}(\theta, f) \quad (7)\end{aligned}$$

where $\Delta h_m^{(p)}(\theta, f)$ is the perturbation of microphone transfer function due to microphone position errors.

Using (7), the array response of the wideband beamformer with microphone position errors can be represented as

$$P(\theta, f) = \mathbf{w}^T \left[\mathbf{g}(\theta, f) + \Delta\mathbf{g}(\theta, f)\right] \quad (8)$$

where

$$\Delta\mathbf{g}(\theta, f) = \Delta\mathbf{h}^{(p)}(\theta, f) \otimes \mathbf{e}(f) \quad (9)$$

with $\Delta\mathbf{h}^{(p)}(\theta, f) = [\Delta h_0^{(p)}(\theta, f), \Delta h_1^{(p)}(\theta, f), \cdots, \Delta h_{M-1}^{(p)}(\theta, f)]^T$.

Theorem 1 *The ARV of farfield wideband beamformer with microphone position errors is*

$$\mathrm{Var}[P(\theta, f)] \simeq (2\pi f\cos\theta/c)^2 \sigma_\delta^2 \mathbf{w}^T[\mathbf{I}_M \otimes \mathbf{\Lambda}(f)]\mathbf{w} \quad (10)$$

where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix, σ_δ^2 is the variance of microphone position errors, and $\mathbf{\Lambda}(f)$ is the $L \times L$ symmetric matrix whose (k, l)th element is $\mathbf{\Lambda}^{(kl)}(f) = \cos[2\pi(k-l)f/f_s]$.

Proof: With (8), we have

$$\begin{aligned}\mathrm{Var}[P(\theta, f)] &= \mathbf{w}^T\mathbb{E}\left\{(\Delta\mathbf{g}(\theta, f) - \mathbb{E}[\Delta\mathbf{g}(\theta, f)])\right. \\ &\quad \left.\times (\Delta\mathbf{g}^H(\theta, f) - \mathbb{E}[\Delta\mathbf{g}^H(\theta, f)])\right\}\mathbf{w} \\ &= \mathbf{w}^T\left\{\mathbb{E}[\Delta\mathbf{g}(\theta, f)\Delta\mathbf{g}^H(\theta, f)]\right. \\ &\quad \left.- \mathbb{E}[\Delta\mathbf{g}(\theta, f)]\mathbb{E}[\Delta\mathbf{g}^H(\theta, f)]\right\}\mathbf{w} \\ &= \mathbf{w}^T\mathbf{Q}(\theta, f)\mathbf{w} \quad (11)\end{aligned}$$

where $(\cdot)^H$ denotes the Hermitian transpose, and

$$\begin{aligned}\mathbf{Q}(\theta, f) \triangleq\ & \mathbb{E}[\Delta\mathbf{g}(\theta, f)\Delta\mathbf{g}^H(\theta, f)] \\ & -\mathbb{E}[\Delta\mathbf{g}(\theta, f)]\mathbb{E}[\Delta\mathbf{g}^H(\theta, f)]. \quad (12)\end{aligned}$$

Substituting (7) and (9) into (12) yields

$$\mathbf{Q}(\theta, f) \simeq \mathrm{diag}\left\{\mathbf{\Theta_0}, \mathbf{\Theta_1}\ldots, \mathbf{\Theta_{M-1}}\right\} \quad (13)$$

where $\mathbf{\Theta_m}$ is the $L \times L$ matrix whose (k, l)th element is

$$\mathbf{\Theta}_m^{(kl)} = (2\pi f\cos\theta/c)^2 \sigma_\delta^2 \exp[j2\pi(k-l)f/f_s]. \quad (14)$$

From (13) and (14), we know that the imaginary part of $\mathbf{Q}(\theta, f)$, denoted as $\mathrm{Im}\{\mathbf{Q}(\theta, f)\}$, is anti-symmetric. Therefore, it holds that

$$\mathbf{w}^T\mathrm{Im}\{\mathbf{Q}(\theta, f)\}\mathbf{w} = 0. \quad (15)$$

With (11) and (15), we have

$$\begin{aligned}\mathrm{Var}[P(\theta, f)] &= \mathbf{w}^T[\mathrm{Re}\{\mathbf{Q}(\theta, f)\} + j\mathrm{Im}\{\mathbf{Q}(\theta, f)\}]\mathbf{w} \\ &\simeq \mathbf{w}^T\mathrm{Re}\{\mathbf{Q}(\theta, f)\}\mathbf{w} \\ &= (2\pi f\cos\theta/c)^2 \sigma_\delta^2 \mathbf{w}^T[\mathbf{I}_M \otimes \mathbf{\Lambda}(f)]\mathbf{w} \quad (16)\end{aligned}$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part. This completes the proof.

From Theorem 1, we have the following remarks.

Remark 1: As expected, the ARV of farfield beamformers increase with microphone position errors increasing.

Remark 2: From (10), it is interesting to note that the ARV is dependent on the source AOA. Explicitly speaking, the ARV will increase if the source is away from the broadside of microphone arrays.

Proof: Consider $|\theta_1 - 90°| < |\theta_2 - 90°|$, $0 \leq \theta_1, \theta_2 \leq 180°$, i.e., θ_1 is closer to the broadside, then we have $|\cos\theta_1| < |\cos\theta_2|$, i.e. $\cos^2\theta_1 < \cos^2\theta_2$. Moreover, from (10), we know $\mathbf{w}^T[\mathbf{I}_M \otimes \mathbf{\Lambda}(f)]\mathbf{w}$ is positive. So, we have $\mathrm{Var}[P(\theta_1, f)] - \mathrm{Var}[P(\theta_2, f)] \simeq \left(\cos^2\theta_1 - \cos^2\theta_2\right)(2\pi f/c)^2$ $\sigma_\delta^2 \mathbf{w}^T [\mathbf{I}_M \otimes \mathbf{\Lambda}(f)]\mathbf{w} < 0$. Therefore, we obtain $\mathrm{Var}[P(\theta_1, f)] < \mathrm{Var}[P(\theta_2, f)]$.

Remark 3: From (10), it reveals that the ARV is dependent of the sound speed. As it is known, the sound speed is not a constant in practice due to temperature variation. When the medium is homogeneous and at rest, we have $c = (331.4 + 0.6t)$ m/s, where t denotes the temperature in centigrade [19]. Therefore, the lower the temperature in homogeneous environments, the higher the ARV will be.

3.2. Microphone Gain and Phase errors

For small microphone gain and phase errors, applying Taylor series expansion with first-order approximation to (6) yields

$$\begin{aligned} \tilde{h}_m^{(gp)}(\theta, f) &\simeq \exp(-j2\pi f x_m \cos\theta/c) + [g_m(\theta, f) \\ &\quad -j\varphi_m(\theta, f)]\exp(-j2\pi f x_m \cos\theta/c) \\ &= h_m(\theta, f) + \Delta h_m^{(gp)}(\theta, f) \end{aligned} \tag{17}$$

where $\Delta h_m^{(gp)}(\theta, f)$ is the perturbation of microphone transfer function due to microphone gain and phase errors.

With (17), the array response of the farfield wideband beamformer in the presence of microphone gain and phase errors can be represented as

$$P(\theta, f) = \mathbf{w}^T [\mathbf{g}(\theta, f) + \Delta\mathbf{g}(\theta, f)] \tag{18}$$

where

$$\Delta\mathbf{g}(\theta, f) = \Delta\mathbf{h}^{(gp)}(\theta, f) \otimes \mathbf{e}(f) \tag{19}$$

with $\Delta\mathbf{h}^{(gp)}(\theta, f) = [\Delta h_0^{(gp)}(\theta, f),\ \Delta h_1^{(gp)}(\theta, f), \cdots, \Delta h_{M-1}^{(gp)}(\theta, f)]^T$.

Theorem 2 *The ARV of farfield wideband beamformer with microphone gain and phase errors is given by*

$$\mathrm{Var}[P(\theta, f)] \simeq (\sigma_g^2 + \sigma_\varphi^2)\mathbf{w}^T[\mathbf{I}_M \otimes \mathbf{\Lambda}(f)]\mathbf{w} \tag{20}$$

where σ_g^2 and σ_φ^2 denote the variance of microphone gain and phase errors, respectively. $\mathbf{I}_M$ and $\mathbf{\Lambda}(f)$ are both defined same as in (10).

Proof: It is similar to the proof of Theorem 1, thus omitted.

From Theorem 2, we have the following remarks.

Remark 4: As also expected, the ARV of farfield wideband beamformers will increase with the microphone gain and phase errors increasing.

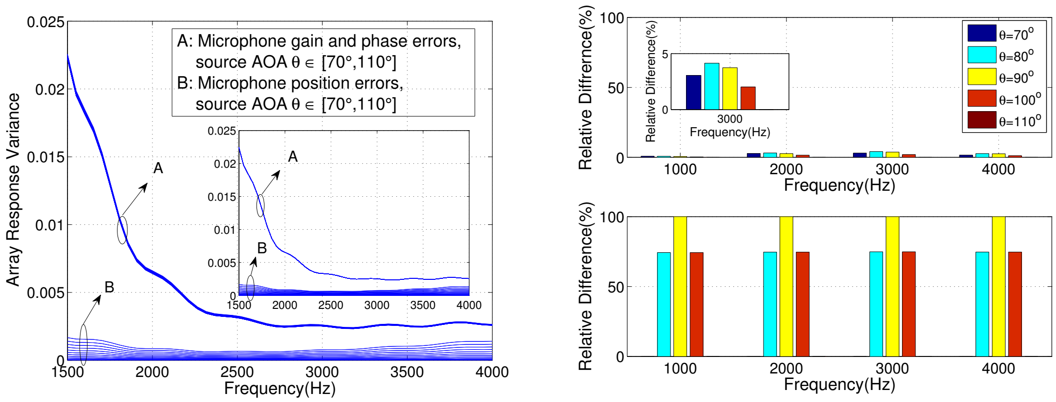
Remark 5: Unlike the case for microphone position errors as discussed above, from (20) it is noted that the ARV of farfield beamformers is independent of the source AOA and sound speed in the presence of microphone gain and phase errors. That is to say, the source AOA and the temperature in homogeneous environments, have little impact on ARV of farfield wideband beamformers.

4. NUMERICAL EVALUATION

In this section, we verify the above theoretical results through some numerical examples with a small-size microphone array. Consider a three-element uniform linear microphone array in farfield with the inter-element spacing 4 cm. The FIR tap length is chosen as $L = 20$, and the sampling frequency is set to $f_s = 8$ kHz. Suppose that the microphone position errors δ_m have a uniform distribution in $[-0.5, 0.5]$ cm. The microphone gain and phase errors g_m and φ_m have a uniform distribution in $[-0.15, 0.15]$ and in $[-5°, 5°]$, respectively. In the following, we consider the well-known wideband beamformer designed by the non-robust least squares method [11]. The design specifications are as follows. The desired passband response is chosen as $P_d(\theta, f) = 1$ while $P_d(\theta, f) = 0$ in the stopband. The passband region is defined as $\{(\theta, f)|70° \leq \theta \leq 110°, 500 \leq f \leq 4000\}$, and the stopband regions are $\{(\theta, f)|0° \leq \theta \leq 60°, 500 \leq f \leq 4000\}$ and $\{(\theta, f)|120° \leq \theta \leq 180°, 500 \leq f \leq 4000\}$, where f is in Hz. All the results below are obtained through 500 trials.

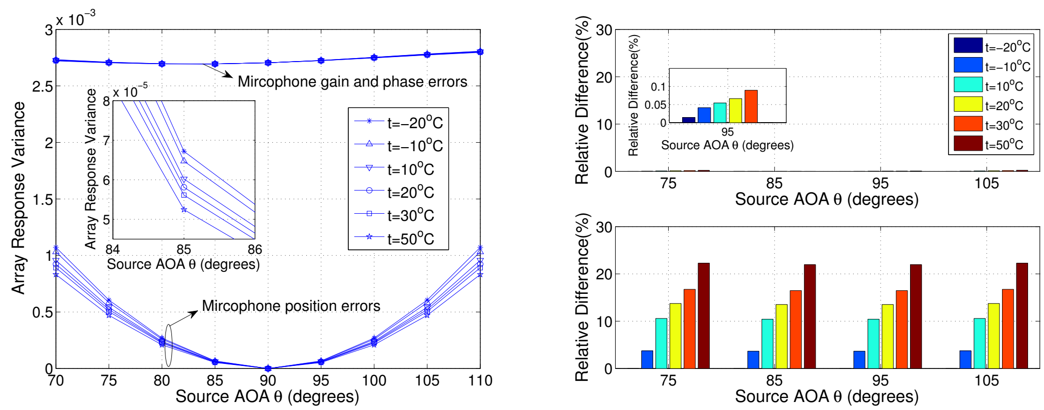
Firstly, we evaluate the effect of microphone mismatches on wideband beamformers under various source AOA. Here the nominal value of sound speed is set to 340 m/s. Fig. 1(a) shows the ARV of wideband beamformers with various source AOA $\theta \in [70°, 110°]$. As can be seen, compared with the case with microphone position errors, the source AOA has little effect on ARV with microphone gain and phase errors. To make a quantitative comparison, the relative differences of ARV with various source AOAs are plotted in Fig. 1(b). Herein, the relative difference is defined with respect to the maximal ARV over $\theta \in [70°, 110°]$. For microphone position errors, the maximum relative difference can attain nearly 100%, while it is just 4.14% for microphone gain and phase errors.

Next we evaluate the effect of microphone mismatches on the wideband beamformer under various temperature t in homogeneous environment. Fig. 2(a) shows the ARV of wideband beamformers with various temperature $t \in [-20°\mathrm{C}, 50°\mathrm{C}]$ and $f = 2500$ Hz. In comparison with microphone position errors, the temperature has little effect on ARV with microphone gain and phase errors, as shown in Fig. 2(a). In Fig. 2(b), the relative difference of the ARV with various temperature is presented. For microphone position



(a) (b)

Fig. 1. The effect of microphone mismatches with various source AOA. (a) The ARV with various source AOA at different frequencies. The theoretical results are shown in the inset. (b) Relative difference of the ARV with various source AOA at different frequencies, the top panel is for gain and phase errors, and the bottom panel for position errors.



(a) (b)

Fig. 2. The effect of microphone mismatches with various temperature. (a) The ARV with various temperature at different source AOA. (b) Relative difference of the ARV with various temperature at different source AOA, the top panel is for microphone gain and phase errors, and the bottom panel for microphone position errors.

errors, the maximum relative difference can attain 22.27%, while it is only 0.24% for microphone gain and phase errors. Moreover, as shown in Fig. 2(a), the ARV increases with the source AOA away from the broadside, i.e., $\theta = 90°$, or with the temperature decreasing, in the presence of microphone position errors. From the simulations at the frequencies other than $f = 2500$, similar conclusions can also be obtained.

To summarize, the simulation results are consistent well with our theoretical analysis. Moreover, the ARV estimated by the simulations also highly approximates to its theoretical values derived in Section 3, as illustrated in the inset of Fig. 1(a) for an example.

5. CONCLUSION

This paper has presented the performance analysis of farfield wideband beamformers with microphone mismatch errors through the variance analysis of array response. Some insights on the properties of farfield wideband beamformers in the presence of microphone mismatches have been obtained. It is revealed that the effects of microphone position errors are dependent on source angle of arrival as well as the sound speed, i.e., the temperature in homogeneous environments, while these factors have little impact on the effects of microphone gain and phase errors. The theoretical results have been further verified by the numerical examples.

6. REFERENCES

[1] B.D. Van Veen and K.M. Buckley, "Beamforming: a versatile approach to spatial filtering," *IEEE ASSP Magazine,* vol. 5, pp. 4-24, April 1988.

[2] M. Brandstein and D. Ward, *Microphone Arrays: Signal Processing Techniques and Applications,* Springer, 2001.

[3] W. Liu and S. Weiss, *Wideband Beamforming: Concepts and Techniques,* John Wiley & Sons, 2010.

[4] S.E. Nordholm, V. Rehbock, K.L. Teo, and S. Nordebo, "Chebyshev optimization for the design of broadband beamformers in the near field," *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.,* vol. 45, no. 1, pp. 141–143, Jan. 1998.

[5] O. Hoshuyama, A. Sugiyama, and A. Hirano, "A robust adaptive beamformer for microphone arrays with a blocking matrix using constrained adaptive filters," *IEEE Trans. Signal Process.*, vol. 47, no. 10, pp. 2677–2684, Oct. 1999.

[6] S. Gannot, D. Burshtein, and E. Weinstein, "Signal enhancement using beamforming and nonstationarity with applications to speech," *IEEE Trans. Signal Process.*, vol. 49, no. 8, pp. 1614–1626, Aug. 2001.

[7] K.F.C. Yiu, X. Yang, S. Nordholm, and K.L. Teo, "Near-field broadband beamformer design via multidimensional semi-infinite linear programming techniques," *IEEE Trans. Speech Audio Process.*, vol. 11, no. 6, pp. 725–732, Nov. 2003.

[8] S. Doclo and M. Moonen, "Design of far-field and near-field broadband beamformers using eigenfilters," *Signal Process.*, vol. 83, pp. 2641–2673, Dec. 2003.

[9] T.P. Hua, A. Sugiyami and G. Faucon, "A new self-calibration technique for adaptive microphone arrays," in *Proc. Int. Workshop Acoust. Echo Noise Contr. (IWAENC),* pp. 237–240, Sep. 2005.

[10] P. Oak and W. Kellermann, "A calibration algorithm for robust generalized sidelobe cancelling beamformers," in *Proc. Int. Workshop Acoust. Echo Noise Contr. (IWAENC),* pp. 97–100, Sep. 2005.

[11] S. Doclo and M. Moonen, "Design of broadband beamformers robust against gain and phase errors in the microphone array characteristics," *IEEE Trans. Signal Process.,* vol. 51, no. 10, pp. 2511–2526, Oct. 2003.

[12] S. Doclo and M. Moonen, "Superdirective beamforming robust against microphone mismatch," *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 15, no. 2, pp. 617–631, Feb. 2007.

[13] H. Chen and W. Ser, Design of robust broadband beamformers with passband shaping characteristics using Tikhonov regularization, *IEEE Trans. Audio Speech Lang. Process.,* vol. 17, no. 4, pp. 665–681, May 2009.

[14] M. Crocco and A. Trucco, "The synthesis of robust broadband beamformers for equally-spaced linear arrays, *J. Acoust. Soc. Amer.,* vol. 128, no. 2, pp. 691–701, Aug. 2010.

[15] M. Crocco and A. Trucco, "Design of robust superdirective arrays with a tunable tradeoff between directivity and frequency-invariance," *IEEE Trans. Signal Process.,* vol. 59, no. 5, pp. 2169–2181, May 2011.

[16] H. Chen, W. Ser, and Z.L. Yu, "Optimal design of nearfield wideband beamformers robust against errors in microphone array characteristics," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 54, no. 9, pp. 1950–1959, Sep. 2007.

[17] H. Chen, W. Ser, and J. Zhou, "Robust nearfield wideband beamformer design using worst case mean performance optimization with passband response variance constraint," *IEEE Trans. Audio Speech Lang. Process.*, vol. 20, no. 5, pp. 1565–1572 , Jul. 2012.

[18] S. Doclo and M. Moonen, "Design of broadband beamformers robust against microphone position errors," in *Proc. Int. Workshop Acoust. Echo Noise Contr. (IWAENC),* pp. 267–270, Sep. 2003.

[19] H. Kuttruff, *Room Acoustics,* Fourth Edition, UK: Spon Press, Taylor & Francis Group, 2000.