ROBUST BEAMFORMING USING SENSORS WITH NONIDENTICAL DIRECTIVITY PATTERNS

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ABSTRACT

The optimal weights for a beamformer that provide maximum directivity, are often found to be severely lacking in terms of robustness. Although an ideal implementation of the beamformer with these weights provides high directivity, minor perturbations of the weights or of sensor placement cause severe degradation. Therefore, a robustness constraint is often imposed during the beamformer's design stage. The classical method of diagonal loading is commonly used for this purpose. There are known results in this field which pertain to an array consisting of sensors with identical directivity-patterns and orientations. We extend these results to account for sensors with nonidentical directivity patterns, and sensors which share placement errors. We show that in such cases, modification of the classical loading scheme to incorporate nonidentical diagonal elements and off-diagonal elements is beneficial.

Index Terms— robust beamforming, maximum directivity

1. INTRODUCTION

Signal acquisition is often conducted in environments which contain noise and interference. For example, a hands-free phone conversation taking place inside a moving automobile may suffer from traffic noise, engine noise, and speech interference from passengers. Designing systems to reduce noise is a primary problem in the realm of signal processing. Typical systems employ a *beamformer* which combines several channels in such a way that noise is reduced.

The choice of weights assigned to each channel affects the beamformer's performance and determines the resulting beampattern. It is often desirable to attain maximum array directivity. The minimum variance distortionless response (MVDR) weights achieve this goal when designed for a diffuse noise filed [1].

The resulting MVDR beamformer often suffers from difficulties in practical implementation. Slight deviations from the assumed scenario can lead to a severe degradation in performance [2]. For instance, small variations in the sensors' gains, phases, or positionings can have a negative impact upon the noise reduction [3]. Similarly, minor deviations from the assumed steering-vector [4, 5] or from the assumed noise correlation matrix of the interference signals [6] can adversely affect the performance.

The detrimental effects of random perturbations are evident upon inspecting their effects on beam-power. Gilbert and Morgan [3] have shown that the introduction of random errors in weights of sensors and their placements induces an increase in the mean beam-power of an array. In effect, the array's mean beam-power consists of two components: a *nominal* beam-power determined by the assumed scenario, and an *excess* beam-power caused by random errors. Since the excess beam-power leads to an increase in the amount of noise received, its level can be used as a measure of sensitivity to errors. It was shown that the excess beam-power level is proportional to the square of the Euclidian norm of the vector containing the beamformer weights.

A number of methods have been proposed to create beamformers which are robust to perturbations from the assumed scenario [4, 5, 7, 8]. Different methods may correspond to somewhat different goals or prior knowledge of the scenario. The method of *diagonal loading* (i.e., regularizing the noise-covariance matrix by adding a multiple of the unity matrix) is possibly the most widely used. In [3], Gilbert and Morgan introduced diagonal loading as a method for increasing robustness towards weight and placement errors while maintaining the smallest possible loss of directivity.

The analysis and the results in [3] pertain to an array consisting of sensors with identical directivity patterns. For an array constructed with sensors of nonidentical directivity patterns, the situation is more complex. Poulsen [9] analyzed the case of an array of vector-sensors (i.e., an array consisting of a monopole collocated with a set of orthogonally oriented dipoles). He showed that sensors with different directivities affect the excess beam-power differently. In the context of vector-sensor arrays, it has been suggested [9, 10] that robust beamforming be preformed by using a diagonal loading matrix with nonidentical values. However, no systematic method for determining the values of this loading matrix is provided.

In the current paper, we extend the analysis of [3, 9] towards an array of sensors with an arbitrary combination of directivity patterns (e.g., cardioid microphones and acoustical vector-sensors). Our results indicate that a modified version of diagonal loading should be employed. The main diagonal of the loading matrix may contain nonidentical elements and off-diagonal elements may be nonzero.

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2. BACKGROUND

Consider an array consisting of N sensors producing N signals $x_{1\ldots N}(\omega)$ in the frequency domain, where ω represents frequency. The response of the n-th sensor to a plane wave (i.e., the sensor's directivity pattern) is denoted $s_n(\mathbf{u},\omega)$, where $\mathbf{u}$ is a unit-vector corresponding to the direction-of-arrival (DOA). The signals and directivity responses are concatenated into the column-vectors $\mathbf{x}(\omega)$ and $\mathbf{s}(\mathbf{u},\omega)$ respectively. The sensor positions are denoted by the matrix $\mathbf{P}$ with dimensions $3\times N$. The steering-vector portrays the overall array response which consists of the individual sensors' directivities and phase shifts due to propagation delays. It is given by:

$$\mathbf{v}(\mathbf{u},\omega)=\mathbf{s}\odot\exp\{j\,k\cdot\mathbf{P}^T\mathbf{u}\}\,, \tag{1}$$

where k is the wavenumber, and the operator $\odot$ represents the Hadamard (element-wise) product. When a signal $g(\omega)$ arrives from the direction $\mathbf{u}_0$, the sensors signals are given by $\mathbf{x}(\omega)=\mathbf{v}(\mathbf{u}_0,\omega)g(\omega)+\mathbf{n}(\omega)$, where $\mathbf{n}(\omega)$ is noise.

The beamformer produces an output signal by performing a weighted sum of the input channels[1]:

$$y=\mathbf{w}^H\mathbf{x}\,, \tag{2}$$

where $\mathbf{w}=[w_1\ldots w_N]^T$ contains the weights corresponding to each sensor. The beampattern is given by:

$$\mathrm{BP}(\mathbf{u})=\mathbf{w}^H\mathbf{v}(\mathbf{u})\,, \tag{3}$$

and the beam-power is given by:

$$\Psi(\mathbf{u})=|\mathbf{w}^H\mathbf{v}(\mathbf{u})|^2\,. \tag{4}$$

Let the matrix $\mathbf{\Phi}$ represent the covariance of the noise components at the different sensors. The noise level at the beamformer's output is given by $\mathbf{w}^H\mathbf{\Phi}\mathbf{w}$. In general, we wish that the beamformer should have a unity response in some desired direction $\mathbf{u}_\mathrm{d}$ while reducing noise. The resulting MVDR beamformer is given as:

$$\mathbf{w}_\mathrm{MVDR}=\frac{\mathbf{\Phi}^{-1}\mathbf{v}_\mathrm{d}}{\mathbf{v}_\mathrm{d}^H\mathbf{\Phi}^{-1}\mathbf{v}_\mathrm{d}}\,, \tag{5}$$

where $\mathbf{v}_\mathrm{d}=\mathbf{v}(\mathbf{u}_\mathrm{d})$. When the covariance matrix corresponds to diffuse noise ($\mathbf{\Phi}=\mathbf{\Phi}_\mathrm{dif}$), (5) maximizes directivity.

Errors in our evaluation of the scenario can increase the mean beam-power and reduce the signal to noise ratio (SNR). When all sensors have identical directivity, the increase in mean beam-power is proportional to $\|\mathbf{w}\|^2$. In order to reduce sensitivity, the level of $\|\mathbf{w}\|^2$ is controlled by the amount of diagonal loading [3, 4]; i.e., the matrix $\mathbf{\Phi}$ of (5) is replaced by $\mathbf{\Phi}+\mu\mathbf{I}$. When $\mu=0$, this is identical to the MVDR beamformer. Increasing μ enhances robustness by reducing $\|\mathbf{w}\|^2$ at the expense of diminished noise reduction.

[1] In the interest of consciences, explicit dependence on frequency is dropped from this point onwards.

3. EXCESS BEAM-POWER

3.1. Perturbation of weights

Let us assume that the weight vector can be decomposed as $\mathbf{w}=\mathbf{w}_0+\mathbf{w}_\mathrm{e}$, where $\mathbf{w}_0$ represents the nominal values and $\mathbf{w}_\mathrm{e}$ represents random perturbations. The vector $\mathbf{w}_0$ is deterministic and $\mathbf{w}_\mathrm{e}$ is a random variable with zero mean. The mean beampattern is:

$$E_\mathbf{w}\{\mathrm{BP}\}=E\{(\mathbf{w}_0+\mathbf{w}_\mathrm{e})^H\mathbf{v}(\mathbf{u})\}=\mathbf{w}_0^H\mathbf{v}(\mathbf{u})\,, \tag{6}$$

where $E_\mathbf{w}\{\cdot\}$ is statistical expectation with respect to the weights. Accordingly, the mean beampattern is not influenced by the perturbations. However, the mean beam-power is:

$$\begin{aligned}E\{\Psi\}&=E_\mathbf{w}\{|(\mathbf{w}_0+\mathbf{w}_\mathrm{e})^H\mathbf{v}(\mathbf{u})|^2\}\\&=|\mathbf{w}_0^H\mathbf{v}(\mathbf{u})|^2+E_\mathbf{w}\{|\mathbf{w}_\mathrm{e}^H\mathbf{v}(\mathbf{u})|^2\}\,.\end{aligned} \tag{7}$$

The first term corresponds to the nominal beam-power, whereas the second term is excess beam-power caused by perturbations. This excess term can be expressed as:

$$\Psi_\mathrm{ex}=E_\mathbf{w}\{|\mathbf{w}_\mathrm{e}^H\mathbf{v}(\mathbf{u})|^2\}=\mathbf{v}^H(\mathbf{u})E_\mathbf{w}\{\mathbf{w}_\mathrm{e}\mathbf{w}_\mathrm{e}^H\}\mathbf{v}(\mathbf{u})\,. \tag{8}$$

If we assume that the perturbations of different sensors are uncorrelated, then $E_\mathbf{w}\{\mathbf{w}_\mathrm{e}\mathbf{w}_\mathrm{e}^H\}$ is a diagonal matrix. The excess term becomes:

$$\begin{aligned}\Psi_\mathrm{ex}&=\sum_{n=1}^{N}E_\mathbf{w}\{|w_{\mathrm{e}_n}|^2\}\cdot|v_n(\mathbf{u})|^2\\&=\sum_{n=1}^{N}E_\mathbf{w}\{|w_{\mathrm{e}_n}|^2\}\cdot|s_n(\mathbf{u})|^2\,,\end{aligned} \tag{9}$$

where w_{e_n} and v_n are the n-th elements of $\mathbf{w}_\mathrm{e}$ and $\mathbf{v}$, respectively. The final stage follows from the absolute value canceling the phase shifts in the steering-vector.

The intuitive interpretation of (9) is that for each sensor, errors in the weight values add to the mean beam-power a component *proportional to the directivity power of that particular sensor*. This generalizes the classical case, in which the contributions of all sensors have identical directivity patterns.

3.2. Perturbation of sensors location

Let us assume that the sensor-placement matrix can be decomposed as $\mathbf{P}=\mathbf{P}_0+\mathbf{P}_\mathrm{e}$, where $\mathbf{P}_0$ represents the nominal locations and $\mathbf{P}_\mathrm{e}$ represents random perturbations. Substituting into (1) yields:

$$\begin{aligned}\mathbf{v}(\mathbf{u})&=\mathbf{s}(\mathbf{u})\odot\exp\{j\,k\cdot(\mathbf{P}_0+\mathbf{P}_\mathrm{e})^T\mathbf{u}\}\\&=\left[\mathbf{s}(\mathbf{u})\odot\exp\{j\,k\cdot\mathbf{P}_0^T\mathbf{u}\}\right]\odot\exp\{j\,k\cdot\mathbf{P}_\mathrm{e}^T\mathbf{u}\}\\&=\mathbf{v}_0(\mathbf{u})\odot\exp\{j\,k\cdot\mathbf{P}_\mathrm{e}^T\mathbf{u}\}\,,\end{aligned} \tag{10}$$

where $\mathbf{v}_0(\mathbf{u})$ is the nominal steering-vector. The mean beampattern is $E_{\mathbf{P}}\{\mathrm{BP}(\mathbf{u})\} = \mathbf{w}^H E\{\mathbf{v}(\mathbf{u})\}$, where $E_{\mathbf{P}}\{\cdot\}$ denotes statistical expectation with respect to positions. Assume that the placements of different sensors have identical spherical distributions (which are not necessarily independent). Let us define:

$$\kappa = E_{\mathbf{P}}\big\{\exp\{j\,k \cdot \mathbf{p}_{\mathrm{e}_m}^T \mathbf{u}\}\big\}\,, \tag{11}$$

where $\mathbf{p}_{\mathrm{e}_m}$ is the m-th column of $\mathbf{P}$. Since the distributions are identical, κ is independent of m, and since the distributions are spherical, κ is also independent of $\mathbf{u}$; hence, $E_{\mathbf{P}}\{\mathrm{BP}(\mathbf{u})\} = \kappa \cdot \mathbf{w}^H \mathbf{v}_0(\mathbf{u})$. This indicates that perturbations of sensor location attenuate the mean beampattern from its nominal value by a factor of κ.

The mean beam-power $E_{\mathbf{P}}\{\Psi\}$ is:

$$E_{\mathbf{P}}\left\{|\mathbf{w}^H\mathbf{v}(\mathbf{u})|^2\right\} = \mathbf{w}^H E_{\mathbf{P}}\left\{\mathbf{v}(\mathbf{u})\mathbf{v}^H(\mathbf{u})\right\}\mathbf{w}\,. \tag{12}$$

The term $E_{\mathbf{P}}\left\{\mathbf{v}(\mathbf{u})\mathbf{v}^H(\mathbf{u})\right\}$ can be simplified:

$$E_{\mathbf{P}}\left\{\mathbf{v}(\mathbf{u})\mathbf{v}^H(\mathbf{u})\right\} \overset{(10)}{=} \tag{13}$$
$$\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u}) \odot E_{\mathbf{P}}\big\{\exp\{j\,k\cdot\mathbf{P}_{\mathrm{e}}^T\mathbf{u}\}(\exp\{j\,k\cdot\mathbf{P}_{\mathrm{e}}^T\mathbf{u}\})^H\big\}\,.$$

Let us inspect the final term $E_{\mathbf{P}}\big\{\exp\{j\,k\cdot\mathbf{P}_{\mathrm{e}}^T\mathbf{u}\}(\exp\{j\,k\cdot\mathbf{P}_{\mathrm{e}}^T\mathbf{u}\})^H\big\}$, which constitutes an $N \times N$ matrix. We assume that if two sensors do not share the same physical packaging then their placement perturbations are independent; otherwise their perturbations are identical. Elements of the matrix corresponding to sensors which do not share packaging will have the value κ^2. All other elements will have unity values (since $|\exp\{j\,k\cdot\mathbf{p}_{e_m}^T\mathbf{u}\}|^2 = 1$). Hence, the matrix equals:

$$\kappa^2 \cdot \mathbf{1}_{N\times N} + (1-\kappa^2)\cdot\boldsymbol{\Xi}\,, \tag{14}$$

where $\mathbf{1}_{N\times N}$ is a matrix of ones and $\boldsymbol{\Xi}$ is a matrix whose elements equal one for sensors with shared packaging and zero otherwise. Substituting (14) back into (13) and then into (12) yields:

$$\begin{aligned}E_{\mathbf{P}}\{\Psi\} = \kappa^2\cdot\mathbf{w}^H\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u})\mathbf{w} \\ + (1-\kappa^2)\cdot\mathbf{w}^H[\boldsymbol{\Xi}\odot\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u})]\mathbf{w}\,.\end{aligned} \tag{15}$$

The first term corresponds to the nominal beam-power attenuated by κ^2, and the second term corresponds to the excess beam-power. If sensors do not share packaging then $\boldsymbol{\Xi}$ becomes the identity matrix, resulting in:

$$\Psi_{\mathrm{ex}} = (1-\kappa^2)\sum_{n=1}^{N} w_n^2\cdot|s_n(\mathbf{u})|^2\,. \tag{16}$$

Here too, each sensor contributes a factor to the excess beam-power which corresponds to its own directivity pattern. Note that for cases of joint packaging, $\boldsymbol{\Xi}$ has off-diagonal elements which signify cross-sensor influence on the beam-power.

When perturbations exist in both the weight values and the sensor locations, then the analyses of subsections 3.1 and 3.2 may be combined. Assuming that the two error types are uncorrelated, we have:

$$\begin{aligned}E_{\mathbf{w},\mathbf{P}}\{\Psi\} =& \kappa^2|\mathbf{w}_0^H\mathbf{v}_0(\mathbf{u})|^2 + \sum_{n=1}^{N} E_{\mathbf{w}}\{|w_{\mathrm{e}_n}|^2\}\cdot|s_n(\mathbf{u})|^2 \\ &+ (1-\kappa^2)\cdot\mathbf{w}_0^H[\boldsymbol{\Xi}\odot\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u})]\mathbf{w}_0\,.\end{aligned} \tag{17}$$

4. ROBUST BEAMFORMER DESIGN

Beamformer design determines the nominal weight values $\mathbf{w}_0$ and thus has a direct impact on the first and third terms of (17) corresponding, respectively, to nominal beam-power and excess beam-power due to placement errors. The second term (excess beam-power due to weight errors) depends on the variances $E_{\mathbf{w}}\{|w_{\mathrm{e}_n}|^2\}$ which are not specified by the design. However, these variance values can be modeled as being related to the nominal design $E_{\mathbf{w}}\{|w_{\mathrm{e}_n}|^2\} = \beta^2\,|w_{0_n}|^2$; i.e., the standard deviation of a weight is proportional to its nominal value with a factor of β. Consequently, (17) can be rewritten as:

$$\begin{aligned}E_{\mathbf{w},\mathbf{P}}\{\Psi\} =& \kappa^2|\mathbf{w}_0^H\mathbf{v}_0(\mathbf{u})|^2 + \beta^2\mathbf{w}_0^H(\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u})\odot\mathbf{I})\mathbf{w}_0 \\ &+ (1-\kappa^2)\cdot\mathbf{w}_0^H[\boldsymbol{\Xi}\odot\mathbf{v}_0(\mathbf{u})\mathbf{v}_0^H(\mathbf{u})]\mathbf{w}_0\,,\end{aligned} \tag{18}$$

and all three terms are now affected by $\mathbf{w}_0$.

In general, noise may arrive from different directions with different levels of intensity. Let $A(\mathbf{u})$ be a function describing the level of noise emanating from each direction. Integrating over steering-vectors, corresponding to all directions, produces the noise covariance matrix $\boldsymbol{\Phi} = \frac{1}{4\pi}\int\int_{\mathbf{u}\in\mathcal{S}} A(\mathbf{u})\mathbf{v}(\mathbf{u})\mathbf{v}^H(\mathbf{u})d\Omega$, where $\mathcal{S}$ is the unit-sphere. Similarly, the mean power of noise at the beamformer's output can be calculated by integrating (18) over all directions to yield:

$$\begin{aligned}\kappa^2\mathbf{w}_0^H\boldsymbol{\Phi}_0\mathbf{w}_0 \;+ \beta^2\mathbf{w}_0^H(\boldsymbol{\Phi}_0\odot\mathbf{I})\mathbf{w}_0 \\ + (1-\kappa^2)\cdot\mathbf{w}_0^H(\boldsymbol{\Xi}\odot\boldsymbol{\Phi}_0)\mathbf{w}_0\,,\end{aligned} \tag{19}$$

where $\boldsymbol{\Phi}_0$ corresponds to the nominal coavariance matrix.

Typical MVDR design, which does not aim for robustness, seeks to minimize the value of the first term of (19) corresponding to nominal noise $\mathbf{w}_0^H\boldsymbol{\Phi}_0\mathbf{w}_0$, while maintaining a unity response in the direction of $\mathbf{u}_{\mathrm{d}}$. A robust design will seek to minimize the excess noise as well [i.e., $\boldsymbol{\Phi}$ of (5) is replaced by $\boldsymbol{\Phi}_{\mathrm{rob}} = \boldsymbol{\Phi}_0\odot[\kappa^2\cdot\mathbf{1}_{N\times N}+\beta^2\cdot\mathbf{I}+(1-\kappa^2)\cdot\boldsymbol{\Xi})]$.

Usually, the values of β and κ are unknown and $\boldsymbol{\Phi}_{\mathrm{rob}}$ cannot be evaluated. In this case, the sensitivity to errors is indicated by $\|\mathbf{w}_0\|_{\mathbf{L}}^2 = \mathbf{w}_0^H\mathbf{L}\mathbf{w}_0$, where

$$\mathbf{L} = \alpha\cdot(\boldsymbol{\Phi}_0\odot\mathbf{I}) + (1-\alpha)\cdot(\boldsymbol{\Xi}\odot\boldsymbol{\Phi}_0)\,, \tag{20}$$

and $\alpha\in[0,1]$ determines the *relative* importance attributed by the designer to the different error types (based on an estimate of $\frac{\beta^2}{\beta^2+1-\kappa^2}$). In the common case where sensors are individually packaged, $\boldsymbol{\Xi} = \mathbf{I}$ such that $\mathbf{L} = \boldsymbol{\Phi}_0\odot\mathbf{I}, \forall\alpha$ and the designer is no longer required to specify α.

A robust design seeks to minimize the nominal noise term while maintaining a unity response towards $\mathbf{u}_{\mathrm{d}}$ *and constraining the magnitude of* $\|\mathbf{w}_0\|^2_{\mathbf{L}}$. Solving this constrained optimization problem with Lagrange multipliers yields:

$$\mathbf{w}_{\mathrm{robust}} = \frac{(\mathbf{\Phi}_0 + \mu\mathbf{L})^{-1}\mathbf{v}_{\mathrm{d}}}{\mathbf{v}_{\mathrm{d}}^H(\mathbf{\Phi}_0 + \mu\mathbf{L})^{-1}\mathbf{v}_{\mathrm{d}}} \,. \tag{21}$$

It should be noted that when all sensors have identical directivity patterns and the perturbations of the positions are i.i.d. then $\mathbf{L}$ becomes a scaled identity matrix. This is the conventional diagonal loading scheme (the extra scaling constant is absorbed by the Lagrange multiplier μ). However, when sensors have different directivity patterns, elements along the main diagonal of $\mathbf{\Phi}_0$ may vary leading to diagonal loading with nonidentical elements. Furthermore, $\mathbf{\Xi}$ may contain nonzero off-diagonal elements (due to shared packaging) leading to nondiagonal loading matrices.

5. EVALUATION

In the following two examples, we compare the performances of (21) with conventional diagonal loading. Robustness towards weight and placement errors will be evaluated with the robustness metric of $\mathrm{RM} = 1/\|\mathbf{w}_0\|^2_{\mathbf{L}}$.

First, we inspect the case of an array operating in a diffuse noise field which consists of a monopole and a collocated dipole[2] oriented towards the desired DOA. The diffuse noise covariance matrix[3] is $\mathbf{\Phi}_{\mathrm{dif}} = \mathrm{diag}([1 \quad \frac{1}{3}])$ and the nominal steering-vector is $\mathbf{v}_0 = [1 \quad 1]^T$. To attain maximum directivity, one uses the MVDR beamformer (5) [with $\mathbf{\Phi} = \mathbf{\Phi}_{\mathrm{dif}}$] yielding a directivity index (DI) of 6 dB using the weight vector $\mathbf{w} = [\frac{1}{4} \quad \frac{3}{4}]^T$. The beampattern of this array corresponds to a hypercardioid. One might attempt to improve robustness by applying conventional diagonal loading. Using $\mu \to \infty$, it would appear that maximum robustness is attained by the weights $\mathbf{w} = [\frac{1}{2} \quad \frac{1}{2}]^T$ (which correspond to a cardioid beampattern) at the expense of reducing the DI to 4.8 dB. However, when RM is used to assess these two beamformers[4], the cardioid beamformer has an RM of 3 dB whereas the hypercardioid beamformer has an RM of 3.6 dB. The diagonal loading is detrimental and actually reduces robustness in this case. In fact, the MVDR hypercardioid weights also happen to have the highest RM, and will be returned by (21) for any (nonnegative) value of μ.

Typically, the use of loading methods for design of robust beamformers produces a set of possible weights resulting from different values of μ. The designer selects from a continuum of possibilities for tradeoff between DI and robustness. In our next example, we demonstrate that the set of beamformers produced by the modified loading method is preferable to that obtained by conventional diagonal loading.

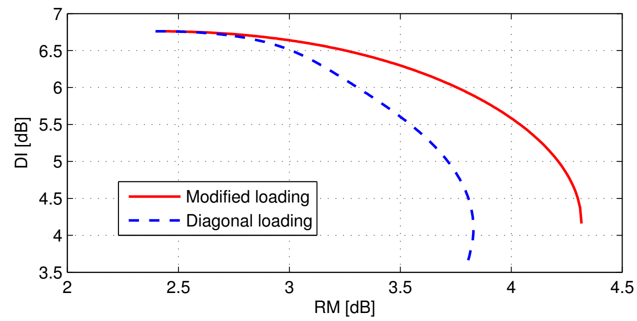

Fig. 1. Options for tradeoff between DI and RM for different robust design methods.

Consider the scenario of an array consisting of two devices – each device comprising two collocated cardioids oriented to the front and rear respectively. These devices are placed side by side separated by a distance of $d = 22$ cm. The scenario described may be viewed as a rather simplistic representation of an array composed of two hearing aid devices.

We examine the performance for a signal arriving from the direction $\theta = 70°$ with respect to the forward direction. Fig. 1 portrays the results of conventional diagonal loading (dashed blue) and modified loading (solid red) in terms of DI and RM. The frequency analyzed is 400 Hz. We have assumed that only perturbations of device placement are significant ($\alpha = 0$). Note that in this case $\mathbf{L}$ is not diagonal. The modified loading method produces a better tradeoff than conventional diagonal loading. For the same level of robustness, a higher DI is attainable. The difference can reach 1.5 dB.

The above two examples illustrate that the use of the proposed modified loading method instead of traditional diagonal loading can be beneficial.

6. CONCLUSION

In this paper, we derived expressions for the mean beam-power of an array with random perturbations in weight values and sensor locations. These random perturbations cause an increase in beam-power which leads to higher noise levels. In a diffuse scenario, the excess noise can be reduced by applying a modified loading scheme. This scheme can differ from conventional diagonal loading by employing a loading matrix with nonidentical elements on the main diagonal (when sensor directivities differ) and off-diagonal elements (when placement errors are correlated). The modified scheme improves robustness with respect to the errors discussed.

[2] This configuration should not be confused with a differential microphone array. The latter can produce beampatterns which closely approximate a combination of monopole and dipole patterns but suffers from poor robustness which characterizes superdirectivity.

[3] References [11] and [12] discuss covariance relations pertinent to the current and subsequent examples.

[4] In this scenario, $\mathbf{\Xi} = \mathbf{1}_{3\times 3}$ and $\mathbf{\Phi}_0 = \mathbf{\Phi}_{\mathrm{dif}}$ is diagonal. Hence, $\mathbf{L}$ of (20) equals $\mathbf{\Phi}_0$ and is independent of α.

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