AN IDEAL INTEGRATOR FOR HIGHER-ORDER INTEGRATED WAVETABLE SYNTHESIS

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ABSTRACT

Higher-order integrated wavetable synthesis (HOIWS) is an efficient technique to reduce aliasing in wavetable and sampling synthesis. A periodic audio signal is integrated repeatedly before it is stored in a wavetable. During playback, the pitch of the audio signal can be changed using interpolation techniques and the resulting signal is differentiated as many times as the wavetable has been integrated. Previous discrete-time integrators approximate ideal integration, which leads to magnitude and phase errors. This paper proposes an ideal integration method, which is applied in the frequency domain with the help of the FFT. Its remarkable advantage is that both the magnitude and the phase errors are completely avoided in the special case of periodic signals. The proposed ideal integrator shows a superior performance over previous digital integration methods. It improves the sound quality of the HOIWS algorithm and helps it to maintain the original waveform after interpolation and differentiation stages.

Index Terms— Acoustic signal processing, alias reduction, discrete-time integrators, interpolation, wavetable synthesis

1. INTRODUCTION

Wavetable synthesis [1, 2, 3, 4] and sampling synthesis [5] are two related sound synthesis techniques. They are based on storing either a single cycle or a longer part of a periodic waveform in memory, the so-called wavetable. Accessing this wavetable with different phase increments, corresponding to a resampling of the waveform, enables the reproduction of differently pitched tones from one or a small set of wavetables. Compared to other synthesis techniques, wavetable synthesis allows the reproduction of complex, spectrally rich sounds with a relatively low complexity. Recent previous research on wavetable synthesis include the reduction of peak sample values in wavetables [6], matching of wavetables to recorded sounds [7, 8], and the compression of sampled data to save memory [9].

Wavetable synthesizers may reproduce a tone with either a lower or higher pitch compared to the wavetable sound. In terms of resource efficiency, wavetable algorithms that raise the pitch are often advantageous, because the rich harmonic content of the sound is retained. Thus, a wide range of pitches can be generated from one wavetable, reducing the memory requirements. On the downside, increasing the pitch results in aliasing artifacts which compromise the sound quality severely [10]. Conceptually, the wavetable signal must be bandlimited with a variable cutoff frequency depending on the pitch change ratio to avoid or reduce aliasing. This corresponds to a resampling process which reduces the sample rate by an arbitrary ratio.

A few resampling methods are applicable to this problem, e.g., [11, 12]. However, when applied to wavetable synthesis, the complexity of these algorithms increases with the pitch ratio, which is unfavorable for realtime application.

A different approach has been proposed by Geiger [13]: Integrating the waveform stored in the wavetable and differentiating it after the table lookup reduces aliasing artifacts. This integration/differentiation scheme is motivated by the differentiated parabolic waveform algorithm [14], which reduces aliasing in the synthesis of classical analog waveforms. Recently, a method termed higher-order integrated wavetable synthesis (HOIWS) has been proposed which extends Geiger's scheme to higher orders of integration and differentiation, yielding significantly better alias reduction [15].

Antialiasing methods, including the use of integration/differentiation schemes [14, 16], are extensively covered in the field of bandlimited synthesis of analog waveforms, e.g., [17, 18, 19, 20]. A recent method based on polynomial transition regions [21] is an attempt to extend the antialiasing synthesis methods to generic audio signals, but they are still not directly applicable to processing of arbitrary sampled waveforms, because the locations of discontinuities are unknown. The higher-order integrated wavetable and sampling synthesis discussed in this paper answers to this need: It can be applied to any periodic waveform.

The present paper focuses on the integration algorithm for creating the integrated wavetables. We show how the choice of the discrete-time integrator influences the quality of the HOIWS method and compare different existing integration filters. To overcome the errors caused by these methods, we propose a novel algorithm to perform ideal integration of wavetables. The design of discrete-time filters for integration is an extensively covered field, with previous research ranging from classic texts [22, 23, 24, 25] to recent publications, e.g., [26, 27, 28]. While realizable integrator algorithms may only approximate the ideal integration process in the general case, ideal integration is possible for the class of periodic wavetable signals. The resulting algorithm is efficient and suitable for large wavetables and high integration orders. It is shown that this optimal integration scheme improves the synthesis quality compared to existing integration methods.

The remainder of this paper is outlined as follows: Section 2 describes the HOIWS method and the application of discrete-time integrators to wavetable signals. The ideal integration algorithm is proposed in Section 3. The performance of the integrator algorithms is compared in Section 4. Section 5 summarizes the results of this paper.

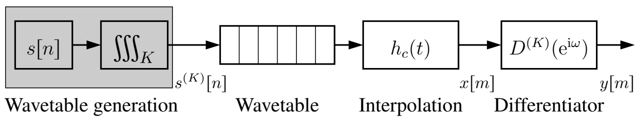

Fig. 1. Signal flow diagram of higher-order integrated wavetable synthesis. The gray area represents the off-line preprocessing.

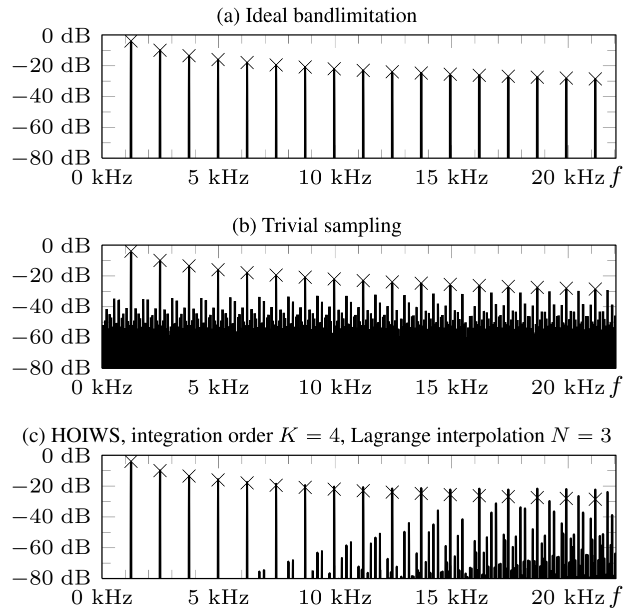

Fig. 2. Discrete sawtooth spectra, wavetable length 48 (input fundamental frequency 918.75 Hz), $f_s = 44.1$ kHz, output fundamental frequency 1245 Hz. Crosses mark ideal magnitudes of harmonics.

2. INTEGRATED WAVETABLE SYNTHESIS

This section provides a brief characterization of the HOIWS algorithm as proposed in [15] and describes the use of existing discrete-time integrators to generate integrated wavetables.

2.1. The HOIWS Algorithm

The signal flow of the higher-order integrated wavetable synthesis algorithm is shown in Fig. 1. The waveform $s[n]$ is K times integrated and stored in the wavetable memory. This integration and any additional preprocessing is performed off-line and therefore does not impact the runtime complexity. Accessing the wavetable generally requires an interpolation filter $h_c(t)$ to suppress spectral images of the discrete-time wavetable signal [5]. Filtering the interpolated signal $x[m]$ with a Kth-order differentiator $D^{(K)}(e^{j\omega})$ yields the output signal $y[m]$. Several designs for this differentiator, including maximally flat and minimax differentiators, are evaluated in [15].

The effect of the HOIWS algorithm is illustrated in Fig. 2. In this example, a single cycle of a sawtooth signal is stored in a wavetable of length 48, corresponding to a fundamental frequency of 918.75 Hz at a sampling frequency $f_s = 44.1$ kHz. This waveform is reproduced with a fundamental frequency of $f = 1245$ Hz (note D#6), that is, a pitch change ratio of about 1.36. The ideally bandlimited spectrum is shown in Fig. 2(a). A trivial table lookup, i.e., rounding the required output times to the nearest sample location, results in considerable aliasing, depicted in Fig. 2(b). The HOIWS algorithm with a simple first-order rectangular integrator, integration order $K = 4$, a Lagrange interpolator of order $N = 3$, and a fourth-order minimax differentiator yields the spectrum displayed in Fig. 2(c). It shows a considerable reduction of aliasing components, but also introduces noticeable magnitude errors of the desired harmonics.

Table 1. Coefficients of IIR integrators, order $K = 1$.

Integrator	Transfer function
Rectangular	$\frac{1}{1-z^{-1}}$
Trapezoidal	$\frac{1}{2}\frac{1+z^{-1}}{1-z^{-1}}$
Simpson's rule	$\frac{1}{3} + \frac{1+4z^{-1}+z^{-2}}{1-z^{-2}}$
Al-Alaoui [25]	$\frac{1}{8}\frac{7+z^{-1}}{1-z^{-1}}$
Upadhyay&Singh [28]	$0.8657\frac{1+0.681z^{-1}+0.0628z^{-2}}{1-0.4975z^{-1}-0.5025z^{-2}}$

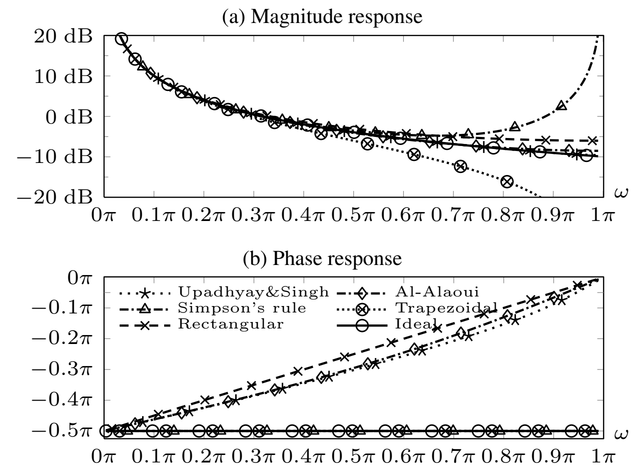

Fig. 3. Frequency response of discrete-time integrators, order K=1.

2.2. Discrete-Time Integrators

To integrate discrete-time wavetables, digital integrators are used. The ideal frequency response of a digital integrator of order K is

$$H_{id}^{(K)}\left(e^{j\omega}\right) = \left(\frac{1}{j\omega}\right)^{K}, \quad (1)$$

which is not realizable by a finite-order discrete-time system. Thus, practical implementations only approximate this response. They are generally implemented as IIR filters. Numerous digital integrators have been proposed, ranging from IIR filter representations of numerical integration formulas such as the rectangular, the trapezoidal, or Simpson's rule [22, 23, 25] to low-order IIR approximations of (1), such as the Al-Alaoui integrator [25] or the design proposed by Upadhyay and Singh [28]. In previous work on the HOIWS algorithm [15], only the rectangular rule has been used. Table 1 shows the transfer functions of the discrete-time integrators considered in this paper for order $K = 1$.

The magnitude responses of these filters are displayed in Fig. 3(a). It is observed that the magnitude responses of the numerical interpolation formulas (rectangular, trapezoidal, and Simpson's rule) exhibit significant magnitude errors with respect to the ideal integrator. In particular, Simpson's rule shows a strong amplification of high frequencies, caused by its transfer pole at $\omega = \pi$. Fig. 3(b) shows the corresponding phase responses. The phase of the trapezoidal rule and Simpson's rule conform to the ideal phase response, which is a constant phase shift of $-\pi K/2$. In contrast, the rectangular rule, the Al-Alaoui integrator, and the Upadhyay&Singh design introduce frequency-dependent phase errors.

2.3. Integration of Wavetables

Although the discrete-time integrator defines the characteristics of the integration, it does not determine the integrated waveform uniquely. As in the integration of continuous functions, a constant of integration has to be specified for each order of integration. In the case of a discrete-time integrator, these constants correspond to the initial states of the IIR filter.

For wavetable synthesis, these free parameters can be determined from the properties of the synthesized sounds. As wavetables are played back repeatedly, the wavetable signal $x[n]$ can be considered as one period of an infinite-duration periodic signal

$$x[n] = x[n-L] = x[n \bmod L] \qquad \text{for } n \in \mathbb{Z}\,. \tag{2}$$

For distortion-free reproduction, the sequence $x[n]$ must be continuous over loop boundaries [2, 9]. This continuity must also hold for the integrated wavetable signals $s^{(k)}[n]$, $1 \leqslant k \leqslant K$, because high-frequency components due to signal discontinuities are disproportionately amplified by the differentiator. The continuity requirement can be fulfilled by restricting the dc (zero-frequency) components of the signals $s^{(k-1)}[n]$ to zero:

$$\sum_{n=0}^{L-1} s^{(k-1)}[n] = 0 \quad \text{for } k = 1, 2, \ldots, K\,, \tag{3}$$

where $s^{(0)}[n] = x[n]$. As described in [15], this can be ensured by subtracting the mean of the sequence before integration.

3. IDEAL INTEGRATOR FOR PERIODIC SIGNALS

In this section, we derive an ideal integration method for periodic discrete-time signals such as wavetables. Due to the inherently infinite impulse response nature of integration [24], the frequency response of a discrete-time integrator (1) cannot be expressed by a finite-length impulse response. However, it can be modeled as an infinite convolution with a real-valued impulse response $h_\infty^{(K)}[n]$

$$s^{(K)}[n] = \sum_{k=-\infty}^{\infty} h_\infty^{(K)}[k]x[n-k] \qquad \text{with } h_\infty^{(K)}[k] \in \mathbb{R}\,. \tag{4}$$

If the sequence $x[n]$ is periodic according to (2), then this impulse response can be transformed into a finite sum

$$s^{(K)}[n] = \sum_{k=0}^{L-1} h^{(K)}[k]x[n-k] \qquad \text{with } h^{(K)}[k] \in \mathbb{R}\,, \tag{5}$$

which represents a circular or periodic convolution [29]

$$s^{(K)}[n] = h^{(K)}[n] \circledast x[n]\ . \tag{6}$$

In the discrete frequency domain, a circular convolution corresponds to a multiplication of the discrete Fourier transforms (DFT) of the two sequences

$$S^{(K)}[l] = X[l] \times H^{(K)}[l] \tag{7}$$

with $X[l] = \mathrm{DFT}\{x[n]\}$, $H^{(K)}[l] = \mathrm{DFT}\left\{h^{(K)}[n]\right\}$, $S^{(K)}[l] = \mathrm{DFT}\left\{s^{(K)}[n]\right\}$. Here, $\mathrm{DFT}\{\cdot\}$ represents the discrete Fourier transform (DFT) operator.

Basically, the discrete frequency response $H_{id}^{(K)}[l]$ is obtained by sampling the ideal frequency response $H^{(K)}(\mathrm{e}^{\mathrm{j}\omega})$ (1) at the discrete frequencies $\omega = 2\pi l/L$

$$H^{(K)}[l] = H^{(K)}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi l}{L}}\right) \quad 0 \leqslant l < L\,. \tag{8}$$

For practical application, however, two additional conditions must be incorporated.

First, to obtain a real-valued impulse response $h^{(K)}[n]$, its DFT must be conjugate symmetric (e.g. [29, 30])

$$H_{id}^{(K)}[L-l] = \overline{H}_{id}^{(K)}[l] \quad 1 \leqslant l < L\,, \tag{9}$$

where $\overline{H}$ denotes complex conjugation. If L is even, this implies $H_{id}^{(K)}(L/2) = 0$.

Second, the component $H^{(K)}[0]$ requires a separate treatment. It cannot be deduced from $H_{id}^{(K)}[l](\mathrm{e}^{\mathrm{j}\omega})$, because this frequency response is not finite at $\omega = 0$. However, the value $H^{(K)}[0]$ describes the translation of dc components from the sequence $x[n]$ to the integrated signal $s^{(K)}[n]$. As argued in Section 2.3, the wavetable integration process should suppress dc components in $x[n]$ to gain a periodic waveform that preserves loop continuity. This property can be enforced by setting $H^{(K)}[0] = 0$.

Combining these considerations, the discrete frequency response is determined by

$$H^{(K)}[l] = \begin{cases} 0\,, & l = 0 \\ (2\mathrm{j}\pi l/L)^K\,, & 1 \leqslant l < L/2 \\ (-2\mathrm{j}\pi l/L)^K\,, & L/2 < l < L \\ 0\,, & l = L/2, L \text{ even} \end{cases}\ . \tag{10}$$

The integrated wavetable $s^{(K)}[n]$ is computed by applying this frequency response in the DFT domain followed by an inverse discrete Fourier transform $\mathrm{DFT}^{-1}\{\cdot\}$

$$s^{(K)}[n] = \mathrm{DFT}^{-1}\left\{H^{(K)}[l] \times \mathrm{DFT}\{x[n]\}\right\}\ . \tag{11}$$

Since these transforms are efficiently implemented by the fast Fourier transform (FFT), the proposed ideal integration method is practical for very large wavetables also. In contrast to IIR integrators, it does not only provide exact magnitude characteristics, but also the ideal phase response.

4. EVALUATION

The effect of the integration method on the quality of the HOIWS algorithm is evaluated using the sawtooth synthesis example of Fig. 2. The resulting spectra and time domain signals are shown in Fig. 4 and Fig. 5, respectively. For comparison, Fig. 4(a) repeats the output spectrum resulting from the rectangular rule already shown in Fig. 2(c). Considering the magnitude responses of the integration algorithms shown in Fig. 3, it is evident that the amplification of high frequencies results from the magnitude error of the rectangular rule. The phase response error of this integrator leads to a misalignment of the time-domain signal as shown in Fig. 5(a). In contrast, the trapezoidal rule does not cause phase errors, but yields

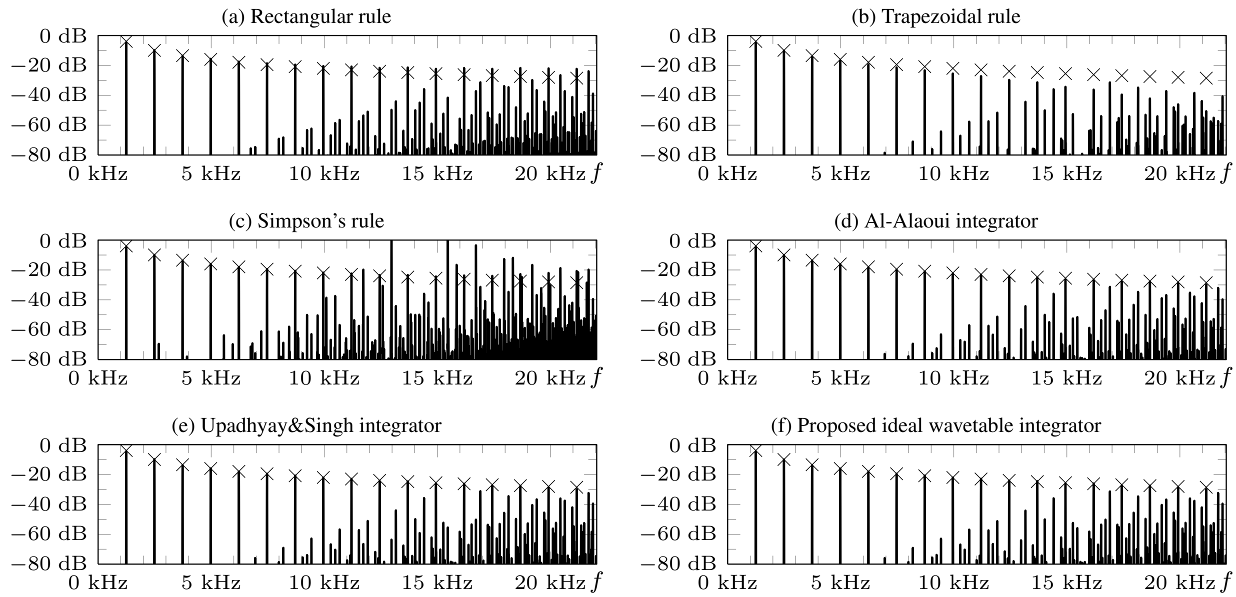

Fig. 4. Effect of the integrator algorithms on the HOIWS algorithm. Sawtooth wave, wavetable length 48 (input fundamental frequency 918.75 Hz), $f_s = 44.1$ kHz, output fundamental frequency 1245 Hz. Integration order $K = 4$, Lagrange interpolation $N = 3$, minimax differentiator. Crosses mark the ideal magnitudes of the harmonics.

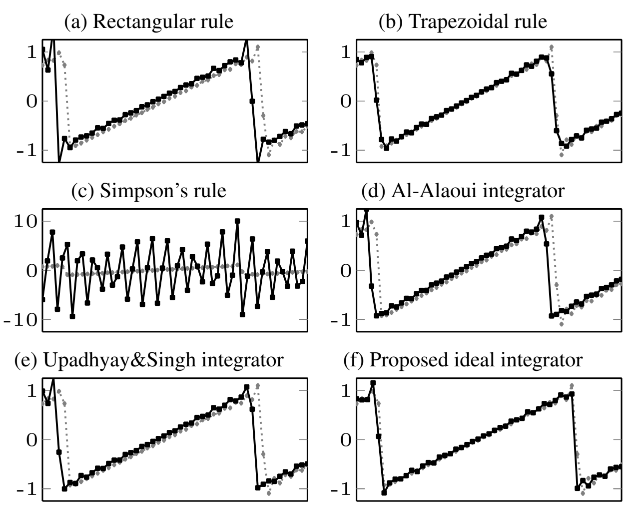

Fig. 5. Time domain signals of the synthesized sawtooth wave. The gray dotted signal represents the ideal bandlimited waveform.

a significant magnitude roll-off towards high frequencies, which also attenuates aliasing and imaging components. Figures 4(b) and 5(b) depict the effect of this integrator in the frequency and time domain, respectively. As shown in Fig. 4(c), the spectrum obtained by using Simpson's rule shows increased magnitudes of higher passband harmonics, but also a disproportionate amplification of aliasing and imaging terms, Both are caused by the large high-frequency gain of the integrator's frequency response. In this example, the maximum level of aliasing terms is about +15 dB. As shown in Fig. 5(c), this has a catastrophic effect on the time-domain signal (note the changed axis scaling).

The synthesized signals based on the Al-Alaoui and the Upadhyay&Singh integrators provide a good match to the magnitude spectrum of the ideal bandlimited sawtooth wave, as shown in Fig. 4(d) and 4(e). However, the corresponding time-domain signals, depicted in Fig. 5(d) and 5(e), exhibit significant misalignment due to phase errors. As shown in Fig. 4(f) and 5(f), the proposed ideal integration method approximates the ideal bandlimited signal closely and without phase errors, thus improving the synthesis quality.

5. CONCLUSION

In this paper, we investigated the influence of the integration method on the quality of the higher-order integrated wavetable synthesis method. It became clear that this choice has a significant effect on the passband quality of the output waveform, and consequently also on the magnitude of aliasing and imaging artifacts. In particular, the integration filters obtained from standard numerical formulas such as the rectangular, trapezoidal, or Simpson's rule yield considerable passband errors. Sophisticated wideband IIR integrators provide an accurate magnitude response, but introduce phase errors.

As a main contribution of this paper, we propose an ideal integration algorithm for periodic wavetable signals. By expressing the integration as a circular convolution of finite-length sequences, the DFT of the proposed integration filter is obtained by sampling the frequency response of the ideal integrator. As an additional advantage, desirable properties such as the suppression of dc components can be directly incorporated into this discrete filter. Integration of wavetables is performed in the discrete frequency domain. Using fast implementations as the FFT, this enables efficient integration for large wavetables and arbitrary orders of integration while avoiding both magnitude and phase errors. In this way, the proposed algorithm improves the quality of the higher-order wavetable and sampling synthesis algorithm.

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