TRANSMIT AND RECEIVE SPACE-TIME-FREQUENCY ADAPTIVE PROCESSING FOR COOPERATIVE DISTRIBUTED MIMO COMMUNICATIONS

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ABSTRACT

In this paper, the problem of informed-transmitter cooperative MIMO communications is addressed. The informed-transmitter link assumes that the distributed transmit nodes have access to channel state information. The channel state information includes the channel between the transmit and receive antenna arrays and a statistical model for interference impinging upon the receive array. In principle, the channel will have resolvable delay spread. In addition, because the distributed sets of nodes do not have a common local oscillator and move independently, the receiving set of nodes may observe resolvable independent frequency offsets and frequency spread from each of the transmit nodes. To compensate for this doubly dispersive channel, a space-time-frequency channel estimate is constructed. The frequency components of the channel estimate enable improved channel prediction capabilities. Space-time-frequency transmit adaptive processing approaches are developed. These techniques are demonstrated in simulation.

1. INTRODUCTION

The concept of transmit beamforming is not new [1, 2]. Conceptually, distributed extensions to transmit beamforming followed quickly; however, practical implementations are difficult. Techniques for distributed receive beamforming are discussed and are experimentally demonstrated with a model of local oscillator errors in Reference [3]. The feasibility of distributed transmit beamforming in a wireless system is discussed in References [4, 5].

The focus of this paper is to describe an approach to construct space-time-frequency transmit beamformers and receivers for a multiple-input multiple-output (MIMO) wireless communication system with a dynamic frequency-selective channel. To construct transmit beamformers, the channel is provided through channel-estimation feedback [6]. Reciprocity techniques do not enable estimation of the interference-plus-noise covariance matrix.

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One of the fundamental challenges in employing channel state information at the transmitter is the intrinsic delay between its estimation and use. In dynamic environments, these stale channel estimates introduce errors in the beamformer. There are a variety of channel-prediction approaches available, for example [7]. Here, the space-time-frequency transmit beamformer is evolved forward in time in an attempt to compensate for the stale channel knowledge.

2. CONSTRUCTION OF TRANSMIT BEAMFORMER

There are numerous metrics for which one might wish to optimize a transmit beamformer. Here, the metric is to maximize the signal-to-interference-plus-noise (SINR) ratio at the output of a receive beamformer. Consequently, to optimize the transmit beamformer, one needs a model of the receive beamformer. In the case of a memoryless static multiple-input single-output (MISO) channel [6], because the capacity of a link increases monotonically with SINR, optimizing this metric maximizes capacity. For a MIMO system, the optimization is slightly more complicated, but in the low signal-to-noise ratio (SNR) limit in the memoryless static channel case, the optimal strategy is to transmit using a single beamformer determined by the dominant right-hand singular vector of the whitened channel matrix [6]. By analogy, we extend this result to time-varying dispersive channels. This approach does not guarantee optimal capacity, but does potentially provide good performance.

To develop a transmit beamformer algorithm, a model for the degrees of freedom available to the transmitter and a model for the degrees of freedom at the receiver are assumed. Both to aid interpretation and in an attempt to make this development clearer, this algorithm is presented under three different assumptions in order of increasing complexity: 1) spatial degrees of freedom at the transmit and receive arrays, 2) spatial and temporal distortion degrees of freedom at both the transmit and receive arrays, and 3) spatial, temporal, and frequency distortion degrees of freedom at both the transmit and receive arrays.

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2.1. Static Flat-Fading Channel

For the sake of introduction, consider the standard block MIMO channel model [6] under the assumption of a static flat-fading environment. For a system with n_t transmit antennas, n_r receive antennas, and blocks of n_s samples or potentially chips (if the signal is critically sampled), the channel model is given by

$$\mathbf{Z} = \mathbf{H}\,\mathbf{S} + \mathbf{N}\,, \tag{1}$$

where the variables are defined as

$$\begin{aligned} \mathbf{Z} &\in \mathbb{C}^{n_r \times n_s} \text{, received data} \\ \mathbf{H} &\in \mathbb{C}^{n_r \times n_t} \text{, flat-fading channel} \\ \mathbf{S} &\in \mathbb{C}^{n_t \times n_s} \text{, transmit matrix, and} \\ \mathbf{N} &\in \mathbb{C}^{n_r \times n_s} \text{, noise plus interference.} \end{aligned} \tag{2}$$

Within this paper, it is assumed that all signals are described by their complex baseband representations.

Under the assumption of a Gaussian probability distribution, $p(\mathbf{Z})$ for the received signal $\mathbf{Z}$ is given by

$$p(\mathbf{Z}) = \frac{1}{|\mathbf{Z}|^{n_s}\,\pi^{n_s\,n_r}}\,e^{-\mathrm{tr}\{(\mathbf{Z}-\mathbf{H}\,\mathbf{S})^\dagger\,\mathbf{R}^{-1}\,(\mathbf{Z}-\mathbf{H}\,\mathbf{S})\}}\,, \tag{3}$$

where $\cdot^\dagger$ indicates the Hermitian conjugate, $|\cdot|$ indicates the determinant, and $\mathbf{R} \in \mathbb{C}^{n_r \times n_r}$ indicates the receive interference-plus-noise covariance matrix,

$$\mathbf{R} = \frac{1}{n_s}\left\langle \mathbf{N}\,\mathbf{N}^\dagger \right\rangle\,, \tag{4}$$

where $\langle\cdot\rangle$ indicates the expectation. By maximizing the likelihood, given some observed received data $\mathbf{Z}$, and under the assumption of a known transmitted training sequence $\mathbf{S}$, the maximum-likelihood estimates for the channel $\hat{\mathbf{H}}$ and interference-plus-noise spatial covariance matrix $\hat{\mathbf{R}}$ are given by

$$\hat{\mathbf{H}} = \mathbf{Z}\,\mathbf{S}^\dagger\,(\mathbf{S}\,\mathbf{S}^\dagger)^{-1}\,, \qquad \hat{\mathbf{R}} = \frac{1}{n_s}\,\mathbf{Z}\,\mathbf{P}^\perp_{\mathbf{S}}\,\mathbf{Z}^\dagger\,, \tag{5}$$

where the matrix $\mathbf{P}^\perp_{\mathbf{S}} = \mathbf{I} - \mathbf{S}^\dagger\,(\mathbf{S}\,\mathbf{S}^\dagger)^{-1}\,\mathbf{S}$ projects onto a basis orthogonal to the row space of the transmitted signal $\mathbf{S}$, and $\mathbf{I}$ indicates the identity matrix.

For a multivariate system, the optimal capacity solution is given by water filling [6, 8]. The capacity c of a MIMO system is the bounding spectral efficiency and is given by

$$c = \log_2\left|\mathbf{I} + \mathbf{R}^{-1}\,\mathbf{H}\,\mathbf{\Sigma}\,\mathbf{H}^\dagger\right|\,, \tag{6}$$

where the spatial transmit covariance is indicated by $\mathbf{\Sigma} \in \mathbb{C}^{n_t \times n_t}$. The water-filling approach puts the most power in those eigenvalues of $\mathbf{\Sigma}$ associated with the best singular values of the spatially whitened channel. The whitened channel is given by $\mathbf{R}^{-1/2}\,\mathbf{H}$, and the singular value decomposition (SVD) [9] of an estimate for it is given by

$$\hat{\mathbf{R}}^{-1/2}\,\hat{\mathbf{H}} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^\dagger\,, \tag{7}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthonormal matrices (that are generally asymmetric), such that $\mathbf{U}^\dagger\,\mathbf{U} = \mathbf{I}$ and $\mathbf{V}^\dagger\,\mathbf{V} = \mathbf{I}$ and $\mathbf{D}$ is a diagonal matrix containing the singular values.

As the power increases, the water-filling approach distributes power across more modes because of the logarithmic compression of the capacity [6]. In the limit of low power, the optimal strategy is to put all of the transmit power in a single eigenvalue of the transmit covariance matrix. This rank-one transmit covariance matrix is constructed from the dominant right-hand singular vector $\mathbf{v}_1$ of the whitened channel matrix estimate, $\mathbf{\Sigma} \propto \mathbf{v}_1\,\mathbf{v}_1^\dagger$. Perhaps more importantly, the transmitted signal matrix is constructed by using a right-hand singular vector from the whitened channel matrix as a beamformer,

$$\mathbf{S}_{data} = \mathbf{v}_1\,\underline{\mathbf{s}}_{data}\,, \tag{8}$$

where $\mathbf{S}_{data} \in \mathbb{C}^{n_t \times n_{data}}$ is the rank-one transmission data matix, and $\underline{\mathbf{s}}_{data} \in \mathbb{C}^{1 \times n_{data}}$ is a row vector containing the data sequence to be transmitted.

2.2. Channel with Transmit and Receive Distortions

To model receive and transmit distortion degrees of freedom, where the distortions can be delay, frequency (such as Doppler) offsets, or both, the following definitions are made:

$$\begin{aligned} \breve{\mathbf{Z}} &\in \mathbb{C}^{(n_r \cdot n_\rho) \times n_s} \text{, distorted received data} \\ \breve{\tilde{\mathbf{H}}} &\in \mathbb{C}^{(n_r \cdot n_\rho) \times (n_t \cdot n_\delta)} \text{, transmit and receive distorted channel} \\ \tilde{\mathbf{S}} &\in \mathbb{C}^{(n_t \cdot n_\delta) \times n_s} \text{, distorted transmit matrix} \\ \breve{\mathbf{S}} &\in \mathbb{C}^{(n_t \cdot n_\beta) \times n_s} \text{, distorted transmit matrix used by projection} \\ \breve{\mathbf{N}} &\in \mathbb{C}^{(n_r \cdot n_\rho) \times n_s} \text{, distorted noise plus interference}\,. \end{aligned} \tag{9}$$

In the above equation, the number transmit distortions is given by n_δ. The model for the number of receive distortions is more complicated. In general, there are a different number of allowed distortions in the channel n_ρ than in the projection matrix used to estimate the covariance n_β in the distortion-extend operation analogous to that found in covariance matrix estimation in Equation (5). Keeping track of the various distortion models is admittedly difficult. To complicate matters further, the actual receiver may use a different numbers of distortion degrees of freedom that those used in the model for constructing the transmit beamformer, but we will not explicitly consider that here.

The receive data matrix with spatial and distortion degrees of freedom is given by

$$\breve{\mathbf{Z}} = \begin{pmatrix} \mathbf{Z}^T_{\rho_1} & \mathbf{Z}^T_{\rho_2} & \cdots & \mathbf{Z}^T_{\rho_{n_\rho}} \end{pmatrix}^T\,, \tag{10}$$

and, similarly, the space-distorted interference-plus-noise matrix is given by

$$\breve{\mathbf{N}} = \begin{pmatrix} \mathbf{N}_{\rho_1}^T & \mathbf{N}_{\rho_2}^T & \cdots & \mathbf{N}_{\rho_{n_\rho}}^T \end{pmatrix}^T . \tag{11}$$

Consequently, the space-distorted receive covariance matrix $\breve{\mathbf{R}} \in \mathbb{C}^{(n_r \cdot n_\rho)\times(n_r \cdot n_\rho)}$ and its estimate $\hat{\breve{\mathbf{R}}}$ are given by

$$\breve{\mathbf{R}} = \frac{1}{n_s}\left\langle \breve{\mathbf{N}}\,\breve{\mathbf{N}}^\dagger \right\rangle \text{ , and } \hat{\breve{\mathbf{R}}}\frac{1}{n_s}\,\breve{\mathbf{Z}}\,\mathbf{P}_{\breve{\mathbf{S}}}^\perp\,\breve{\mathbf{Z}}^\dagger . \tag{12}$$

Because the distortions assumed for the channel estimation and the projection operator $\mathbf{P}_{\breve{\mathbf{S}}}^\perp$ are not the necessarily the same, the dimensions of the distorted signal data matrices are not necessarily the same, $\dim\{\breve{\mathbf{S}}\} \neq \dim\{\tilde{\mathbf{S}}\}$,

$$\breve{\mathbf{S}} = \begin{pmatrix} \mathbf{S}_{\beta_1}^T & \mathbf{S}_{\beta_2}^T & \cdots & \mathbf{S}_{\beta_{n_\beta}}^T \end{pmatrix}^T . \tag{13}$$

Now the channel model must take into account degrees of freedom of potential distortions at both the transmitter and the receiver, leading to the somewhat cumbersome form for the distorted channel $\breve{\breve{\mathbf{H}}}$,

$$\breve{\breve{\mathbf{H}}} = \begin{pmatrix} \mathbf{H}_{\rho_1,\delta_1} & \mathbf{H}_{\rho_1,\delta_2} & \cdots & \mathbf{H}_{\rho_1,\delta_{n_\delta}} \\ \mathbf{H}_{\rho_2,\delta_1} & \mathbf{H}_{\rho_2,\delta_2} & \cdots & \mathbf{H}_{\rho_2,\delta_{n_\delta}} \\ & & \vdots & \\ \mathbf{H}_{\rho_{n_\rho},\delta_1} & \mathbf{H}_{\rho_{n_\rho},\delta_2} & \cdots & \mathbf{H}_{\rho_{n_\rho},\delta_{n_\delta}} \end{pmatrix} . \tag{14}$$

The estimate of this transmit and receive distorted channel is given by

$$\hat{\breve{\breve{\mathbf{H}}}} = \breve{\mathbf{Z}}\,\tilde{\mathbf{S}}^\dagger\,(\tilde{\mathbf{S}}\,\tilde{\mathbf{S}}^\dagger)^{-1} , \tag{15}$$

and by using the same singular value decomposition,

$$\hat{\breve{\mathbf{R}}}^{-1/2}\,\hat{\breve{\breve{\mathbf{H}}}} = \breve{\breve{\mathbf{U}}}\,\breve{\breve{\mathbf{D}}}\,\breve{\breve{\mathbf{V}}}^\dagger . \tag{16}$$

The optimized solution is given by the dominant right-hand singular vector. By considering a model in which both the transmitter and the receiver are allowed to compensate for channel and interference distortions, the total energy coupled from the transmitter to the receiver is increased.

2.2.1. Delay Distortion Assumption

If the channel is static, then the distortions are interpreted as delays, and the transmission approach is given by

$$\mathbf{S}_{data} = \left(\mathbf{1}_{n_\delta}^T \otimes \mathbf{I}_{n_t}\right) \left[(\breve{\breve{\mathbf{v}}}_1\,\mathbf{1}_{n_s}^T) \odot \left(\begin{bmatrix} \underline{\mathbf{s}}_{-\delta_1} \\ \underline{\mathbf{s}}_{-\delta_2} \\ \vdots \\ \underline{\mathbf{s}}_{-\delta_{n_\delta}} \end{bmatrix} \otimes \mathbf{1}_{n_t} \right) \right] , \tag{17}$$

where $\underline{\mathbf{s}}_{-\delta_1}$ indicates a compensating distortion for δ_1, $\otimes$ indicates the Kronecker product, and $\odot$ indicates the Hadamard (element-by-element) product.

2.2.2. Delay-Frequency Distortion Assumption

If the channel and interference are dynamic, then there are time-varying effects that disrupt the use of the space-time transmit beamformer. In particular, the space-time transmit beamformer is typically implemented at some delay relative to when it is estimated. Consequently, the estimated beamformer is stale. To compute a prediction for a beamformer based on current channel estimates, for some time offset T, the diagonal matrix $\mathbf{T}_T$ for the transmit distortions are constructed such that for some set of frequency distortions $f_{\delta_1}, f_{\delta_2}, \cdots$,

$$\mathbf{T}_T = \mathrm{diag}\{e^{-2\pi i\, f_{\delta_1} T},\, e^{-2\pi i\, f_{\delta_2} T},\, \cdots\} \tag{18}$$

can be used to evolve the source model forward in time. To be clear, because the frequency taps are being used after they are estimated, the effective frequency resolution of the entire coherent processing interval is finer than than the intrinsic resolution during estimation. Consequently, the estimates of the degrees of freedom associated with frequency distortions need to be oversampled by an appropriate factors, and some regularization, such as diagonal loading, needs to be applied in the matrix inversion in Equation (15). The distortion degrees of freedom can be interpreted as a set of both delay and frequency offsets that produces a transmit data sequence

$$\begin{aligned} \mathbf{S}_{data} &= \left(\mathbf{1}_{n_\delta}^T \otimes \mathbf{I}_{n_t}\right)\,\mathbf{T}_T \\ &\cdot \left[(\breve{\breve{\mathbf{v}}}_1\,\mathbf{1}_{n_s}^T) \odot \left(\begin{bmatrix} e^{-i\,2\pi\,\underline{\mathbf{t}} f_{\delta_1}} \odot \underline{\mathbf{s}}_{-\delta_1} \\ e^{-i\,2\pi\,\underline{\mathbf{t}} f_{\delta_2}} \odot \underline{\mathbf{s}}_{-\delta_2} \\ \vdots \\ e^{-i\,2\pi\,\underline{\mathbf{t}} f_{\delta_{n_\delta}}} \odot \underline{\mathbf{s}}_{-\delta_{n_\delta}} \end{bmatrix} \otimes \mathbf{1}_{n_t} \right) \right] , \end{aligned} \tag{19}$$

where for some sampling interval Δt, the row vector $\underline{\mathbf{t}} = \Delta t\,\{0 \quad 1 \quad \cdots \quad n_{data}-1\}$ contains the set of time offsets corresponding to the number of transmitted samples, and f_{δ_m} is the frequency offset associated with the δ_m^{th} distortion.

3. SIMULATION

As an example, we consider doubly dispersive channel between a moving distributed 10 node (antenna) transmit array and a static coherent 2 antenna receiver. A single, static, 20 dB interference-to-noise ratio per receive antenna interferer is present that has a channel free of delay spread. This example allows for easier interpretation of the results. It is assumed that training occurs for the first 5000-sample epoch and the beamformer estimated during that period is applied during the following 5000-sample epoch. The space-time-frequency channel between transmit and receive arrays is characterized by circularly symmetric Gaussian channel fading with average power weighting $w(f,d)$ for frequency offset f and delay offset d given by an exponential

$$w(f,d) = e^{-\alpha\left[([2T]\,f)^2 + (B\,d)^2\right]} \tag{20}$$

where $2T$ is the total (training plus data) channel usage duration, and B is the bandwidth of the signal. In the example the weighting exponential coefficient is given by $\alpha = 1/2$. The average incoherent SNR per transmit antenna is -20 dB at each receive antenna. The weighing function is displayed in Figure 1 in normalized units of delay offset ($B = 1$) and frequency offset ($2T = 1$). In the example, we assume that the receiver only performs spatial processing, so $n_\rho = 1$. In addition, the orthogonal signal projection during the training epoch is performed ideally. We employ 5 delay and 5 frequency (so $n_\delta = 25$) degrees of freedom for each transmit antenna. With the training data, spatial, space-time, and space-time-frequency beamformers are constructed. These transmit beamformers are applied during the second epoch. A spatial receive beamformer is constructed that maximizes SINR at its output for each transmit beamformer.

The performance of each transmit beamformer is characterized by the SINR at the receive beamformer output for a given transmit beamformer approach relative to the average SINR at the receive beamformer output for a random transmit beamformer. The relative performances of various transmit beamforming approaches is displayed in Figure 2. In the figure, the distributions of performance over a 1000 random channels are displayed. Because the receiver must mitigate the large interferer, there is a single receive degree of freedom remaining. In this case, one would expect that the coherent transmit gain of the 10 transmit nodes to be 10 dB stronger than the incoherent gain of the random transmit beamformer. We see here that the space-time-frequency transmit beamformer achieves this gain, while the spatial and space-time transmit beamformers have relatively poor performance because of their inability to compensate for channel dynamics.

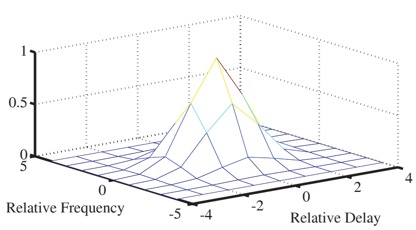

Fig. 1. Channel amplitude weighting $w(f, d)$, as a function of delay and frequency offset in units of resolution cells.

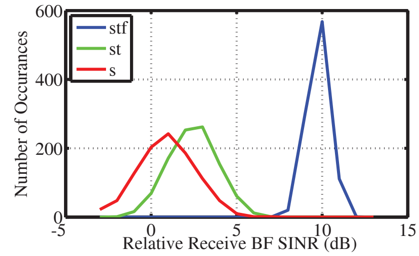

Fig. 2. For a doubly dispersive 10×2 MIMO channel, histograms of received power at a maximum SINR receive beamformer are displayed for space-time-frequency (stf), space-time (st), and spatial-only (s) transmit beamformers relative to a random transmit beamformer. The received signal is in the presence 20 dB flat-fading, static interferer.

4. CONCLUSION

By extending the low-SNR optimal capacity beamforming approach for an informed transmitter MIMO system in the presence of interference, a space-time-frequency adaptive processing approach is developed. This approach enables increased received power and an increased coherence interval compared to spatial-only approaches.

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