NETWORK LOCALIZATION FOR DISTRIBUTED TRANSMIT BEAM FORMING WITH MOBILE RADIOS

Troy Stevens and Keith Kastella

SRI International, 2100 Commonwealth Blvd., Ann Arbor, MI 48105
Troy.Stevens@SRI.com Keith.Kastella@SRI.com

ABSTRACT

This paper describes open-loop and reference-aided methods to localize and synchronize a network of mobile radios to perform distributed transmit beamforming. To steer beams to a desired location, the radios must (a) synchronize their independent clocks, (b) estimate their positions relative to each other, and (c) estimate the relative angular orientation of their constellation to the steering direction. To estimate (a) and (b), messages containing transmission and reception time stamps are transmitted between the beamforming radios in a round-robin fashion. Inter-radio ranges and clock offsets are estimated from these measurements. To estimate (c), we can use GPS measurements alone to direct the beam to a known location (open-loop operation), or we can use GPS and transmissions from the intended recipient at a known location to aid beam-steering (reference-aided operation). In both cases, a Kalman filter is used to blend the measurements and propagate the node states through the beamforming process.

Index Terms— distributed transmit beamforming, ad-hoc communications network, wireless communications, self-localization and synchronization

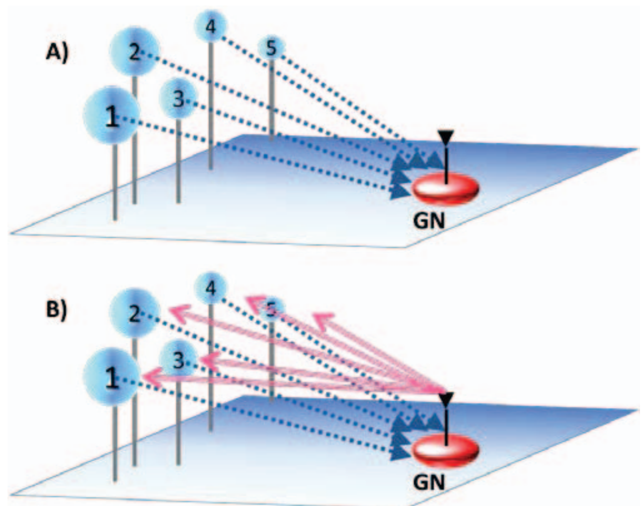

Figure 1 – System model for A) open-loop (OL) and B) reference-aided (RA) distributed transmit beamforming (DTB) with N=5 beamforming nodes (BFNs) and one ground node (GN).

1. INTRODUCTION

Distributed transmit beamforming (DTB) is a cooperative communication scheme in which two or more information sources simultaneously transmit a common message with controlled phase so that the signals constructively combine at an intended destination. Rapid advancement in this area is being enabled by developments in computing, wireless communications, navigation technologies and algorithms. The main advantage of DTB is that with fixed radiated power at each emitter, ideal beamforming with N emitters yields an N^2 gain in received power. This power gain can be translated into a dramatic increase in range, rate, or energy efficiency. It can also provide benefits in terms of security and interference reduction since less power is transmitted in unintended directions. DTB can use feedback from a cooperating receiver, use a reference beacon at or near an intended receiver, or not use reference-based feedback in an open-loop (OL) mode. A number of challenges must be addressed before these benefits are realized, including: 1) message coordination; 2) transmitter coordination at the symbol or pulse level; 3) carrier frequency synchronization; and 4) carrier phase synchronization.

A number of techniques have been proposed that that use varying levels of cooperative control with the receiver including full feedback [1], one-bit feedback [2], master-slave synchronization [3], round-trip synchronization [4], two-way synchronization [5] and receiver-coordinated beamforming with kinematic tracking [6]. For a recent review of DTB, see [7].

2. SYSTEM MODEL

In this paper, we estimate the performance that can be achieved in a less-structured environment consisting of either OL or reference-aided (RA) DTB for the system shown in Figure 1. The system under study consists of a set of $\boldsymbol{N}$ beamforming nodes (BFN) (1-5 in Figure 1), consisting of, say, tethered aerostats. The node array is to communicate with the ground node (GN) at a known location through a duplex channel. Each node is equipped with 1) a beamforming radio, 2) a navigation system that provides coarse positioning but does not provide sufficient sub-wavelength accuracy required for beam-forming, 3) an inter-node ranging system, and 4) a backchannel inter-node communications system. We assume that the system performs fine-resolution inter-node ranging and synchronization that can cohere the radios to a small fraction of the carrier frequency wavelength [8]. In the OL application, the BFNs use their coarse positioning system to direct

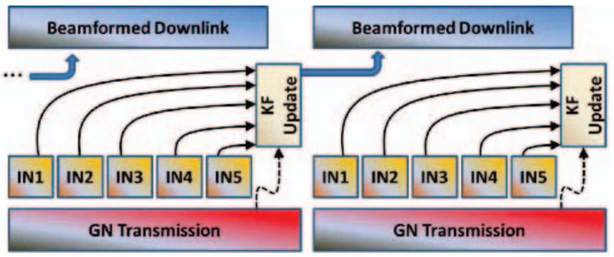

Figure 2 – Timing schema for open-loop (OL) and reference-aided (RA) distributed transmit beamforming (DTB).

the beam at the GN. In the RA case, the BFNs augment the inter-node measurements and coarse positioning with time-difference of arrival (TDOA) information derived from the ground node uplink. This may be obtained from codes embedded within the uplink message or from cross-correlating the uplink message as observed by all of the nodes. While this technique has some similarity to the case studied in [6], it differs in that the downlink and uplink frequencies are not the same and the precise estimation of the relative positioning of the nodes is required.

3. COMMUNICATIONS PROTOCOL

We consider a wireless communications system consisting of a number of BFNs on tethered aerostats in duplex communication with a ground-based receiver GN (Figure 1). The aerostats have low-quality positioning sensors onboard (conventional GPS) that are not accurate enough to support transmit beamforming. To beamform, the BNs perform TDOA measurements to estimate their relative positions and times on a regular round-robin schedule. Within its scheduled timeslot, each BFN transmits a timing signal and its current local clock state to all of the other nodes. Each BFN measures the receipt time using its local clock and shares the result of these measurements across the network. The measurements are combined with the GPS data to provide a fine-grained position and orientation solution for the aerostat swarm. For OL operation, we assume that the position of the receiver is known and a coherent beam can be directed at the receiver.

With RA operation, we assume that the system operates with a full-duplex link in which the uplink occurs at one frequency and the distributed transmit beamformed downlink occurs on another frequency. The advantage of the RA approach is that the BFNs use the uplink signal to refine the estimate of their orientation relative to GN, significantly improving performance.

4. DTB EXTENDED KALMAN FILTER

To formulate the Kalman filter for this problem, let u denote the universal time. The physical 3D East/North/Up (ENU) position and velocity of node n at time u are $\mathbf{X}_n(u)$ and $\dot{\mathbf{X}}_n(u)$ and its clock output and drift are $t_n(u) = u + T_n(u)$ and $\dot{t}_n(u) = 1 + \dot{T}_n(u)$. We construct a filter on the joint position and time states of all N nodes. The state of node n is $\mathbf{x}_n = (x_n, \dot{x}_n, y_n, \dot{y}_n, z_n, \dot{z}_n, \tau_n, \dot{\tau}_n)$, $n = 1, \ldots, N$ where x, y, z are position components in an ENU frame, $\dot{x}, \dot{y}, \dot{z}$ are the velocity components, τ_n is the clock offset and $\dot{\tau}_n$ the clock rate.

The state vector of the N-node system is the 8-N dimensional vector $\mathbf{x} = \left(\mathbf{x}_1^{\mathrm{T}}, \ldots, \mathbf{x}_N^{\mathrm{T}}\right)^{\mathrm{T}}$ where T denotes transpose. We assume that the time evolution of the nodes is independent and governed by a stochastic difference equation. We develop a discrete time model for this system at times u^k and $u^{k+1} = u^k + \Delta^{k+1}$, where $k = 1, \ldots, K$ indexes time. Note that the increments of universal time Δ^k are irregular for our problem.

To track the node positions, times and rates relative to the ground receiver, we construct an extended Kalman filter on the joint system state. The system dynamics are modeled as linear with state estimates

$$\hat{\mathbf{x}}^{k|k-1} = \mathbf{F}\hat{\mathbf{x}}^{k-1|k-1}$$
$$\mathbf{P}^{k|k-1} = \mathbf{F}\mathbf{P}^{k-1|k-1}\mathbf{F}^{\mathrm{T}} + \mathbf{Q}\,.$$

The nonlinear TDOA measurements are

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\xi}\,.$$

In this notation, the standard expressions for the first-order extended Kalman filter are [9]

$$\mathbf{H}^k \equiv \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}^{k|k-1}}$$
$$\mathbf{v}^k = \mathbf{z}^k - \mathbf{h}\left(\hat{\mathbf{x}}^{k|k-1}\right)$$
$$\mathbf{S}^k = \mathbf{H}^k\mathbf{P}^{k|k-1}(\mathbf{H}^k)^{\mathrm{T}} + \mathbf{R}$$
$$\mathbf{W}^k = \mathbf{P}^{k|k-1}(\mathbf{H}^k)^{\mathrm{T}}(\mathbf{S}^k)^{-1}$$
$$\hat{\mathbf{x}}^{k|k} = \hat{\mathbf{x}}^{k|k-1} + \mathbf{W}^k\mathbf{v}^k$$
$$\mathbf{P}^{k|k} = (\mathbf{1} - \mathbf{W}^k\mathbf{H}^k)\mathbf{P}^{k|k-1}\,.$$

5. KINEMATICS AND MEASUREMENT MODELS

We model the tethered aerostat beamforming node motion as a damped harmonic oscillator with independent motion for each component $\mathbf{y} = (x, \dot{x})^{\mathrm{T}}$. This obeys the stochastic differential equation $\dot{\mathbf{y}}(t) = \mathbf{A}\mathbf{y}(t) + \mathbf{w}(t)$, where $A = \begin{pmatrix} 0 & 1 \\ -\kappa & -\gamma \end{pmatrix}$ with restoring force $\kappa \geq 0$ and damping factor $\gamma \geq 0$. This corresponds to a unit-mass system with undamped angular frequency $\omega_0 = \sqrt{\kappa}$. The system oscillates at angular frequency $\omega_d = \omega_0\sqrt{1-\zeta^2}$, where $\zeta = \gamma/2\omega_0$ determines whether the system is overdamped ($\zeta > 1$), critically damped $\zeta = 1$), underdamped ($0 < \zeta < 1$, or undamped $\zeta = 0$).

The system is driven by white Gaussian noise $\mathbf{w}(t)$ with covariance $C(t, t') = \mathrm{diag}[0\ 1]q^2\delta(t-t')$. For finite time interval Δ, the system evolves as $\mathbf{y}(t+\Delta) = \mathbf{F}(\Delta)\mathbf{y}(t) + \mathbf{v}(\Delta)$ where $\mathbf{F}(\Delta) = \exp(\mathbf{A}\Delta)$,

$$\mathbf{F}(\Delta) = e^{-\frac{\gamma\Delta}{2}} \times$$
$$\begin{pmatrix} \cos\omega_d\Delta + \frac{\gamma\sin\omega_d\Delta}{2\omega_d} & \frac{\sin\omega_d\Delta}{\omega_d} \\ -\frac{\kappa\sin\omega_d\Delta}{\omega_d} & \cos\omega_d\Delta - \frac{\gamma\sin\omega_d\Delta}{2\omega_d} \end{pmatrix}.$$

The plant noise $\mathbf{v}(\Delta) = \int_0^{\Delta}\mathbf{F}(\Delta - t)\mathbf{w}(t)dt$ is a zero-mean Gaussian vector with covariance $\mathbf{Q}(\Delta) = E[\mathbf{v}(\Delta)\mathbf{v}(\Delta)^{\mathrm{T}}]$.

5.1. Clock Dynamics Model

We assume that the clock and kinematic states evolve independently. The clock offset and rate for each node are characterized by state vector $\boldsymbol{\tau} = (\tau, \dot{\tau})^{\mathrm{T}}$ with discrete-time state transition matrix $\mathbf{F}_\tau(\Delta) = \begin{pmatrix} 1 & \Delta \\ 0 & 1 \end{pmatrix}$. The clock plant noise is

$$\mathbf{Q}_\tau(\Delta) = q_1^2 \begin{pmatrix} \Delta & 0 \\ 0 & 0 \end{pmatrix} + q_2^2 \begin{pmatrix} \Delta^3/3 & \Delta^2/2 \\ \Delta^2/2 & \Delta \end{pmatrix}.$$

The plant noise parameters q_1 and q_2 can be derived from measurements of the Allan variance [10].

5.2. Beamforming Node TDOA Measurements

For the inter-node TDOA measurements, BFN l emits a signal at time u_l^k. If l is one of the BFNs, then it also communicates its transmission time $t_l(u_l^k) \equiv t_l^k$. One or more BFNs m measure the message receipt time. The transmission delay from l to m is d_{lm}^k so that node m receives the message at time $u_m^k = u_l^k + d_{lm}^k$ and its clock reads $t_m(u_m^k) \equiv t_m^k$. The delay is related to the node separation through $cd_{lm}^k = |\mathbf{X}_m(u_m^k) - \mathbf{X}_l(u_l^k)|$, where c is the speed of light and $|\cdot|$ denotes three-vector norm.

For measurement k, node l^k transmits and the $N-1$ element set of nodes $\mu = \{m_1^k, \dots, m_{N-1}^k\} = \{1, \dots, N\} \backslash l^k$ receive. Let $t_{l^k}^k$ denote the transmit time broadcast by l^k and $t_{m_j^k}^k$ the receipt times. The $N-1$ dimension time-difference measurement vector is $\mathbf{z}^k$ with components $\mathbf{z}_j^k = c\left(t_{m_j^k}^k - t_{l^k}^k\right)$.

The measurement function is the $N-1$ dimensional vector function $\boldsymbol{h}(\mathbf{x})$ with j^{th} component is

$$\mathbf{h}_j(\mathbf{x}) = \left|\mathbf{X}_{m_j^k} - \mathbf{X}_{l^k}\right| + c\left(\tau_{m_j^k} - \tau_{l^k}\right)$$

where, $\mathbf{X}_n$ denotes the 3D ENU vector extracted from $\mathbf{x}$ and τ_n is the corresponding time offset.

5.3. Reference Node TDOA Measurements

We assume that the GN is non-cooperative and its signal does not include the timing information available from the BFNs so that the TDOA must be computed by double-differencing. With each emission there are $N(N-1)/2$ measurement pairs $\mathbf{z}_{ij}^k = c(t_i^k - t_j^k)$ with measurement equation

$$\mathbf{h}_{ij}(\mathbf{x}) = |\mathbf{X}_i - \mathbf{X}_0| - |\mathbf{X}_j - \mathbf{X}_0| + c(\tau_i - \tau_j),$$

which are concatenated to form the reference node measurement function $\mathbf{h}(\mathbf{x})$. When reference node and inter-node measurements occur at the same time, then the measurements are stacked and processed as a single joint measurement. Inter-node and reference node measurements are assumed to have uncorrelated zero-mean Gaussian noise in each component (see Table 1 below).

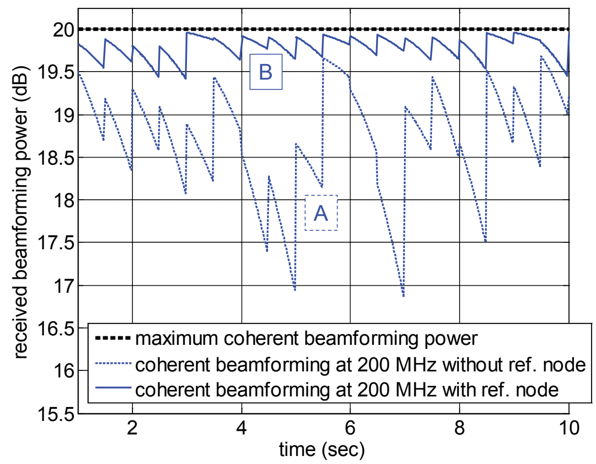

Figure 3 – Coherent DTB power received by a GN without a GN reference signal. B) With GN reference signal. The reference-signal TDOA measurement uncertainty is 50 ps in B).

6. RESULTS

We used the parameters listed in Table 1 in the simulations. Clock and plant-noise values were derived from fits to data collected from a Rakon RFPO45 oven-controlled oscillator [11] and tethered balloon measurements. The GPS values are representative of differential GPS performance. The DTB performance metric is the power received by the GN, which is shown in Figure 3 for OL and RA modes of operation at 200 MHz; the ideal beamforming power limit of $N^2 = 20$ dB is also shown for reference. The receive power has a characteristic sawtooth-like variation, the falls and rises of which result from divergence of the actual BFN states from the Kalman filter state projections and from measurement updates, respectively. We observe that the mean transmit power is higher for RA than OL operation. As stated above, the primary reason for this is the GN reference signal provides the BFNs with a refined orientation estimate. The inter-node TDOA uncertainty σ_1 is small enough to ensure that BFN-to-BFN localization is accurate to within a few millimeters. This produces an RMS beamforming phase error on the order of a few degrees and degrades the maximum coherent gain by a negligible amount [6]. Because the GPS measurements necessary to provide orientation information to the BFNs have relatively large uncertainties, however, the pointing uncertainty of the BFN array is sufficiently large that its boresight is not always directed precisely towards the intended destination in OL operation.

Figure 4 shows the power received by the GN for RA operation as a function of GN signal TDOA measurement uncertainty σ_3. The received powers for OL are shown for reference. RA operation is superior to that of OL for $\sigma_3 \leq 1$ ns. This value is significantly larger than the inter-node TDOA measurement uncertainty $\sigma_1 = 6.7$ ps. The reason good DTB can still be achieved is that range phase errors up to about 30° RMS only degrade beamforming gain modestly, consistent with results in [6], provided beam pointing errors are limited by the use of a reference node.

Table 1 – Simulation parameters.

number of nodes N	10
DTB frequency f	$f = 200$ MHz $f = 450$ MHz
mean BFN array horizontal width $\mathbf{E}_{horz}[\mathbf{X}_m - \mathbf{X}_l]$	30 m
mean BFN array vertical height $\mathbf{E}_{vert}[\mathbf{X}_m - \mathbf{X}_l]$	10 m
mean BFN-GN range $\mathbf{E}[\mathbf{X}_m - \mathbf{X}_0]$	1 km
inter-node TDOA measurement period T_1 and uncertainty σ_1	$T_1 = 0.5$ s $\sigma_1 = 6.7$ ps
differential GPS position and time measurement period T_2 and uncertainties σ_{2a} and σ_{2b}	$T_2 = 0.5$ s $\sigma_{2a} = 0.25$ m $\sigma_{2b} = 3$ ns
GN reference signal TDOA measurement period T_3 and uncertainty σ_3	$T_3 = 0.18$ s 5 ps $\leq \sigma_3 \leq$ 100 ns
harmonic oscillator restoring force κ, damping factor γ and covariance σ_{HO}^2	$\kappa = 0.005$ s^{-2} $\gamma = 0.1$ s^{-1} σ_{HO}^2=0.1 m^2
clock Gaussian process covariance q_1^2 and Wiener process covariance q_2^2	$q_1^2 = 6 \times 10^{-13}$ s$^{1/2}$ $q_2^2 = 2 \times 10^{-14}$ s$^{-1/2}$

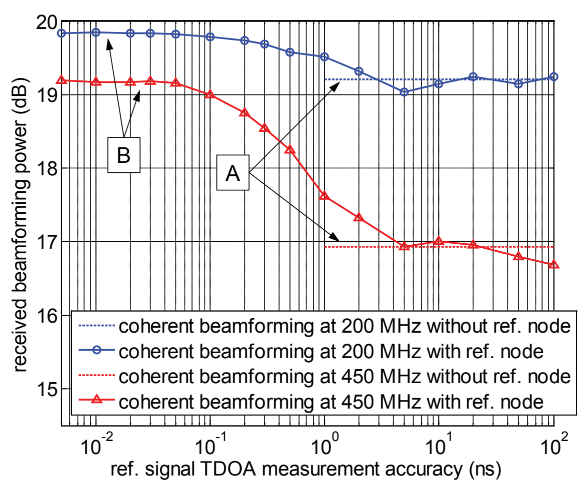

Figure 4 – A) Coherent DTB power received by a GN without a GN reference signal. B) With GN reference signal.

7. CONCLUSION

This paper describes open-loop and reference-aided distributed transmit beamforming. The critical input assumptions are 1) the bulk of the protocol burden falls on the beamforming array; and 2) internode positions can be measured with an accuracy that is a small part of the message carrier wavelength. The system must be able to both form a beam and then point the beam at the receiver accurately. Open-loop beamforming performance quickly degrades as frequency increases, which seems to be primarily due to the inability of the open-loop scheme to accurately point the beam. When the system incorporates information from the receiver uplink in the reference-aided scheme, performance is significantly enhanced.

8. REFERENCES

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