Estimation Performance and Resource Savings: Tradeoffs in Multiple Radars Systems

Hana Godrich†, Athina Petropulu°, and H. Vincent Poor†
†Princeton University, Princeton, NJ 08544
°Rutgers University, Piscataway, NJ 08854

***Abstract*—In widely distributed multiple radar systems, employing larger numbers of transmit and receive antennas supports better target parameter estimation. Increased dimensions results in higher communication needs, synchronization overhead, and processing complexity. In our previous studies, resource-aware operational schemes have been introduced for a given localization estimation mean-square error (MSE) threshold requirement. Power allocation scheme that minimizes the total transmitted power for a given MSE goal has been derived. As most of the transmitted power was allocated to a few of the available transmit antennas, a subset selection scheme has been proposed to identifying a minimal set of transmit and receive antennas that offer the required accuracy performance. The study indicates that some transmit and receive antenna pairs contribute more than others to the localization performance. Based on this, a different approach to resource-aware operation is proposed in this paper. The objective is to identify an antenna subset that offers an optimal tradeoff between performance loss in term of localization MSE and the active subset size. By setting an acceptable loss threshold, relative to the best performances achievable with all antennas active, joint optimization of subset size and power allocation is performed to maximize the trade-off gains. A mixed optimization problem is defined, based on the Cramer-Rao bound (CRB), and fast approximation algorithm is proposed, maximizing the trace of the Fisher information matrix (FIM) while minimizing the number of active antennas. The closed-form expression of the CRB offers additional understanding of the relation between the geometric layout of the transmit and the receive antennas with respect to the target and their relative contribution to the performance.**

***Index Terms*—MIMO radar, Multistatic radar, CRB, resource allocation, target localization.**

I. Introduction

There is a growing awareness of the issue of resource management even in systems that were not considered natural candidates in the past. The global requirement for "greening" the operation of any type of system, including government and military ones, sets new challenges when it comes to systems such as radars. There are growing numbers of radar architectures that use multiple, widely spread, antennas and thus call for attention as for the efficient use of their available resources. The deployments of portable lightweight systems that may be carried by one person or mounted on a vehicle introduce new requirements in terms of the communication load, processing needs and synchronization process due to the distributed nature of these systems. Multiple radars systems, such as multiple-input multiple-output (MIMO) radar systems [1] and multistatic/multisite radar systems [2], benefit from the extended number of radars however may be utilized more efficiently using methods such as optimal power management and subset selection. MIMO radar systems offer enhanced target localization capabilities by exploiting increased spatial spread [1]. The more transmit and receive radars involved the better the estimation performance of a target's parameters is [3]- [4]. The drawback is in the power needs, computational complexity, and communication requirements for larger systems.

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Power management is one way to address resource-aware operation. In [5], we propose methods for allocating power among the transmit antennas such that localization performance is optimized while the total transmitted power is minimized. The power allocation schemes introduced in [5] adapt the transmitted energy to the system characteristics, such as physical locations of transmit and receive antennas with respect to the target, target reflectivity, and propagation paths loss. Another aspect of resource-aware operation is the use of the available infrastructure. A given localization accuracy threshold may be met by using a smaller subset of the available transmit antennas. In addition to its operational savings, selecting a subset of active radars offers reduced communication needs and computational complexity. Motivated by this, two new operational policies are proposed in [6]. The first deals with the problem of identifying the smallest subset of transmit and receive antennas, out of the available M transmit and N receive ones, that achieves a given target localization estimation mean-square error (MSE) threshold. The second is concerned with the case in which the active set must have a specific size. In this case, the goal is to select a subset of predetermined size K such that the estimation capability of the subset is maximized. The choice of the appropriate subset of antennas for both cases depend on the system parameters, such as the topology of the system with respect to the target, the SNR over the different propagation paths, the effective bandwidth, and the transmitted power.

In this paper the relation between the geometry of the system and the best transmit and receive antenna pairs for the purpose of localization is investigated through the Cramer-Rao bound (CRB) expression [7]. The analysis is extended to derive a mechanism for the selection of an optimal subset of radars that deliver the closest to maximum performance. It has been shown that most of the available total power is allocated to a smaller subset of transmitters while others are kept to a minimal power [5]. A method of identifying these so-called *best* sets is proposed. Previous studies used preset absolute performance thresholds and optimized either the power allocation or the active subset selection such that these constraints are met. In this study, an adaptive threshold is set, determined relative to the best estimation performance that may be obtained with a given, fully operational, system. The total transmit power budget is optimally allocated among the subsets' transmit antennas such that the MSE attained by the subset is minimized. Thus, the proposed scheme jointly optimizes the size of the subset and the power allocation in the subset to deliver an acceptable MSE performance loss with the minimal active subset.

The paper is organized as follows: The system model is introduced in Section II, including the formulation of the CRB. A resource allocation problem that finds an appropriate trade-off between performance and resource use is given in Section III. Numerical analysis is provided in Section IV, for the selection algorithm. Finally, Section V concludes the paper.

II. System Model

We consider a distributed multiple radar system with M transmit and N receive radars, forming an $M \times N$ distributed multiple radar system. The transmit radar set is defined as $\mathcal{S}_{Tx} := \{(x_{1_{tx}}, y_{1_{tx}}), (x_{2_{tx}}, y_{2_{tx}}), ..., (x_{M_{Tx}}, y_{M_{Tx}})\}$ and the receive radar set as $\mathcal{S}_{Rx} := \{(x_{1_{Rx}}, y_{1_{Rx}}), (x_{2_{Rx}}, y_{2_{Rx}}), ..., (x_{N_{Rx}}, y_{N_{Rx}})\}$. An extended target, with a center of mass located at position (x, y), is assumed. The variation in the location of the targets' center of mass, as viewed by the set of radars, is assumed to be small with respect to the system resolution capabilities. The system is tracking the target's location and has available estimates for unknown parameters, such as the target RCS, from previous cycles. The search cell is confined to $(x_c \pm kc/\beta, y_c \pm kc/\beta)$, where k is an integer, c is the speed of light, and β is the waveform effective bandwidth. The transmit and receive radars are located in a two dimensional plane. The M transmit radars are arbitrarily located at coordinates $(x_{m_{Tx}}, y_{m_{Tx}})$, $m = 1, \ldots, M$, and the N receiver radars are arbitrarily located at coordinates $(x_{n_{Rx}}, y_{n_{Rx}})$, $n = 1, \ldots, N$. A set of orthogonal waveforms is transmitted, each with a lowpass equivalent $s_m(t)$, where $\int_{\mathcal{T}_m} |s_m(t)|^2 dt = 1$, and $\mathcal{T}_m$ is the duration of the m-th transmitted signal. The waveform effective bandwidth is denoted by β_m. The waveforms' transmitted powers $p_{m_{tx}}$ are constrained by $\mathbf{p}_{tx} = [p_{1_{tx}}, p_{2_{tx}}, ..., p_{M_{tx}}]^T$.

Let $\tau_{m,n}(x, y)$ denote the propagation time of a signal transmitted by radar m, reflected by the target, and received by radar n. The baseband representation for the signal transmitted from radar m received at radar n is

$$r_{m,n}(t) = \sqrt{\alpha_{m,n} p_{m_{tx}}} h_{m,n} s_m(t - \tau_{m,n}) + w_{m,n}(t). \quad (1)$$

The term $\alpha_{m,n} \propto \frac{1}{R^2_{m_{Tx}} R^2_{n_{Rx}}}$ represents the variation in the signal strength due to path loss effects, $R_{m_{Tx}}$ is the range from transmitter m to the target and the range from the target to receiver n is denoted by $R_{n_{Rx}}$. The target RCS $h_{m,n}$ is modeled as deterministic and complex, and is assumed to be unknown. The term $w_{m,n}(t)$ represents circularly symmetric, zero-mean, complex Gaussian noise, spatially and temporally white with autocorrelation function $\sigma^2_w \delta(\tau)$. The propagation path from transmitter m to the target and from the target to receiver n is defined as channel (m, n). The SNR of channel (m, n) is defined as $\mathrm{SNR}_{m,n} = \frac{\alpha_{m,n} |h_{m,n}|^2 p_{m_{tx}}}{\sigma^2_w}$.

We define a vector of unknown parameters as $\mathbf{u} = [x, y, \mathbf{h}^T]^T$,where $\mathbf{h} = [h_{1,1}, h_{1,2}, ..., h_{M,N}]^T$. The following vector notation is defined for later use: $\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_M]^T$, $\boldsymbol{\alpha} = [\alpha_{1,1}, \alpha_{1,2}, ..., \alpha_{M,N}]^T$, $\mathbf{p}_{tx} = [p_{1_{tx}}, p_{2_{tx}}, ..., p_{M_{tx}}]^T$, and $\boldsymbol{\tau} = [\tau_{1,1}, \tau_{1,2}, ..., \tau_{M,N}]^T$.

The CRB is known to be asymptotically tight to the maximum likelihood estimator (MLE) MSE at high SNR [7], thus, it is used here to represent the localization MSE. Herein, the CRB expression, $\mathbf{C}_{x,y}(\mathbf{u}, \mathbf{p}_{tx})$, derived in [5], is used, resulting in the following expression:

$$\mathbf{C}_{x,y}(\mathbf{u}, \mathbf{p}_{tx}) = \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} \mathbf{J}_{m,n} \right\}^{-1}, \quad (2)$$

where the submatrix $\mathbf{J}_{m,n}$ is the Fisher information matrix (FIM) defined as

$$\mathbf{J}_{m,n} = \begin{bmatrix} u_{a_{m,n}} & u_{c_{m,n}} \\ u_{c_{m,n}} & u_{b_{m,n}} \end{bmatrix}, \quad (3)$$

and the elements $u_{a_{m,n}}$, $u_{b_{m,n}}$, and $u_{c_{m,n}}$, are defined as

$$u_{a_{m,n}} = \xi_{m,n} (\cos\gamma_m + \cos\phi_n)^2, \quad (4)$$

$$u_{b_{m,n}} = \xi_{m,n} (\sin\gamma_m + \sin\phi_n)^2, \quad (5)$$

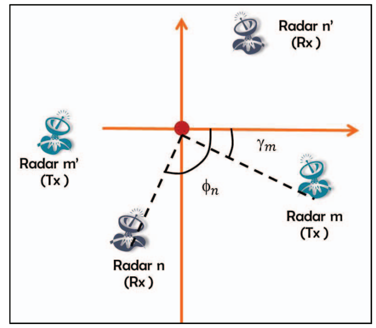

Fig. 1. System model for multiple radar systems with widely separated antennas.

and

$$u_{c_{m,n}} = \xi_{m,n} (\cos\gamma_m + \cos\phi_n) \times (\sin\gamma_m + \sin\phi_n), \quad (6)$$

where $\xi_{m,n} = \frac{8\pi^2 \beta_m^2 \mathrm{SNR}_{m,n}}{c^2}$, the phase γ_m is the bearing angle of the transmit antenna m to the target measured with respect to the x axis, and the phase ϕ_n is the bearing angle of the receive antenna n to the target measured with respect to the x axis, as demonstrated in Figure 1.

The trace of the matrix $\mathbf{C}_{\mathcal{S}_A}$ represents a lower bound on the sum of the MSEs for the target location estimation, i.e., $\mathrm{tr}(\mathbf{C}_{\mathcal{S}_A}) \leq \sigma^2_x + \sigma^2_y$, where σ^2_x and σ^2_y are the target's x and y location estimation MSE, respectively. The traced of the CRB matrix represents the *overall* estimation MSE. It may be formulated as

$$\mathrm{tr}(\mathbf{C}_{x,y}(\tilde{\mathbf{u}}, \mathbf{p}_{tx})) = \sum_{m=1}^{M} \sum_{n=1}^{N} (u_{a_{m,n}} + u_{b_{m,n}}) \times \left[\left(\sum_{m=1}^{M} \sum_{n=1}^{N} u_{a_{m,n}} \right) \left(\sum_{m=1}^{M} \sum_{n=1}^{N} u_{b_{m,n}} \right) - \left(\sum_{m=1}^{M} \sum_{n=1}^{N} u_{c_{m,n}} \right)^2 \right]^{-1}. \quad (7)$$

The CRB matrix, $\mathbf{C}_{x,y}(\tilde{\mathbf{u}}, \mathbf{p}_{tx})$, is evaluated at $\tilde{\mathbf{u}} = [\tilde{x}, \tilde{y}, \tilde{\mathbf{h}}^T]^T$; a vector of prior estimates of the target location and RCS, obtained in previous cycles. The search cell center coordinates, (x_c, y_c), may also be used instead of an estimated target location$(\tilde{x}, \tilde{y})$. The expression given in (7) offers a metric that represents the MLE MSE in the analysis provided next.

III. Resource Allocation

The power allocation scheme introduced in [5] adapts the transmitted energy to the system characteristics. A resource operation aware approach is proposed in [6], deriving subset selection schemes for the identification of an optimal subset of transmit and receive antennas, chosen such that a predetermine MSE goal is achieved.

For a given scenario, some transmit/receive antenna pairs contribute more to the localization performance than others. Thus, a given localization accuracy threshold may be obtained by using a smaller subset of the available transmit and receive antennas. The relative contribution of a transmit/receive antenna pair m, n to the overall MSE is dependent of a several factors. Some are given system characteristics, such as the path loss on the m, n propagation path (inversely proportional to the range from the transmit/receive antennas to the target), the target reflectivity on the m, n path, and the geometric layout of each transmit/receive antenna pairs with respect to the target. Others are design parameters, such as the power allocated to the m-th transmit antenna, the signal bandwidth, and the operational set. In some cases, the location of the transmit and receive antennas with respect to a given search cell may be considered as a design problem. For mobile applications, one may optimize the locations of the antennas to maximize the localization performance.

Hereafter, an extended analysis of the CRB expression, given in (7), is used in order to gain a better understanding of the relationship between the contribution of a transmit/receive antenna pair to the localization accuracy and its geometric spread, i.e., angular view of the target. This expression is than utilized to define a trade-off optimization problem, where transmit/receive antenna pairs are systematically add to bring the localization accuracy as close as needed to the MSE obtained by a fully operational system while the operational costs are kept to a minimum.

Expanding and rearranging the overall MSE in (7) while introducing a set of binary variables

$$q_{tx_m} = \begin{cases} 1 & \text{if transmit radar } m \text{ is selected;} \\ 0 & \text{otherwise} \end{cases}, \quad m = 1, ..., M, \tag{8}$$

and

$$q_{rx_n} = \begin{cases} 1 & \text{if receive radar } n \text{ is selected;} \\ 0 & \text{otherwise} \end{cases}, \quad n = 1, ..., N, \tag{9}$$

to the trace of the CRB for a set of radars, $\mathcal{S}_A = \{\mathbf{x}_{m_{Tx}} \in \mathcal{S}_{tx}, \mathbf{x}_{n_{Rx}} \in \mathcal{S}_{Tx} \mid q_{tx_m} = 1,\ q_{rx_n} = 1\}$, results in the following expression

$$\begin{aligned} &\operatorname{tr}\left(\mathbf{C}_{x,y}\left(\mathbf{p}_{tx}, \mathbf{q}_{tx}, \mathbf{q}_{rx}\right)\right) = \qquad (10) \\ &\left\{\sum_{m=1}^{M}\sum_{n=1}^{N} 2 q_{tx_m} q_{rx_n} \xi_{m,n}\left(\mathbf{p}_{tx}\right)\left[1 + \cos\left(\gamma_m - \phi_n\right)\right]\right\} \\ &\times \Bigg\{\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{n'=n+1}^{N} q_{tx_m} q_{rx_n} q_{rx_{n'}} \xi_{m,n}\left(\mathbf{p}_{tx}\right) \xi_{m,n'}\left(\mathbf{p}_{tx}\right) \\ &\quad \times \left[\cos\left(\gamma_m + \gamma_m\right) + \cos\left(\gamma_m + \phi_{n'}\right) + \cos\left(\gamma_m + \phi_n\right) + \cos\left(\phi_n + \phi_{n'}\right)\right]^2, \\ &+ \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{m'=m+1}^{M}\sum_{n'=1}^{N} q_{tx_m} q_{rx_n} q_{tx_{m'}} q_{rx_{n'}} \xi_{m,n}\left(\mathbf{p}_{tx}\right) \xi_{m',n'}\left(\mathbf{p}_{tx}\right) \\ &\quad \times \left[\cos\left(\gamma_m + \gamma_{m'}\right) + \cos\left(\gamma_m + \phi_{n'}\right) + \cos\left(\gamma_{m'} + \phi_n\right) + \cos\left(\phi_n + \phi_{n'}\right)\right]^2\Bigg\}^{-1}. \end{aligned}$$

The overall MSE is reliant on the sums and differences between the viewing angles of the target by each transmit-receive antenna pairs, γ_m and ϕ_n (see Figure 1). The numerator is a weighted sum of cosine functions $\cos\left(\gamma_m - \phi_n\right) = \cos\gamma_m \sin\phi_n + \cos\phi_n \sin\gamma_m$ while the denominator is a weighted sum cosine functions $\left[\cos\left(\gamma_m + \gamma_{m'}\right) + \cos\left(\gamma_m + \phi_{n'}\right) + \cos\left(\gamma_{m'} + \phi_n\right) + \cos\left(\phi_n + \phi_{n'}\right)\right]^2$. A common relaxation approach to minimize the trace of the CRB matrix is to maximize the trace of the FIM. For a given total transmit power budget, the minimal MSE achievable with a fully operational system may be obtained by

$$\begin{aligned} \underset{\mathbf{p}_{tx}}{\text{maximize}} \quad & \sum_{m=1}^{M}\sum_{n=1}^{N} \xi_{m,n}\left(\mathbf{p}_{tx}\right) \cos\left(\gamma_m - \phi_n\right), \\ s.t. \quad & \mathbf{1}^T \mathbf{p}_{tx} \leq p_{total}, \\ & p_{\min m} \leq p_{tx_m} \leq p_{\max m}, \quad m = 1, .., M. \end{aligned} \tag{11}$$

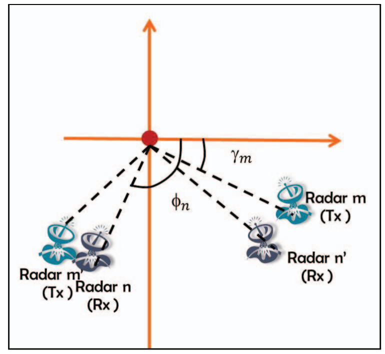

Fig. 2. Advantageous locations for a set of two transmit antennas and two receive antennas for target localization.

Transmit/receive antenna pairs with the largest sum of $\cos\left(\gamma_m - \phi_n\right)$, $m = 1, .., M$, $n = 1, .., N$, elements are preferable over others. For localization, a minimum of four transmit/receive antennas are required. For a given system layout, an optimal set of four antennas is selected out of the given set. Assume two transmit antennas and two receive antennas, located in equal ranges from a target with uniform reflectivity. Uniform transmit power allocation is applied $p_{tx} = \min\left[\frac{p_{total}}{2}, p_{\max t1}, p_{\max t2}\right]$. The set of antennas that offer the lowest MSE is the one that maximizes

$$\begin{aligned} \arg \max_{(t1,t2)\in\mathcal{S}_{Tx},(r1,r2)\in\mathcal{S}_{Rx}} \xi_{m,n}\left(p_{tx}\right) \{&\cos\left(\gamma_{t1} - \phi_{r1}\right) \\ + \cos\left(\gamma_{t1} - \phi_{r2}\right) + \cos\left(\gamma_{t2} - \phi_{r1}\right) &+ \cos\left(\gamma_{t2} - \phi_{r2}\right)\}. \end{aligned} \tag{12}$$

This is a combinatorial optimization problem that can be solve by exhausting all possible combinations of two transmit antennas out of the existing M and two receive antennas out of the existing N (overall $\frac{MN(M-1)(N-1)}{4}$ possibilities). The best two pairs are such that each transmitter has one receiver closed by, thus having very low angular difference $\left(\gamma_1 - \phi_1\right) \approx 0$, and one receiver at around $\left(\gamma_1 - \phi_2\right) \approx 90^\circ$, as illustrated in Figure 1. This spread maximizes the expression in (12) while minimizing the MSE in (10). For the case when the locations of the antennas are a design parameters, the angular locations set $\mathcal{S}_{A_4} = \{\gamma_{t1}, \gamma_{t2}, \phi_{r1}, \phi_{r2}\}$ are optimized such that the expression in (12) is maximized.

For the case of unequal ranges and reflectivity coefficients, the weight values $\xi_{m,n}\left(\mathbf{p}_{tx}\right)$ need to be factored into the selection. Thus, a combination of angular spread and channel quality determine the best four antennas to participate in the subset. Transmit/receive antenna pairs that have lower path losses and better angular spread are advantageous over ones with higher path losses, low target reflectivity, and narrow angular spread. Power allocation is determined to optimize (12) while constraining the solution to $\mathbf{1}^T \mathbf{p}_{tx} \leq p_{total}$ and $p_{tx_m} \leq p_{\max m}$. This is a mixed integer optimization problem of the form

$$\begin{aligned} \underset{\mathbf{q}_{tx}, \mathbf{q}_{rx}, \mathbf{p}_{tx}}{\text{maximize}} \quad & \sum_{m=1}^{M}\sum_{n=1}^{N} q_{tx_m} q_{rx_n} \xi_{m,n}\left(p_{tx_m}\right) \cos\left(\gamma_m - \phi_n\right), \\ s.t. \quad & \sum_{m=1}^{M} q_{tx_m} + \sum_{n=1}^{N} q_{rx_n} = L \\ & \mathbf{1}^T \mathbf{p}_{tx} \leq p_{total}, \\ & p_{\min m} \leq p_{tx_m} \leq p_{\max m}, \quad m = 1, .., M, \\ & q_{tx_m} \in \{0, 1\}, \quad q_{rx_n} \in \{0, 1\}, \quad n = 1, .., N. \end{aligned} \tag{13}$$

The objective of this resource aware operation is to minimize the number of operating elements $\frac{L}{M+N}$ while minimizing the accuracy loss $\frac{\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right)}{\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right)} \leq \varepsilon$ - optimizing the trade-off between performance and infrastructure related savings. This may be solved by iterations. An initial selection of four transmit and receive antennas, that maximizes (13), is made. Optimal selection of the basic subset of four offers a specific MSE $\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right)$ that is than compared with $\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right)$, obtained by optimal power allocation for the full system. A threshold condition $\frac{\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right)}{\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right)} \leq \varepsilon$ is evaluated and, if needed, the subset size is updated, $L = L + 1$, repeating the optimization problem in (13) and the threshold condition verification steps for all possible combinations of L antennas. This process has a very high computational complexity, growing exponentially with the size of the system. An efficient approximation algorithm is proposed next.

Approximation algorithm for trade-off optimization:

1) Set up an initial subset size: $L = 4$.
2) Select a subset of L transmit and receive antennas out of the available $M + N$ ones.
3) For each combination, maximize (13) for $\mathbf{p}_{tx}$. Out of all solutions, select the one with the maximal FIM trace, resulting in $\mathbf{q}^*_{tx_L}$, $\mathbf{q}^*_{rx_L}$, and $\mathbf{p}^*_{tx_L}$.
4) Evaluate $\frac{\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right)}{\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right)} \leq \varepsilon$. If threshold conditions are reached than select $\mathbf{q}^*_{tx_L}$, $\mathbf{q}^*_{rx_L}$, and $\mathbf{p}^*_{tx_L}$ as the final solution. If not, increase the subset size by one, $L = L + 1$.
5) Add one transmitter or receiver out of the non-active antennas set to the existing subset selections, $\mathbf{q}_{tx_L} = \mathbf{q}^*_{tx_{L-1}} + \mathbf{q}_{tx_\Delta}$, $\mathbf{q}_{rx_L} = \mathbf{q}^*_{rx_L} + \mathbf{q}_{rx_\Delta}$, such that (13) is maximized. repeat step 4.

The proposed heuristic method offers a reduced complexity of $\sim O\left(LMN\left(M+N\right)^2\right)$, where L is the number of antennas in the final subset. Comparatively, an exhaustive search has an exponential complexity. For large numbers of antennas, significant computational savings are obtained through the use of the proposed algorithm.

IV. Numerical Analysis

The spatially diverse multiple propagation paths between the transmit and receive radars have different error characteristics, reliant on the specific path loss, target reflectivity, effective bandwidth, and transmitted power. In this section, numerical analysis is provided for the proposed resource-aware scheme. A 5×7 MIMO radar system ($M = 5$ and $N = 7$) is chosen for this analysis. To evaluate the effect of the radars spread, two different angular spreads of radar with respect to the target are chosen, as illustrated in Figure 3 in Case 1 and Case 2. The antennas are set at equal ranges with respect to the target position. The angular positions of the transmit antennas are $[10°; 82°; 154°; 262°; 334°]$ and the receive antennas $[0°; 51°; 102°; 154°; 205°; 257°; 308°]$.

Two target RCS is modeled as having uniform reflectivity. Each of the transmit radars is assumed to transmit a maximum allowable power defined by $\mathbf{p}_{tx_{\max}} = 150 \times \mathbf{1}_{5\times1}$ with a total transmit budget of $p_{totla} = 500$. The threshold condition is set to $\varepsilon = 2$. The proposed subset selection algorithm is applied to the system layouts given in Figure 3. For Case 1 seven antennas are utilized, $\frac{L}{M+N} = \frac{7}{12} = 0.58$, and only four out of the available seven receivers. The resulting subset is $\mathcal{S}_{\min}$ (Case 1) $= \{\mathbf{x}_{1_{Tx}}, \mathbf{x}_{2_{Tx}}, \mathbf{x}_{5_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{2_{Rx}}, \mathbf{x}_{6_{Rx}}, \mathbf{x}_{7_{Rx}}\}$. The minimal MSE achievable with a fully operational system is $\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right) = 1.47\text{m}^2$ and with subset $\mathcal{S}_{\min}$ (Case 1) it is

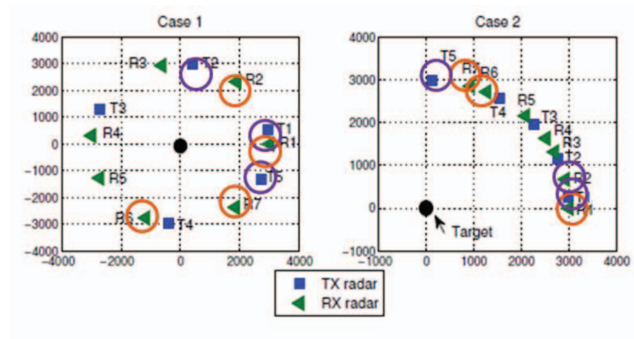

Fig. 3. Examples of different antennas spreads and subset selection that optimize performance and resources.

$\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right) = 2.6\text{m}^2$, i.e., $\frac{2.6}{1.47} = 1.77 \leq \varepsilon$. For Case 2 six antennas are utilized, $\frac{L}{M+N} = \frac{6}{12} = 0.5$, and only three out of the available seven receivers. The resulting subset is $\mathcal{S}_{\min}$ (Case 2) $= \{\mathbf{x}_{1_{Tx}}, \mathbf{x}_{2_{Tx}}, \mathbf{x}_{5_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{6_{Rx}}, \mathbf{x}_{7_{Rx}}\}$. The minimal MSE achievable with a fully operational system is $\eta\left(\mathbf{1},\mathbf{1},\mathbf{p}^*_{tx_{M+N}}\right) = 2.44\text{m}^2$ and with subset $\mathcal{S}_{\min}$ (Case 1) it is $\eta\left(\mathbf{q}^*_{tx_L},\mathbf{q}^*_{rx_L},\mathbf{p}^*_{tx_L}\right) = 3.88\text{m}^2$, i.e., $\frac{3.88}{2.44} = 1.6 \leq \varepsilon$. In Case 1 subset $\{\mathbf{x}_{1_{Tx}}, \mathbf{x}_{2_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{2_{Rx}}\}$ is the initial one while for Case 2 subset $\{\mathbf{x}_{1_{Tx}}, \mathbf{x}_{5_{Tx}}, \mathbf{x}_{1_{Rx}}, \mathbf{x}_{7_{Rx}}\}$ is the initial selection. These initial selection follow the conclusions with respect to the optimal selection of the first two transmit/receive antenna pairs, demonstrated in Figure 2. In the two cases, these pairs are the closest ones to the optimal position.

V. Conclusions

A resource aware operational concept, balancing the trade-off between localization accuracy and infrastructure utilization, has been proposed. The problem has been formulated as a mixed optimization problem and a fast approximation algorithm has been proposed, offering significantly reduced complexity. Preferable locations for the transmit and receive antenna pairs have been identified based on the closed-form expression of the FIM. These locations offer better MSE estimation for a given system size. Numerical examples have been used to demonstrate the selection process and trade-offs.

References

[1] H. Godrich, A. M. Haimovich and R. S. Blum, "Concepts and applications of a MIMO radar system with widely separated antennas," in *MIMO Radar Signal Processing*, edited by J. Li and P. Stoica, New York: John Wiley, 2008.

[2] V. S. Chernyak, *Fundamental of Multisite Radar Systems: Multistatic Radars and Multi radar Systems,* London: CRC Press, 1998.

[3] Q. He, R. S. Blum, and A. M. Haimovich, "Noncoherent MIMO radar for location and velocity estimation: More antennas means better performance," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3661–3680, Jul. 2010.

[4] H. Godrich, A. M. Haimovich, and R. S. Blum, "Target localisation techniques and tools for multiple-input multiple-output radar," *IET Radar, Sonar, and Nav.,* vol. 3, no. 4, pp. 314-327, Aug. 2009.

[5] H. Godrich, A. Petropulu, and H. V. Poor, "Power allocation strategies for target localization in distributed multiple-radar architectures," *IEEE Trans. Signal Process.* vol. 59, no. 7, pp. 3226-3240, July 2011.

[6] H. Godrich, A. Petropulu, and H. V. Poor, "Sensor selection in distributed multiple-radar architectures for localization: A knapsack problem formulation," *IEEE Trans. Signal Process.* to appear.

[7] H. V. Poor, *An Introduction to Signal Detection and Estimation*, New York; Springer, 2nd ed, 1994.