

TARGET VELOCITY ESTIMATION WITH DISTRIBUTED MIMO RADAR USING MULTIPLE PULSE REPETITION FREQUENCIES

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ABSTRACT

In this paper, we propose estimating the velocity of a target using a widely distributed multiple-input multiple-output radar that employs multiple pulse repetition frequencies. In a MIMO radar, it is possible to use different PRFs in different transmitters without added complexity. This allows one to increase the number of pulses for estimation without decreasing the unambiguous range or increasing the time the target needs to be illuminated. We derive a maximum likelihood estimator for the velocity of the target under the assumptions that the scattering is independent and noise spatially and temporally white.

Index Terms— MIMO radar, Maximum likelihood estimation

1. INTRODUCTION

In a distributed multiple-input multiple-output (MIMO) radar, the transmitters and receivers are spatially distributed so that a target is seen from several aspects simultaneously creating angular diversity in the system[1]. This approach differs from the MIMO radar with colocated antennas[2, 3], although both MIMO radar types use multiple waveforms to improve the performance.

Target velocity estimation with a widely distributed MIMO radar system has been previously studied in [4–6], but the velocity was estimated from the Doppler shift in a single pulse. Such estimation procedure might not be accurate enough in many radar system and, consequently, many operational radars are pulse Doppler radars. An estimator using several pulses was derived in [6], but pulse repetition frequency (PRF) was not considered at all.

Pulse Doppler radars operate by transmitting a sequence of pulses. The radial velocity of the target can then be determined from the phase differences of the received pulses.

The PRF that is used has an impact on both the range and the velocity estimates. When the PRF is increased, the maximum unambiguous velocity increases but the largest unambiguous range decreases. This phenomenon is commonly called range–Doppler ambiguity problem.

Many different methods have been proposed and successfully applied to reduce the ambiguities in range and velocity in the pulse Doppler radar. The methods can be divided mainly into staggered PRF, in which pulse repetition time is varied, or multiple PRF that transmits several pulse trains with different PRFs. When the multi-PRF measurements have been obtained, there are several methods to obtain the unambiguous estimates[7–9]. However, clutter cancellation is a problem in the former and false coincidences in the latter[10].

On the other hand, the architecture of a distributed MIMO radar lends itself to using multiple PRFs naturally. In this paper, we examine the use of multiple PRFs in the estimation of the target velocity using a distributed MIMO radar. This differs from the methods used for mitigation of ambiguities in networked radar systems proposed, for example, in [11]. We derive a maximum likelihood estimator for the target velocity under the assumptions that the waveforms are orthogonal, the scattering from the target is independent, and noise is spatially and temporally white.

This paper is organized as follows: The signal model is discussed in Section 2 and velocity estimation in Section 3. Numerical results will be provided in Section 4. Finally, conclusions are drawn in Section 5.

2. SIGNAL MODEL

We consider a distributed radar system with M transmitters and N receivers. The k -th pulse transmitted by the m -th transmitter and received by the n -th receiver can be written in baseband as

$$r_{nm}(t, k) = \sqrt{P_{nm}c_{nm}}s_m(t - kT_m - \tau_{nm}(k)) \times e^{j2\pi f_{nm}t} e^{-j2\pi f_c \tau_{nm}(k)} + w_n(t), \quad (1)$$

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where P_{nm} is a power parameter, c_{nm} the scattering coefficient, s_m the signal transmitted by the m -th transmitter, T_m the pulse repetition time, $\tau_{nm}(k)$ the pulse-dependent time delay, f_{nm} the Doppler shift, f_c the carrier frequency, and w_n is the noise and interference term. Any unknown oscillator phase terms can be included into c_{nm} . Denoting the transmitter position by $\mathbf{x}_m$, the receiver position by $\mathbf{x}_n$, the target position by $\mathbf{x}_b$, and the target velocity by $\mathbf{v}$ in 3-D space, the Doppler frequency can be written as

$$f_{nm} = \frac{1}{\lambda} \mathbf{v}^T \left(\frac{\mathbf{x}_b - \mathbf{x}_m}{\|\mathbf{x}_b - \mathbf{x}_m\|} + \frac{\mathbf{x}_b - \mathbf{x}_n}{\|\mathbf{x}_b - \mathbf{x}_n\|} \right), \quad (2)$$

where λ is the carrier wavelength.

We assume that the scattering coefficients c_{nm} are i.i.d. zero-mean circular complex Gaussian random variables with unit variance corresponding the Case 1 in the Swerling scattering model[12], as was discussed in [13]. We also assume that the noise $w_n(t)$ is temporally and spatially white with variance σ_w^2 , for the sake of simplicity.

Let us assume without loss of generality that the lowest pulse repetition frequency is $1/T_1$. Furthermore, there are K_1 pulses with this PRF that can be used for target parameter estimation. The number of pulses available from the m -th transmitter is then

$$K_m = \left\lfloor K_1 \frac{T_1}{T_m} \right\rfloor. \quad (3)$$

We assume that the transmitted signals have sufficient orthogonality properties and that the Doppler frequency has negligible effect on the waveforms so that matched filtering can be done with signal $s_m(t - pT_m - \hat{\tau}_{nm})e^{j2\pi f_{nm}t}e^{j2\pi f_c \hat{\tau}_{nm}}$. The estimated time delay $\hat{\tau}_{nm}$ can be obtained by dividing the entire search area into sufficiently small bins and testing if a target can be found in a particular bin. We assume that f_{nm} is obtained from a bank of matched filters, but it is not accurate enough to estimate the velocity of the target with sufficient precision.

In addition, it is assumed that

$$s_m(t + \hat{\tau}_{nm} - \tau_{nm}(k)) \approx s_m(t) \quad (4)$$

which means that the target remains in the same range bin over the observation period. The same does not hold for the phase of the pulse however, as the target is moving within the bin. Since the motion of the target is small during the period between two pulses compared to the distance to the transmitter and receiver, it is possible to write

$$\begin{aligned} & \hat{\tau}_{nm} - \tau_{nm}(k) \\ & \approx \frac{kT_m}{\lambda f_c} \left(\mathbf{v}^T \frac{\mathbf{x}_b - \mathbf{x}_m}{\|\mathbf{x}_b - \mathbf{x}_m\|} + \mathbf{v}^T \frac{\mathbf{x}_b - \mathbf{x}_n}{\|\mathbf{x}_b - \mathbf{x}_n\|} \right) \end{aligned} \quad (5)$$

by using the Taylor series of square root. This approximation can also be thought as only considering the movement of the

target along the line between the transmitter and the target as well as along the line between the target and the receiver. Thus,

$$e^{j2\pi f_c(-\tau_{nm}(k) + \hat{\tau}_{nm})} \approx e^{j2\pi kT_m f_{nm}}, \quad (6)$$

and the signal after matched filtering can be written as

$$y_{nm}(k) = \sqrt{P_{nm}} G c_{nm} e^{j2\pi f_{nm} k T_m} + \tilde{w}_{nm}(k). \quad (7)$$

G is a gain resulting from matched filtering. Due to the orthogonality of the filters, the filtered noise $\tilde{w}_n(t)$ is assumed to have the same statistics as the noise before the filtering.

Next, we stack the matched filter outputs from different MIMO branches into a column vector

$$\mathbf{y} = [y_{11}(0), y_{11}(1), \dots, y_{11}(K_1 - 1), y_{12}(0), \dots, y_{12}(K_2 - 1), \dots, y_{NM}(0), \dots, y_{NM}(K_M - 1)]^T.$$

This vector can be written as

$$\mathbf{y} = \sqrt{G} \mathbf{F} \mathbf{P}^{1/2} \mathbf{c} + \tilde{\mathbf{w}}, \quad (8)$$

where F is a matrix with $\sum_{m=1}^M K_m$ rows and MN columns defined as

$$\mathbf{F} = \begin{bmatrix} e^{j2\pi f_{11} 0 T_1} & 0 & \dots & \\ \vdots & & & \\ e^{j2\pi f_{11} (K_1 - 1) T_1} & 0 & \dots & \\ 0 & e^{j2\pi f_{12} 0 T_2} & 0 & \dots \\ \vdots & & & \\ 0 & \dots & e^{j2\pi f_{NM} (K_M - 1) T_M} & \end{bmatrix},$$

$$\mathbf{P} = \text{diag}(GP_{11}, GP_{12}, \dots, GP_{NM}),$$

$\mathbf{c}$ is a vector containing the scattering coefficients defined as

$$\mathbf{c} = [c_{1,1} \quad c_{1,2} \quad \dots \quad c_{N,M}]^T$$

and $\mathbf{n}$ is the filtered noise vector formed similarly as $\mathbf{y}$.

3. TARGET VELOCITY ESTIMATION

In this section, we derive a maximum likelihood estimator for the target velocity using the matched filter output vector.

Since both $\mathbf{n}$ and $\mathbf{c}$ are Gaussian, the matched filter output vector is distributed as

$$\mathbf{y} \sim \mathcal{CN}(0, \mathbf{F} \mathbf{P} \mathbf{F}^H + \sigma_w^2 \mathbf{I}). \quad (9)$$

As $\mathbf{P}$ consists of the filtering gain and line-of-sight propagation attenuation coefficients, we can assume it to be known

since we are dealing with a known location of the target. The log-likelihood function can be written as

$$\mathcal{L}(\mathbf{y}; \mathbf{v}) = -\mathbf{y}^H (\mathbf{F}\mathbf{P}\mathbf{F}^H + \sigma_w^2 \mathbf{I})^{-1} \mathbf{y} - \log \left[\pi \sum_m K_m \det(\mathbf{F}\mathbf{P}\mathbf{F}^H + \sigma_w^2 \mathbf{I}) \right]. \quad (10)$$

Looking at the covariance matrix, we can see that it is in fact block-diagonal:

$$\mathbf{F}\mathbf{P}\mathbf{F}^H + \sigma_w^2 \mathbf{I} = \begin{bmatrix} GP_{11}\mathbf{f}_{11}\mathbf{f}_{11}^H + \sigma_w^2 \mathbf{I} & & \\ & GP_{12}\mathbf{f}_{12}\mathbf{f}_{12}^H + \sigma_w^2 \mathbf{I} & \\ & & \ddots \end{bmatrix},$$

where

$$\mathbf{f}_{nm} = [e^{j2\pi f_{nm} 0 T_m} \quad \dots \quad e^{j2\pi f_{nm} (K_m - 1) T_m}]^T,$$

i.e. each block is a sum of a diagonal and rank-1 matrix. Because $\mathbf{f}_{nm}^H \mathbf{f}_{nm} = K_m$ as the modulus of each element in the vector equals one, the eigenvalues of this matrix are σ_w^2 and $K_m + \sigma_w^2$, $m = 1 \dots M$. Therefore, the determinant is independent of the target velocity $\mathbf{v}$.

Using the matrix inversion lemma, the inverse of the block-diagonal covariance matrix can be written as

$$(\mathbf{F}\mathbf{P}\mathbf{F}^H + \sigma_w^2 \mathbf{I})^{-1} = \frac{1}{\sigma_w^2} \begin{bmatrix} \ddots & & \\ & \mathbf{I} - \frac{GP_{nm}}{\sigma_w^2} \frac{\mathbf{f}_{nm}\mathbf{f}_{nm}^H}{1 + \sigma_w^{-2} GP_{nm} \mathbf{f}_{nm}^H \mathbf{f}_{nm}} & \\ & & \ddots \end{bmatrix}. \quad (11)$$

This simplifies the log-likelihood function, which can be written as

$$\mathcal{L}(\mathbf{y}; \mathbf{v}) = A - \frac{1}{\sigma_w^2} \sum_{m=1}^M \sum_{n=1}^N \|\mathbf{y}_{nm}\|^2 - \frac{GP_{nm} |\mathbf{f}_{nm}^H \mathbf{y}_{nm}|^2}{\sigma_w^2 + GP_{nm} K_m}, \quad (12)$$

where A is a constant and $\mathbf{y}_{nm}$ is a vector containing the matched filter output for the pulses from transmitter m to receiver n . Therefore, in order to get the ML estimate, it suffices to maximize

$$J(\mathbf{v}) = \sum_{m=1}^M \sum_{n=1}^N \frac{GP_{nm}}{\sigma_w^2 + GP_{nm} K_m} |\mathbf{f}_{nm}^H \mathbf{y}_{nm}|^2 \quad (13)$$

with respect to $\mathbf{v}$, i.e.

$$\hat{\mathbf{v}}_{ML} = \arg \max_{\mathbf{v}} J(\mathbf{v}). \quad (14)$$

The maximization can be understood as trying to find $\mathbf{v}$ such that the vectors $\mathbf{f}_{nm}$ are as parallel as possible to matched filter output vectors while giving more weight to the pulses

Table 1. Difference to the baseline SNR in dB for each Tx–Rx branch in the example.

	Tx 1	Tx 2	Tx 3
Rx 1	0	0.8522	3.5999
Rx 2	−1.3637	−0.5115	2.2362
Rx 3	−2.3924	−1.5402	1.2075
Rx 4	0.4458	1.2980	4.0457

with high SNR. This is a simpler and more intuitive solution than the one derived in [6] by considering the scattering coefficients $\mathbf{c}$ to be unknown deterministic parameters. The objective function J is nonconvex, but if SNR is sufficiently high, there is a single maximum within the unambiguous velocity range. A numerical example of using the ML estimate is shown in the next section.

4. NUMERICAL EXAMPLES

A numerical example demonstrating the use on the multiple PRFs and the maximum likelihood velocity estimate is shown in this section. This example uses the signal model for the matched filter output developed in Section 2 and the derived ML estimator to estimate the velocity of a single target on a 2-D plane.

In this example, the widely distributed MIMO radar systems consists of three transmitters located at $(0, 0)$, $(500, 500)$, and $(700, 2000)$. There are four receivers at $(-500, 200)$, $(0, 0)$, $(600, 0)$, $(700, 3000)$. The target is at $(-700, 2000)$ and its velocity is $(20, -20)$. All the transmitters use 0.5 GHz carrier frequency.

In a single PRF case, all the transmitters use a PRF of 1.2kHz and transmit eight pulses. In the multiple PRF case, the PRF of the second and third transmitters is twice as high doubling the number of samples available for estimation from these transmitters. The unambiguous range remains the same, however, as the PRF of the first transmitter is not changed. It was assumed that the path loss is inversely proportional to the distance squared. The SNR of signal from first transmitter received at the first receiver was chosen as the baseline, and the difference in SNR of each received signal to this baseline is shown in Table 1. Nelder–Mead simplex method available in Matlab was used to do the optimization given in (14) to get ML estimate for the velocity of the target.

Figure 1 shows the Mean square error of the velocity estimate. The MSE was calculated by averaging over 2000 independent trials for each SNR value. We have established the Cramér–Rao bound of an unbiased estimator for this problem but the derivation is omitted due to limited space. The CRB is also shown in the figure. It can be seen in Figure 1 that using multiple PRFs to increase the amount of samples available decreases the velocity estimation error. This benefit is seen in both the CRB and the MSE.

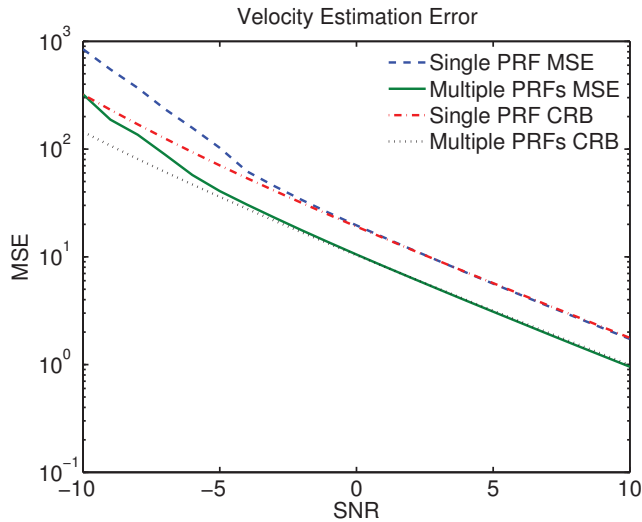


Fig. 1. Mean square error of the velocity estimate plotted with the Cramér–Rao bound. It can be seen that using multiple PRFs improves the actual estimate as well as the performance bound

5. CONCLUSIONS

In this paper, we have considered using multiple pulse repetition frequencies in a widely distributed MIMO radar to estimate the velocity of the target. In a MIMO radar, the different pulse repetition frequencies can be employed at the different transmitters without switching the pulse repetition time in a single transmitter. An advantage of this scheme is that number of pulses available for estimation can be increased without decreasing the unambiguous range or complicating detection or estimation methods.

Starting from the matched filter outputs, we derived the maximum likelihood estimator for the target velocity under independent scattering and spatially as well as temporally white noise. The derived estimator has an intuitive form and is computationally efficient.

Several simplifying assumptions were made in the signal model considered in this paper. A less simplified model should be used in future work. The impact of the multiple PRFs on the locating the target should also be addressed while considering joint detection and estimation.

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