CRAMÉR-RAO BOUNDS AND THEIR APPLICATION TO SENSOR SELECTION

Pietro Stinco[1], Maria Greco[1], Fulvio Gini[1] and Alfonso Farina[2]

[1]Department of "Ingegneria dell'Informazione", University of Pisa, Via G. Caruso 16, Pisa, 56122, Italy
[2]SELEX-Sistemi Integrati,Via Tiburtina Km 12, Rome, Italy

ABSTRACT

This work deals with the analysis of a multisensor radar system in the context of maritime border control scenario. The system is composed by a receiver that exploits the signal emitted by two non co-operative transmitters of opportunity: a UMTS base station and a FM radio station. An algorithm for selecting the transmitter for the tracking of a radar target is proposed. This algorithm can provide a significant aid to harbour protection and can reduce the computational load of surveillance operations.

***Index Terms*—** PCL, Multistatic, Tracking.

1. INTRODUCTION

In the last years multisensor systems have become very important for harbour protection and homeland security, for their improved system reliability and robustness, extended coverage, shorter response time, and improved resolution. The main mission of networking multiple sensor is to extend coverage and improve detection, tracking and identification of targets entering the region under surveillance. This can be done exploiting the different sources of illumination that are employed in the network and the spatial diversity provided by different channel of observation. The information gain obtained through this spatial diversity, combined with some level of distributed signal processing, can give rise to a number of advantages in typical radar functions. The multisensor radar system analyzed in this work is composed of two PCL (Passive Coherent Location) systems, that is, passive bistatic radars where the transmitter of opportunity is a non-radar transmitter (TX), such as broadcast or communications signals TXs. The analyzed system is a passive receiver in Leghorn harbour that exploits the signal emitted by a UMTS Base Station and a FM commercial radio station. This kind of system has some significant attractions because it can allow the use of parts of the RF spectrum (VHF/UHF) and, since broadcast transmissions at these frequencies can have substantial transmit powers, they can have excellent coverage. Furthermore, advantage should be drawn utilising the UMTS sources of illumination. In fact, the FM system could detect targets at relatively far range and use its fine Doppler resolution to build a crude track. Once the target appears within the coverage area of the UMTS system, this can be exploited for its superior range resolution properties. In this work we will evaluate the performance of each bistatic channel in estimating the target trajectory of a target approaching Leghorn harbour. We will also propose an algorithm for optimally selecting the transmitter of opportunity that can significantly improve the performance of the whole system.

2. PCL MULTISTATIC SCENARIO

The analyzed multistatic scenario is showed in Fig. 1 and is composed of one receiver and two transmitters. The receiver is placed in "via Michelangelo", between "darsena Petroli" and "darsena Pisa" of Leghorn harbour. Its antenna has a gain G of 10dB and an Half Power Beam Width (HPBW) of 3°. It exploits the signals emitted by two non co-operative transmitters. The first one is a UMTS Base Station located in "Piazzale Marmi", 453 m away the receiver in the South-East direction. The second transmitter is a FM commercial radio station located in "Monte Serra", 36 km away the receiver in the North-East direction. The complex envelop of the signal emitted by the UMTS transmitter has been modelled as a Quadrature Phase Shift Keying (QPSK) signal [1] with i.i.d. symbols. This signal is compliant with the Third Generation Partnership (3GPP) specifications [5]. The unitary energy QPSK signal is given by:

$$u_U(t) = \frac{1}{\sqrt{N_U}} \sum_{n=0}^{N_U-1} c_n g(t - nT_U) \tag{1}$$

where $g(t)$ is a RRC (Root Raised Cosine) with roll-off factor α=0.22, T_U=0.26μsec, c_n are the independent identical distributed QPSK symbols and $N_U T_U$ is the integration time fixed to 0.1 sec. The UMTS carrier frequency f_C is equal to 2100 MHz according to the Italian standards.

On the other hand, the unitary energy complex envelope of the FM signal has been modelled as [2]:

$$u_F(t) = \frac{1}{\sqrt{T_F}} e^{j\beta \sin(2\pi f_0 t + \pi/2)} rect\left(\frac{t}{T_F}\right) \tag{2}$$

that is a pulse whose instantaneous frequency is a sinusoidal oscillation. In fact, for observation times of the order of a second the radio content can be approximated as a sinusoidal oscillation. In (2), T_F is the observation time fixed to 0.5 sec, β is the modulation index equal to 5 and f_0 is the instantaneous frequency equal to 15 kHz. This value belongs to the range of audible frequency to model the program content of a FM radio which is speech and/or music.

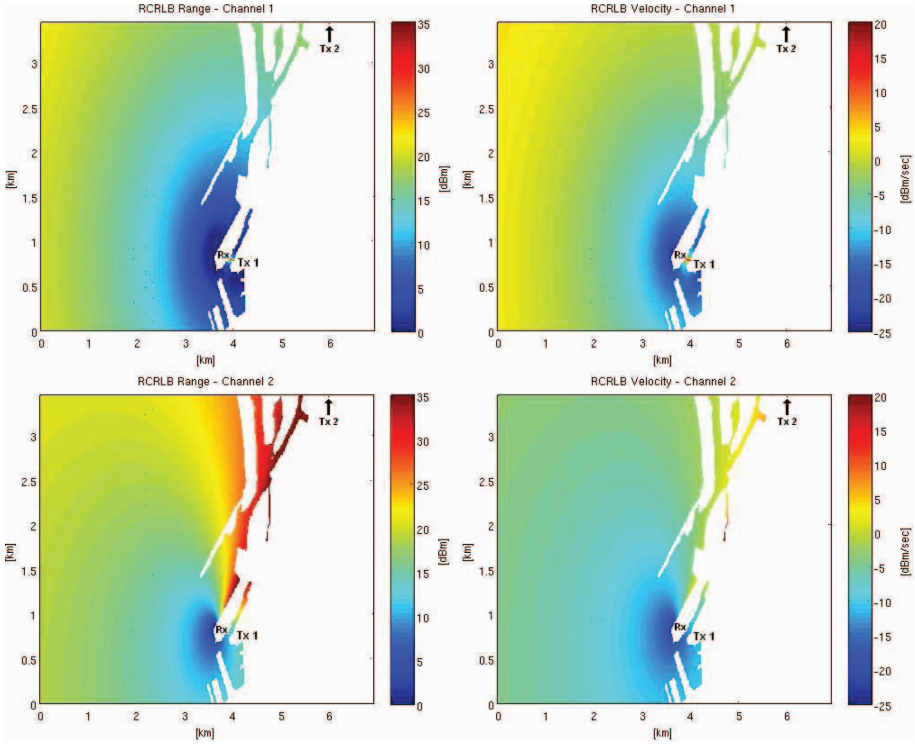

Figure 1 – RCRLB of range (left hand side) and bistatic velocity (right hand side) for the UMTS channel (top) and FM channel (bottom).

According to the Italian standards the FM carrier frequency f_C has been fixed to 100 MHz. To measure the possible global resolution of the target parameters estimates, the ambiguity function (AF) is often used. The AF is related to the accuracy on the estimation of target echo delay and its Doppler shift.

In [3] a relationship between the ambiguity function and the Fisher information matrix (FIM) was derived, based on the observation that the FIM is derived by the received data log-likelihood function (LLF) and the AF is the LLF.

The inverse of the Fisher information is the Cramér-Rao Lower Bound (CRLB), which bounds the error variance of the estimates obtained from the radar measurements.

In [3] it is shown that the CRLB is dependent on both the SNR (Signal to Noise Ratio) and the second derivatives of the AF, that is, the sharpness of the AF mainlobe in its maximum.

By these considerations and using the results showed in [1],[2], the FIM elements of the UMTS signal are given by:

$$\begin{aligned}
[\mathbf{J}_{UMTS}]_{1,1} &= 2\mathrm{SNR}\frac{\pi^2+3\alpha^2\pi^2-24\alpha^2}{3T^2}\\
[\mathbf{J}_{UMTS}]_{2,2} &= 2\mathrm{SNR}\left(\frac{\pi^2T^2}{4\alpha}+\frac{\pi^2T^2\left(N^2-1\right)}{3}\right)\\
[\mathbf{J}_{UMTS}]_{1,2} &= [\mathbf{J}_{UMTS}]_{2,1} = 0
\end{aligned} \tag{3}$$

On the other hand, the FIM elements of the FM signal are:

$$\begin{aligned}
[\mathbf{J}_{FM}]_{1,1} &= 2\mathrm{SNR}\frac{\beta^2 2\pi f_0}{T}\left[2\pi f_0 T+\sin\left(2\pi f_0 T\right)\right]\\
[\mathbf{J}_{FM}]_{2,2} &= 2\mathrm{SNR}\frac{\pi^2T^2}{3}\\
[\mathbf{J}_{FM}]_{1,2} &= -2\mathrm{SNR}\frac{2\beta}{Tf_0}\left[\pi f_0 T\cos\left(\pi f_0 T\right)-\sin\left(\pi f_0 T\right)\right]
\end{aligned} \tag{4}$$

The diagonal elements of the inverse of the FIM are the CRLBs of the time-delay and the Doppler shift of the target echo. In monostatic configuration, estimation of the time-delay and Doppler shift directly provides information on target range and velocity. This information can be retrieved also in a bistatic radar configuration, even if the relations between measured delay and Doppler frequency and target distance and velocity, respectively, are not linear and depend on the geometry [2]. These relationships are given by:

$$\tau_k = \frac{r_k+\sqrt{r_k^2+L^2+2r_kL\sin\theta_k}}{c}, \tag{5}$$

$$\xi_k = 2\frac{f_C}{c}v_k\sqrt{\frac{1}{2}+\frac{r_k+L\sin\theta_k}{2\sqrt{r_k^2+L^2+2r_kL\sin\theta_k}}} \tag{6}$$

where, c is the speed of light, τ_k is the time delay, ξ_k is the Doppler shift, r_k is the range from receiver to target, L is the baseline between the transmitter and the receiver, θ_k is the look angle of the receiver and v_k is the bistatic velocity. Note that in eq. (5) and (6) we highlight the dependence of the parameters on the time index k. In fact they depend on the bistatic geometry that changes while the target is moving along its trajectory. Using the derivative chain rule [2], it is possible to demonstrate that the FIM of the range and bistatic velocity is given by the equation $\mathbf{R}_k^{-1}=\mathbf{P}_k\mathbf{J}_M\mathbf{P}_k^T$ where $\mathbf{R}_k$ is the CRLB matrix of the range and velocity, also referred as the measurements covariance matrix. Matrix $\mathbf{P}_k$ is defined as:

$$\mathbf{P}_k=\begin{bmatrix} \frac{\partial \tau_k}{\partial r_k} & \frac{\partial \xi_k}{\partial r_k} \\ \frac{\partial \tau_k}{\partial v_k} & \frac{\partial \xi_k}{\partial v_k} \end{bmatrix} \quad (7)$$

and takes into account only the effect of the bistatic geometry. The matrix $\mathbf{J}_M$ is the FIM defined in (3) and (4) that depends only on the transmitted waveform and the SNR. The SNR is given by the bistatic radar equation:

$$\mathrm{SNR}_k=\frac{GP_T\sigma^2c^2}{N_0 f_C^2(4\pi)^3\left(r_k r_{t_k}\right)^2} \quad (8)$$

where r_{t_k} is the transmitter to target range, N_0 is the noise power, σ^2 is the RCS set to 10 m^2 and P_T is the transmitted power equal to 20dBW for the UMTS channel and to 40dBW for the FM channel. Note that the SNR depends on the geometry through the energy pass loss factor $(r_k r_{t_k})^2$ due to propagation. Figure 1 shows the Root Cramér-Rao lower bounds (RCRLBs) of target range and bistatic velocity, that is the root of the diagonal elements of $\mathbf{R}_k$. At the top of Figure 1 the RCRLBs of the UMTS channel are showed, while at the bottom the RCRLBs of the FM channel. From the results it is clear that the performance in measuring the target range and the bistatic velocity strongly depends both on the geometry and the transmitted waveform. It is also interesting to note that the performance in estimating the target velocity is better in the FM channel due to the longer time of observation. On the other hand, the range resolution is better in the UMTS channel. Note also that the FM channel operates with very high transmitted power giving a better coverage and appreciable resolution also in the far range.

3. SEQUENTIAL CRAMER RAO LOWER BOUNDS

This section deals with the Sequential Cramér-Rao Lower Bounds (SCRLB) for the recursive target state estimates produced from the radar measurements. The target trajectory is described by the state vector $\mathbf{x}_k=[x_k,\ \dot{x}_k,\ y_k,\ \dot{y}_k]^T$, where (x_k, y_k) is the position of the target at time k, while $(\dot{x}_k,\ \dot{y}_k)$ are the velocity components along the main axes of the coordinate system. Assuming that the target is moving with constant velocity and that the evolution of the state vector is deterministic, it is possible to write $\mathbf{x}_{k+1}=\mathbf{F}\mathbf{x}_k$, where:

$$\mathbf{F}=\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

and T is the radar sampling interval.

The objective of target tracking is to estimate recursively the target state from a set of measurement $\mathbf{z}_k$ that, as showed in precious section, are the range from receiver to target and the bistatic velocity. The measurement equation can be put in the following vectorial form:

$$\mathbf{z}_k=\mathbf{h}(\mathbf{x}_k)+\mathbf{w}_k \quad (10)$$

where $\mathbf{h}$ is a known function of the state vector and $\mathbf{w}_k$ is a measurement noise sequence. Assuming that the receiver is at the origin of the coordinate system, the function $\mathbf{h}$ is given by $\mathbf{h}(\mathbf{x}_k)=[r_k, v_k]^T=[h_r(\mathbf{x}_k), h_v(\mathbf{x}_k)]^T$ where:

$$h_r(\mathbf{x}_k)=\sqrt{x_k^2+y_k^2}\ ,\ h_v(\mathbf{x}_k)=\frac{\tilde{x}_k\dot{x}_k+\tilde{y}_k\dot{y}_k}{\sqrt{\tilde{x}_k^2+\tilde{y}_k^2}}, \quad (11)$$

$$(\tilde{x}_k,\tilde{y}_k)=\frac{L(x_k,y_k)+r_k(x_T,y_T)}{L+r_k+rt_k}-(x_k,y_k) \quad (12)$$

and (x_T, y_T) are the coordinate of the transmitter.

As apparent from (10), the bistatic measurements are affected by additive noise $\mathbf{w}_k$ that is assumed to be Gaussian with zero mean and covariance matrix $\mathbf{R}_k$.

Let indicate with $\mathbf{J}_k$ the filtering information matrix of the state vector at time k. Its inverse is the SCRLB that bounds the error variance of the target state estimate at time k, that is $\mathbf{J}_k^{-1}\leq \mathrm{E}\{(\hat{\mathbf{x}}_{k|k}-\mathbf{x}_k)(\hat{\mathbf{x}}_{k|k}-\mathbf{x}_k)^T\}$ where $\hat{\mathbf{x}}_{k|k}$ is an unbiased estimator of the state vector based on all the available measurements up to time k. In [4], the authors provided a method of recursively computing matrix $\mathbf{J}_k$ using the following equation:

$$\mathbf{J}_{k+1}=\left[\mathbf{F}^{-1}\right]^T\mathbf{J}_k\mathbf{F}^{-1}+\mathbf{H}_{k+1}^T\mathbf{R}_{k+1}^{-1}\mathbf{H}_{k+1} \quad (13)$$

where $\mathbf{H}_{k+1}$ is the Jacobian of $\mathbf{h}(\mathbf{x}_{k+1})$ evaluated at the true state $\mathbf{x}_{k+1}$; that is:

$$\mathbf{H}_{k+1}=\begin{bmatrix} \frac{\partial h_r}{\partial x_{k+1}} & \frac{\partial h_r}{\partial \dot{x}_{k+1}} & \frac{\partial h_r}{\partial y_{k+1}} & \frac{\partial h_r}{\partial \dot{y}_{k+1}} \\ \frac{\partial h_v}{\partial x_{k+1}} & \frac{\partial h_v}{\partial \dot{x}_{k+1}} & \frac{\partial h_v}{\partial y_{k+1}} & \frac{\partial h_v}{\partial \dot{y}_{k+1}} \end{bmatrix} \quad (14)$$

The recursion in (13) starts with the initial FIM $\mathbf{J}_0$ that, assuming the initial distribution of $\mathbf{x}_0$ is Gaussian, is equal to the inverse of the covariance matrix of $\mathbf{x}_0$.

The recursive equation in (13) is the sum of two terms: the first one $[\mathbf{F}^{-1}]^T\mathbf{J}_k^{-1}\mathbf{F}^{-1}$ is the a priori information given by the previous target state, while $\mathbf{H}^T_{k+1}\mathbf{R}^{-1}_{k+1}\mathbf{H}_{k+1}$ is the information gained by the radar measurements.

4. CHANNEL SELECTION ALGORITHM

As showed, the SCRLB depends on target trajectory, sensor accuracy, transmitted waveform and the bistatic geometry. This dependence is given by the measurement information term that depends on the measurement covariance matrix $\mathbf{R}_k$. Figure 1 shows for each point of the surveillance area the range and velocity resolutions of each bistatic channel. Using this information, it is possible to select, for each point of the analyzed area, the channel having the best performances. Figure 2 shows the channel which has the better resolution for each point of the analyzed area for range and velocity measurements. The scale of this figure is quantized into 2 levels, level 1 refers to the UMTS channel while level 2 to the FM channel. In Figure 2 we also plotted a red line that represents the trajectory of a ship approaching the harbour. The ship is moving with constant velocity equal to 3 m/sec. Using (13) it is possible to calculate the SCRLBs of each bistatic channel. The obtained results are showed in Figure 3. Knowing the transmitter positions, the receiver is able to calculate for each point of the target trajectory the measurement error covariance matrix of each channel and therefore it is able to evaluate the channel with the best performance. A common operator that can be used to evaluate the performance of each channel is the determinant of $\mathbf{R}_k$, the lower the determinant the better the performance. Being the target trajectory deterministic, it is easy to verify that the SCRLBs of the receiver that dynamically select the best channel are given by:

$$\mathbf{J}_{k+1} = \left[\mathbf{F}^{-1}\right]^T \mathbf{J}_k \mathbf{F}^{-1} + \mathbf{J}_{k+1}^Z \qquad (15)$$

where

$$\mathbf{J}_{k+1}^Z = \begin{cases} \left[\mathbf{H}_{k+1}^{(1)}\right]^T \left[\mathbf{R}_{k+1}^{(1)}\right]^{-1} \mathbf{H}_{k+1}^{(1)} & \det\left(\mathbf{R}_{k+1}^{(1)}\right) < \det\left(\mathbf{R}_{k+1}^{(2)}\right) \\ \left[\mathbf{H}_{k+1}^{(2)}\right]^T \left[\mathbf{R}_{k+1}^{(2)}\right]^{-1} \mathbf{H}_{k+1}^{(2)} & \det\left(\mathbf{R}_{k+1}^{(1)}\right) > \det\left(\mathbf{R}_{k+1}^{(2)}\right) \end{cases} \qquad (16)$$

that is, at each step k+1 the measurement information term $\mathbf{J}^Z_{k+1}$ is given by selecting the channel with the lowest determinant of the measurement covariance matrix. In (16) the upper index (1) refers to the UMTS channel while the index (2) to the FM channel. The SCRLBs of the receiver that select the best transmitter are also shown in Figure 3. From the results it is evident that there is a substantial gain with respect to each bistatic channel and the obtained performance is equal or one order of magnitude better than the performance of the channel with the lowest SCRLB. As apparent, initially the performance are the same as the FM channel. This is due to the fact that in the far range the FM channel has the best performance thanks to the higher SNR. When the target approaches the harbour, the UMTS channel has a better resolution and, exploiting this channel, the proposed receiver is able to increase the performance in estimating the target trajectory.

5. REFERENCES

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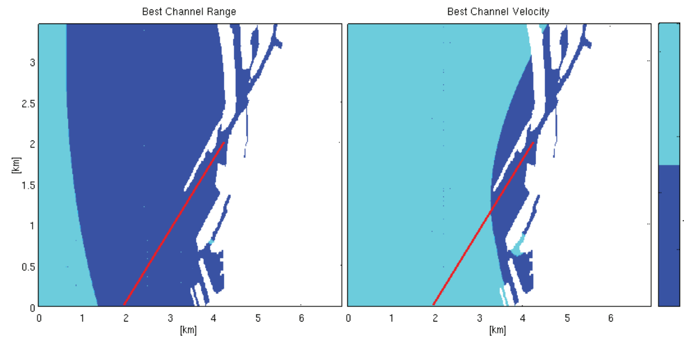

Figure 2 – Channel with the best resolution and target trajectory

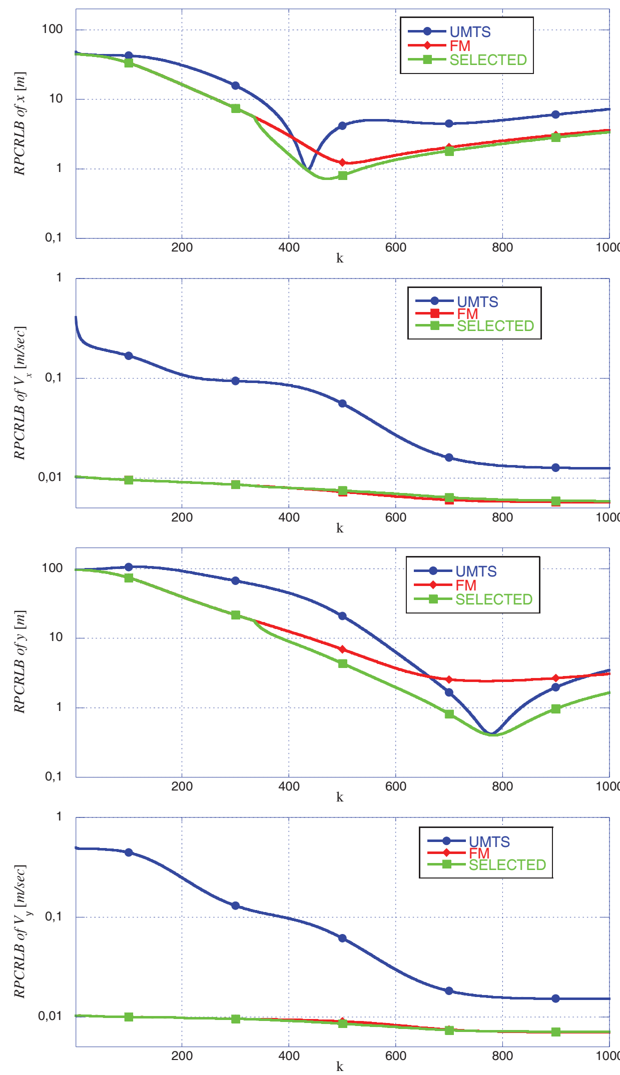

Figure 3 - Root of the SCRLB of target state. UMTS channel, FM channel and dynamic selection channel.