

PRICE THEORY FRAMEWORK FOR TARGET TRACKING USING MULTI-MODAL SENSORS

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ABSTRACT

We propose a unified framework for jointly solving the problems of sensor selection (SS), resource allocation (RA) and data fusion (DF) in multi-modal sensor networks. Our framework is inspired by the trading behavior in commercial markets. We develop an iterative double-auction mechanism for computing the equilibrium of the market. The equilibrium point corresponds to the joint solution of SS, RA, DF problems. To illustrate the framework, we consider a scenario where the objective is to track an unknown variable number of moving targets of different categories, using a sensor network comprising of a multistatic radar, infrared camera, and a human scout. Numerical examples demonstrate the effectiveness of the proposed method.

Index Terms— multi-modal sensors, tracking, resource allocation, sensor selection, data fusion, price theory, auctions

1. INTRODUCTION

The development of surveillance systems for unknown environments has been an active area of research in the past few years. The ability to track unknown number of moving targets is essential in several military and commercial applications such as air traffic control and battle-field surveillance. The problem of multitarget tracking [1] poses several major challenges. The number of targets in the region of interest is unknown and the sensing system should be capable of automatically determining this number. Also, not all the targets can be observed by the sensing system at all the times. Tracking systems that rely on a single sensor cannot overcome these difficulties and they are vulnerable to errors. In order to improve the performance of the overall sensing system, multiple sensors of different modalities can be employed [2]. When the complementary information from different modalities is appropriately combined, the performance of the overall system improves significantly compared to the performance of each modality separately. However, employing multi-modal sensors results in other problems.

First, the sensing system should understand how to combine the diverse and sometimes inconsistent information provided by multiple sensors. This is called data fusion [3] (DF)

problem. Second, the system must efficiently manage its resources in a dynamic and an adaptive way [4]. This consideration gives rise to the sensor selection (SS) and the resource allocation (RA) problems. All the three problems have been studied independently in the past, and several methods and techniques have been proposed to solve them. However, a unified framework for jointly obtaining the solutions to all the three problems has been lacking. In this paper, we attempt to provide the framework for obtaining a joint solution to these problems.

Our framework for solving the SS, RA, DF problems is inspired by the trading behavior of agents in a commercial market and we use an economic price theory based approach [5]. We model each sensor as a producer, who wishes to sell the data it produced, and the sensing system as a consumer who wants to buy the data from the sensors, in order to maximize its utility. The producers and consumers will then interact among themselves to reach an equilibrium. We propose an iterative double auction algorithm to find the equilibrium of the market. We then show how the equilibrium point can be related to the solutions of SS, RA and DF.

2. PRICE THEORY

2.1. Walrasian Equilibrium

Price theory is a branch of economics that explains the trade of goods and services between different economic agents [5]. Agents fall into two different categories; consumers and producers. Consumers can buy and sell various goods in the market whereas producers can transform goods of some sort into goods of a different sort. Consider an economic market with N agents and K indivisible goods. Out of the N agents, N_c are the consumers and $N_p = N - N_c$ are the producers. For each consumer i , the preference for consuming various combinations of goods is specified by a utility function, $u_i : \mathbb{N}^K \rightarrow \mathbb{R}$. We define the vector consisting of various combinations of the goods as the demand vector and denote it using x_i , with x_{ik} representing the amount of k^{th} good that i^{th} consumer buys or sells. If $x_{ik} > 0$, then the consumer buys the good and if $x_{ik} < 0$, the consumer sells the good. Each consumer starts with an initial endowment of goods e_i , with e_{ik} representing the quantity of k^{th} good available for trade with the i^{th} consumer. Given a price vector $p = [p_1, \dots, p_K]^T$, the objective of the consumer is to find a demand vector that maximizes his utility such that

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the total wealth he spends is less than the total wealth that the consumer can generate by selling his initial endowment. Thus, the consumer's demand can be obtained by solving the following constrained optimization problem:

$$\mathbf{x}_i^* = \arg \max_{\mathbf{x}_i \in B_i(\mathbf{p}, \mathbf{e}_i)} u_i(\mathbf{x}_i)$$

where $B_i(\mathbf{p}, \mathbf{e}_i) = \{\mathbf{x}_i \in \mathbb{N}^K : \mathbf{p}^T \mathbf{x}_i \leq \mathbf{p}^T \mathbf{e}_i\}$ (1)

Agents of the second type, the producers, will take as input goods from consumers and convert them into goods of different sort. For each producer j , a vector $\mathbf{y}_j = [y_{j1}, y_{j2}, \dots, y_{jK}]^T$ called the production plan vector, where $y_{jk} > 0$ if k^{th} good is an output and $y_{jk} < 0$ if it is an input, defines the amount of goods that the producer takes as input and produces as output. The maximum amount of k^{th} good produced by the j^{th} producer will be a function of his input goods and the available technology to produce the good. This is represented by a production function, $v_{jk} : \mathbb{N}^K \rightarrow \mathbb{N}$. Each producer has an initial wealth w_j required to start the production. Given a price vector $\mathbf{p} = [p_1, \dots, p_K]^T$, the objective of the producer is to choose a production plan vector that maximizes his profit, subject to the constraints on the maximum amount of goods that he can produce. Define $\mathbf{y}_j^+ = \{y_{jk} \in \mathbf{y}_j | y_{jk} > 0\}$ and $\mathbf{y}_j^- = \{y_{jk} \in \mathbf{y}_j | y_{jk} < 0\}$. The optimal production plan vector can be obtained by solving the following constrained optimization problem:

$$\mathbf{y}_j^* = \arg \max_{\mathbf{y}_j \in \mathbb{N}^K} \mathbf{p}^T \mathbf{y}_j$$

s. t. $y_{jk}^+ \leq v_{jk}(\mathbf{y}_j^-)$ and $(\mathbf{p}_j^-)^T \mathbf{y}_j^- \leq w_j \quad \forall j, k$, (2)

where $\mathbf{p}_j^- = \{p_k \in \mathbf{p} | y_{jk} < 0\}$.

Walrasian Equilibrium : The tuples $(\{\mathbf{x}_i^*\}_{i=1}^{N_c}, \{\mathbf{y}_j^*\}_{j=1}^{N_p}, \mathbf{p}^*)$ of the demand vector, production plan vector and the price vector in an economy form a Walrasian equilibrium if and only if: (i) \mathbf{x}_i is a solution to the constrained optimization problem given in Eq. (1) at price \mathbf{p} ; (ii) \mathbf{y}_j is a solution to the constrained optimization problem given in Eq. (2) at price \mathbf{p} ; and (iii) the market is clear at price \mathbf{p} , i.e., $\sum_{i=1}^{N_c} x_{ik} = \sum_{j=1}^{N_p} y_{jk} \forall k$. Under some mild assumptions on the continuity and the monotonicity of the utility and production functions, it was shown that the Walrasian equilibrium exists for all economies [6] using a fixed point argument. The key result of the price theory is that the Walrasian equilibria, although defined as a solution to utility maximization and the profit maximization problems of individual agents, will produce *Pareto-optimal* allocations. This result is called the first fundamental welfare theorem [5].

The market equilibrium problem is to compute a price vector, the corresponding demand vectors and the production plan vectors for all the agents in the economy such that they form a Walrasian equilibrium. However, finding a computationally efficient polynomial time algorithm to compute the equilibrium prices and allocations for a general economy is still an open problem and forms a major research area. Auctions are a class of algorithms that are useful for finding the approximate equilibrium.

Table 1. Finding Walrasian Equilibrium using Auctions

Iterative CDA Algorithm

Initialize $p_k^1 = 0, \forall k$, and $n = 1$
Repeat until ϵ -approximate equilibrium is satisfied or $n > N_{th}$
 $\forall i, j$, find \mathbf{x}_i^n and \mathbf{y}_j^n by solving Eqs. (1) and (2), respectively
if $(\sum_{i=1}^{N_c} x_{ik}^n > \sum_{j=1}^{N_p} y_{jk}^n)$
set $p_k^{n+1} = p_k^n (1 + \delta p_k^n |(\sum_{i=1}^{N_c} x_{ik}^n - \sum_{j=1}^{N_p} y_{jk}^n)|)$
else if $(\sum_{i=1}^{N_c} x_{ik}^n < \sum_{j=1}^{N_p} y_{jk}^n)$
set $p_k^{n+1} = p_k^n / (1 + \delta p_k^n |(\sum_{i=1}^{N_c} x_{ik}^n - \sum_{j=1}^{N_p} y_{jk}^n)|)$
else set $p_k^{n+1} = p_k^n$

2.2. Auctions and Price discovery

Auction algorithms originated first as a method for finding solutions to an assignment problem where several agents were competing for various resources [7]. Since then, auctions have been used for solving a variety of problems in the areas of computer science, Economics and finance. Combinatorial double auctions (CDA) [8] are a class of auction algorithms where both buyers and sellers can place bids on combinations of goods that they want to buy or sell, instead of being limited to bidding on single item. We present an iterative CDA mechanism that can be used to achieve an approximate Walrasian equilibrium for the market scenario described in the previous section. The market is called ϵ -approximate if for $0 < \epsilon < 1$, the equilibrium solution $(\{\mathbf{x}_i^*\}_{i=1}^{N_c}, \{\mathbf{y}_j^*\}_{j=1}^{N_p}, \mathbf{p}^*)$ is such that $\sum_{i=1}^{N_c} x_{ik}^* = (1 - \epsilon) \sum_{j=1}^{N_p} y_{jk}^* \forall k$, i.e., the market clearing condition is approximately satisfied. The detailed algorithm is shown in Table 1. We continue the iterations until the approximate market clearing condition is satisfied for an initially chosen value of ϵ , or the number of iterations exceed a pre-determined threshold. The algorithm outputs the final demand vectors, production plan vectors, and the equilibrium prices.

3. PROPOSED FRAMEWORK

In this section, we propose our framework for jointly solving the RA, SS and DF problems. We will illustrate the framework using an example where the goal is to track the positions, velocities and categories of an unknown number of targets, moving in a region of interest. Although the description is specific to the example, the framework can be easily extended to other scenarios. We describe the state-space model first, and then the measurement model for the sensors.

3.1. Multiple Target State-Space Model

We assume that the two-dimensional region of interest, $\mathcal{R} \subset \mathbb{R}^2$, has multiple moving targets which belong to different categories. At each time t , we are interested in estimating the number of targets $N_t \in \mathbb{N}$, their categories $\alpha_t \in \mathcal{A}^{N_t}$, their positions $\boldsymbol{\rho}_t = [\rho_{t1}^T, \rho_{t2}^T, \dots, \rho_{tN_t}^T]^T \in \mathbb{R}^{2 \times N_t}$ and their velocities $\dot{\boldsymbol{\rho}}_t = [\dot{\rho}_{t1}^T, \dots, \dot{\rho}_{tN_t}^T]^T \in \mathbb{R}^{2 \times N_t}$. Hence, the state vector

at time t is $\theta_t = [N_t, \alpha_t^T, \rho_t^T, \dot{\rho}_t^T]^T$. For simplicity, we assume that there can be at most one birth or one death at each state transition, and represent the probabilities of death and birth of a target using p_d and p_b , respectively. We have

$$p(N_{t+1}|N_t = n_t) = \begin{cases} p_b & \text{if } N_{t+1} = n_t + 1, \\ n_t p_d (1 - p_d)^{n_t - 1} & \text{if } N_{t+1} = n_t - 1, \\ 1 - p_b - n_t p_d (1 - p_d)^{n_t - 1} & \text{if } N_{t+1} = n_t. \end{cases} \quad (3)$$

Let $\alpha_t = \{\alpha_{t,1}, \alpha_{t,2}, \dots, \alpha_{t,N_t}\}$ denote the categories of each of the N_t targets, and assume that the number of categories is finite, i.e., $\text{card}(\mathcal{A}) = M < \infty$. Then for $\alpha_{t,i} \in \alpha_t$, and $\alpha^* \in \mathcal{A}$, the state transition for α_{t+1} given α_t and N_{t+1} can be written as

$$p(\alpha_{t+1}|\alpha_t, N_{t+1}) = \begin{cases} \frac{1}{M} & \text{if } N_{t+1} = N_t + 1, \alpha_{t+1} = \alpha_t \cup \alpha^*, \\ \frac{1}{N_t} & \text{if } N_{t+1} = N_t - 1, \alpha_{t+1} = \alpha_t - \alpha_{t,n_t}, \\ \frac{p_d p_b}{N_t M} & \text{if } N_{t+1} = N_t, \alpha_{t+1} = \alpha_t - \alpha_{t,n_t} \cup \alpha^*, \\ 1 - \frac{p_d p_b}{N_t M} & \text{if } N_{t+1} = N_t, \alpha_{t+1} = \alpha_t. \end{cases} \quad (4)$$

For a target $n_t \in \alpha_{t+1} \cap \alpha_t$, let $\xi_{t+1,n_t} = [\rho_{t+1,n_t}^T, \dot{\rho}_{t+1,n_t}^T]^T$. Then, given ξ_{t,n_t} , we have

$$p(\xi_{t+1,n_t}|\xi_{t,n_t}) = \mathcal{N}(\xi_{t+1,n_t}; F_{n_t} \xi_{t,n_t}, \Sigma_{n_t}), \quad (5)$$

where F_{n_t} is the state transition matrix and Σ_{n_t} is the covariance matrix of the process noise. The overall state transition can be obtained by multiplying these individual transitions.

$$p(\theta_{t+1}|\theta_t) = p(\xi_{t+1}^*) p(\alpha_{t+1}|\alpha_t, N_{t+1}) p(N_{t+1}|N_t) \prod_{n_t \in \alpha_{t+1} \cap \alpha_t} p(\xi_{t+1,n_t}|\xi_{t,n_t}), \quad (6)$$

where $p(\xi_{t+1}^*)$ is the probability density of the new targets initiated at time $t + 1$.

3.2. Measurement Model

We consider a multi-modal sensor network with three types of sensors; a multistatic radar system with one transmit and three receive antennas, an infrared camera and an intelligence report provided by a human scout. At time t , let $Y_t = \{y_t^{\text{rad}}, Y_t^{\text{ir}}, y_t^{\text{hs}}\}$, where $y_t^{\text{rad}}, Y_t^{\text{ir}}, y_t^{\text{hs}}$ are the measurements from the radar, infrared camera and the human scout, respectively. We briefly describe the statistical models for these measurements.

Multistatic Radar: We use the standard model for the received radar signal in which the measured signal is a delayed and Doppler shifted version of the transmitted signal. The likelihood of the received signal at the p^{th} receive antenna $p = 1, 2, 3$ at time t , given the state of the system can be expressed as

$$p(y_{p,t}^{\text{rad}}|\theta_t) = \mathcal{CN}(y_{p,t}^{\text{rad}}; \Phi_{p,t} \beta_{p,t}, \Sigma_{p,t}^{\text{rad}}), \quad (7)$$

where the matrix $\Phi_{p,t}$ has the information about the delay and Doppler of the targets, $\beta_{p,t}$ is a vector of target scattering coefficients, which is a function of target category, and $\Sigma_{p,t}^{\text{rad}}$

is the covariance matrix of the measurement noise.

Infrared Camera: The output Y_t^{ir} of an infrared camera, a matrix of pixel values, is modeled as a noisy version an ideal image I_0 convolved with the point-spread function of the camera. For simplicity, we assume the point-spread function to be a delta function. The likelihood can then be expressed as

$$p(Y_t^{\text{ir}}|N_t, \alpha_t, \rho_t) = \mathcal{N}(Y_t^{\text{ir}}; I_0, \Sigma^{\text{ir}} \otimes \Sigma^{\text{ir}}), \quad (8)$$

where I_0 is the ideal image with $I_0(x, y) = T_{n_t}$, if the n_t^{th} target is present at (x, y) and is 0 otherwise, and Σ^{ir} is the variance matrix of the measurement noise. In the above T_{n_t} is a constant that depends on target category.

Human scout: The measurement report given by the scout is an M -dimensional vector with its m^{th} entry giving the count of number of targets of type m . The total number of targets at time t as counted by the scout is given by $N_t^{\text{hs}} = \sum_{m=1}^M y_{t,m}^{\text{hs}}$. Let p_c be the probability that the scout counts at least one target incorrectly, and N_{max} be the upper bound on N_t^{hs} . We obtain the probability masses for N_t^{hs} , denoted $g(N_t^{\text{hs}})$, by evaluating a Gaussian density of mean N_t and variance σ_t^2 and then normalizing. We use the following likelihood for scout's measurement report.

$$p(y_t^{\text{hs}}|\alpha_t, N_t) = \begin{cases} (1 - p_c) + p_c [g(N_t^{\text{hs}}) \frac{N_t^{\text{hs}}!}{y_{t,1}^{\text{hs}}! \dots y_{t,M}^{\text{hs}}!} q_{1,t}^{\text{hs}} \dots q_{M,t}^{\text{hs}}] & \text{if } y_t^{\text{hs}} \sim \alpha_t, \\ p_c \sum_{k=1}^{N_{\text{max}}} g(k) \frac{k!}{y_{t,1}^{\text{hs}}! \dots y_{t,M}^{\text{hs}}!} q_{1,t}^{\text{hs}} \dots q_{M,t}^{\text{hs}} & \text{else,} \end{cases} \quad (9)$$

where \sim refers to a case where the measurement y_t^{hs} is such that all the targets are identified correctly, according to α_t . The parameters $(q_{1,t}, \dots, q_{M,t})$ depend on the true value of the vector α_t .

3.3. Target Tracking Algorithm

We use a standard particle filter, which computes a discrete weighted approximation to the true posterior density using a set of particles, that characterize the posterior probability distribution. We generate M_s particles, $\{\theta_t^s\}_{s=1}^{M_s}$, of the state vector θ_t using importance sampling where the importance function was chosen to be the transitional prior described in Sec. 3.1. The associated weights corresponding to j^{th} sensor are updated using $w_{j,t}^s \propto w_{j,t-1}^s p(y_{j,t}^s|\theta_t^s)$.

3.4. Sensor Management

We consider each sensor as a producer in the market and the tracker as a consumer. The goods correspond to the data collected by each of the sensors and the power allocated to each sensor by the tracker. The demand vector of the tracker corresponds to the number of measurements that the tracker seeks from each sensor and the total power that it would like to allocate to the sensors. If $x_k = 0$ for some k , then the tracker does not request any measurements from the k^{th} sensor. The utility function of the tracker quantifies the uncertainty reduced

by the observations made up to time $t - 1$. We consider the following utility function at time t

$$u(\mathbf{x}) = \sum_{j=1}^5 \frac{1}{d_j} x_j \sum_{s=1}^{M_s} w_{j,t-1}^{(s)} \log(w_{j,t-1}^{(s)}), \quad (10)$$

where d_j is the dimension of the subspace that the j^{th} sensor observes. The production function for each producer defines the number of measurements that each sensor can take as a function of the power allocated to it. We consider a linear function for each producer.

$$v_j(y_j^-) = c_j y_j^-, \quad (11)$$

where $v_j(y_j^-)$ represents the maximum number of measurements that j^{th} sensor can obtain and y_j^- is the power allocated to the j^{th} sensor. With these definitions, the multi-modal sensor network is similar to the market described in Sec. 2. We use the algorithm described in Table 1 to obtain an equilibrium solution $(\{\mathbf{x}^*\}, \{\mathbf{y}_j^*\}_{j=1}^{N_p}, \mathbf{p}^*)$ to this market. In this equilibrium solution, \mathbf{x}^* is the solution to the SS problem, $(\mathbf{y}_j^*)^-$ is the solution to the RA problem, and \mathbf{p}^* is the solution to the DF problem. After \mathbf{p}^* is known, we compute the overall likelihood of the data by combining the individual likelihoods in a linear fashion according to the price of the data.

4. NUMERICAL EXAMPLES

In this section, we use numerical examples to demonstrate the performance improvement obtained due to the proposed framework. We evaluated the performance of the system using four metrics; the average number of targets detected in the scene (ATD), the average number of targets identified incorrectly (ATII), and the root mean-squared error in the position (RMSE-range) and velocity (RMSE-velocity) estimates of correctly identified targets. In Fig. 1, we plot the four metrics for two methods. In the first method, the standard approach, we used all sensors at all times and equally distributed the power among them. We computed the global posterior as a linear combination of local posteriors at each modality. In the second method, the price theory approach, we computed the optimal sensors and power to be used in each interval, and computed the global posterior as a weighted linear combination of local posteriors, where the weight of each modality was chosen in proportion to the price of the data collected by that modality. For Figs. 1(a) and 1(b), we considered a scenario where there were 3 targets with $\alpha_t = \{1, 2, 4\}$, for $t = 1, \dots, 8$, 4 targets with $\alpha_t = \{1, 2, 4, 5\}$, for $t = 9, \dots, 15$, and 3 targets again with $\alpha_t = \{1, 2, 5\}$, for $t = 15, \dots, 20$. For Figs. 1(c) and 1(d), there were 3 targets, with $\alpha_t = \{1, 2, 4\}$, for $t = 1, \dots, 20$. We used $p_d = p_b = 0.01$ for the simulations. We considered $M_s = 2000$ particles and averaged the results over 50 Monte Carlo iterations. It can be seen that using the proposed approach, the system was able to accurately identify the number of targets, produce higher correct identifications, and lower RMSE in range and velocity estimates.

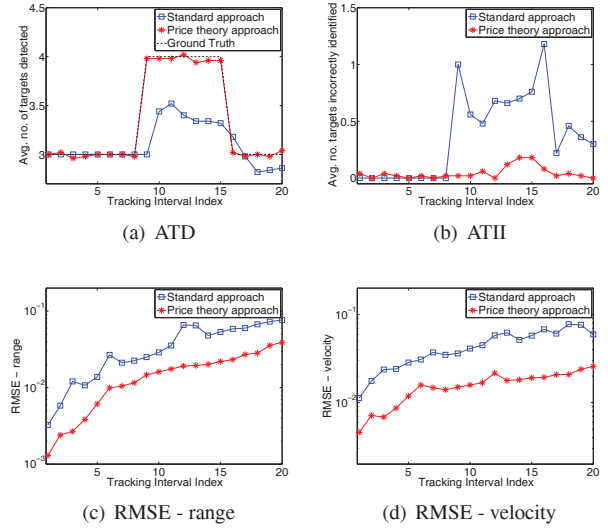


Fig. 1. Performance comparison of the two approaches

5. SUMMARY

We developed a framework, based on economic price theory, to jointly solve the problems of sensor selection (SS), resource allocation (RA) and data fusion (DF) in multi-modal sensor networks. We illustrate this framework using sensor management for multiple target tracking as an example. Numerical examples demonstrate the effectiveness of the proposed framework. In the future, we will compare the performance of the proposed framework with other algorithms that solve SS, RA, and DF.

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