ORDERING FOR ENERGY EFFICIENT COMMUNICATIONS FOR NONCOHERENT MIMO RADAR NETWORKS

Qian He, Rick S. Blum†, and Ziad N. Rawas†*

*EE Department, Univ. of Electron. Sci. and Tech. of China, Chengdu, Sichuan 611731 China
†ECE Department, Lehigh University, Bethlehem, PA 18015 USA

ABSTRACT

In order to reduce the number of transmissions between a set of sensors and a fusion center in signal detection applications, we propose an algorithm based on ordering and halting the transmissions wisely, which can reduce the data transmission, and thus expended energy and data rate, without sacrificing signal detection performance. Here we consider the specific case of noncoherent signal detection, where the log-likelihood ratio turns out to be nonnegative, with independent observations form sensor to sensor. For this specific case, we design a new ordering algorithm which provides very large savings for some example MIMO radar systems considered for almost all false alarm probabilities and signal-to-noise ratios (SNRs). While these savings are demonstrated numerically, we also prove analytically that savings of $(N-1)/N \times 100\%$ are achieved for sufficiently small or large false alarm probabilities and sufficiently large distance measures, a generalization of SNR, for a very large class of signal detection problems which employ N total sensors.

Index Terms— Energy efficient, MIMO radar, noncoherent signal detection, ordered transmissions.

1. INTRODUCTION

In centralized noncoherent MIMO radar networks, the fusion center generally collects data from all sensors. Sensors transmitting data to the fusion center causes energy consumption and increases communication requirements. Thus, when energy or communication resources are limited, for example when the radar sensors are equipped self-contained energy sources etc., we want to minimize the data transmissions between the sensors and the fusion center to save energy.

In most of the previous work on energy-efficiency for sensor networks, energy saving is achieved at the expense of detection performance. More recently, in [1], an approach for energy and communication-efficient signal detection was developed in which sensor transmissions are ordered according to the informativeness of their observations. The work in [1] considered a hypothesis testing problem where the sensor log-likelihood ratios, the optimum sensor statistic for cases with independent observations, could take on both nonnegative and negative values. Further, a Bayesian hypothesis testing approach was considered that assumed prior probabilities to be known.

In this paper, we describe an ordering approach for a class of noncoherent signal detection problems where the sensor log-likelihood ratios at each sensor can take on nonnegative values. There are many practical binary hypothesis where this is the case. Noncoherent MIMO radar target detection is an important practical problems of this type. We show that the new ordering approach significantly reduces the average number of transmissions, while achieving exactly the same performance as if all data has been transmitted to the fusion center. It is interesting that the ordering approach is only slightly different from the one in [1], the upper threshold in the stopping rule is altered, but the resulting savings generally greatly exceed those in [1] for the numerical examples considered. Analytical results explain the reason for this by focusing on some important limiting situations. We focus on the Neyman-Pearson criteria. We note that these savings extend to cases other than signal detection where a sum of sensor metrics are computed which satisfy the assumptions made.

This material is based on research supported by the National Science Foundation of China under Grant 61102142 and the National Science Foundation under Grant CCR-0829958.

2. NEW ORDERED TRANSMISSION ALGORITHM

Consider a system with N networked sensors attempting to solve a binary hypothesis problem under the following assumption.

Assumption 1 *Assume all sensors receive independent and identically distributed (iid) observations, conditioned on the binary hypothesis. For simplicity, assume each sensor uses an orthogonal noise-free communication channel to transmit its data to a fusion center.*

Under Assumption 1, the optimum unconstrained (all N sensors transmit) approach requires that each sensor computes its sensor log-likelihood L_k, $k=1,\dots,N$ based on its own observation and sends L_k to a fusion center to form the overall test statistic for binary hypothesis testing. The optimum unconstrained test statistic $\sum_{k=1}^{N} L_k$ is compared to a fixed threshold τ. A decision for H_1 is made if the overall test statistic is larger than τ, otherwise a decision for H_0 is made[1].

[1]The results in this paper are also valid for randomized tests.

The next assumption quantifies what we mean by noncoherent detection in this paper.

Assumption 2 *The local metric computed at every sensor is nonnegative such that $L_k \geq 0, \forall k$.*

We propose the following algorithm.

Algorithm 1 *Consider an approach where the sensors order their transmissions to the fusion center so that the sensor with the largest sensor metric transmits its data to the fusion center first. Denote the ordered sensor metrics as $L_{[1]} > L_{[2]} > \cdots > L_{[N]}$. Thus the first sensor to transmit will transmit $L_{[1]} = max(L_1, \ldots, L_N)$ and the next $L_{[2]}$, the next largest in the set $\{L_1, \ldots, L_N\}$, and so on. In fact, the sensors can decide when to transmit in a completely distributed manner. The k^{th} sensor could transmit[2] after a time equal to C/L_k for some common constant C. After each transmission, the fusion center accumulates the sum of all sensor metrics that have been transmitted so far and compares this sum to two thresholds τ and t_L. Assume that after a given time all but n_{UT} sensors have transmitted. Denote the last sensor transmission as $L_{[N-n_{UT}]}$ and define $t_L = \tau - n_{UT}L_{[N-n_{UT}]}$, where τ is the previously defined fixed threshold for the unconstrained optimum N-sensor test. If*

$$\sum_{k=1}^{N-n_{UT}} L_{[k]} > \tau, \tag{1}$$

we halt transmissions and decide for H_1. Alternatively, if

$$\sum_{k=1}^{N-n_{UT}} L_{[k]} < t_L \tag{2}$$

we halt transmissions and decide for H_0. Otherwise we continue to transmit until all transmissions occur. Then we use the optimum unconstrained (N-sensor) test.

The following theorem describes the capability of *Algorithm 1* to save transmissions.

Theorem 1 *Consider a sensor network with N sensors which is employing* Algorithm 1 *to solve a binary hypothesis testing problem. Under* Assumptions 1 *and* 2*, the approach described in* Algorithm 1 *will always make the same decision as the optimum approach where all N sensors transmit, while generally using a smaller average number of sensor transmissions.*

Note that if all the sensors hear all the transmissions, then each sensor can perform the computations to know when to stop the transmissions. Otherwise, a single stop signal can be broadcast to all sensors from the fusion center to halt the transmissions.

The following assumption and theorem analytically address the magnitude of the savings in some cases of great interest.

Assumption 3 *For the binary hypothesis testing problems considered, we assume the existence of a distance measure s, which describes the distance between observations under H_0 and H_1, such that $Pr(L_k > \tau | H_1) \to 1$ as $s \to \infty$ for any finite τ.*

Intuitively, *Assumption 3* implies that as s gets larger it becomes easier to tell the difference between observations under H_0 and H_1. Such distance measures can be found for many hypothesis testing problems of practical interest. For example, in many cases where we are testing noise only, H_0, versus signal-plus-noise, H_1, the signal-to-noise-ratio (SNR) plays the role of s. Later in Section 3, we show that for noncoherent MIMO radar target detection, the SNR is an example of the distance measure s as described in *Assumption 3*.

Define N_s as the number of transmissions saved and N_t the number of transmissions after which a decision can be made using *Algorithm 1*, then $N_s = N - N_t$.

Theorem 2 *Under* Assumptions 1-3 *for the case where H_1 is true, the probability that only a single transmission is needed when* Algorithm 1 *is employed with a finite τ approaches* 1 *as $s \to \infty$. It follows that $\mathbb{E}\{N_t | H_1\} = 1$ as $s \to \infty$.*

In the rest of this section, we focus on the Neyman-Pearson hypothesis test, where the threshold τ is set by the probability of false alarm, P_{FA}.

Assumption 4 *For the binary hypothesis testing problem considered, $P_{FA} \to 1$ as $\tau \to 0$, and $P_{FA} \to 0$ as $\tau \to \infty$. Further, $0 < L_k < \infty$, $\forall k$ with probability one under H_0 or H_1.*

Theorem 3 *Assume the Neyman-Pearson criterion is employed and* Assumptions 1-4 *hold. If* Algorithm 1 *is employed, the probability that only a single transmission is needed approaches 1 as $P_{FA} \to 0$, under the case where H_0 or H_1 is true. It follows that* $\Pr(N_s = N - 1 | H_j) \to 1$ *as $P_{FA} \to 0$ for $j = 0, 1$.*

Theorem 4 *Assume the Neyman-Pearson criterion is employed and* Assumptions 1-4 *hold. If* Algorithm 1 *is employed, the probability that only a single transmission is needed approaches 1 as $P_{FA} \to 1$, under the case where H_0 or H_1 is true. It follows that $Pr(N_s = N - 1 | H_j) \to 1$ as $P_{FA} \to 1$ for $j = 0, 1$.*

3. ORDERING FOR NONCOHERENT MIMO RADAR TARGET DETECTION

3.1. Problem Formulation

Consider a MIMO radar system that has M transmit and N receiver antennas under the simplified assumtions we now describe[3]. The transmit antennas are placed at the known positions (x_l^t, y_l^t), $l = 1, \cdots, M$ and the receive antennas are placed at the known positions (x_k^r, y_k^r), $k = 1, \cdots, N$ in a two-dimensional Cartesian coordinate system. The low-pass equivalent of the signal transmitted from the l-th transmitter is $\sqrt{E}s_l(t)$, where E denotes the transmitted energy per transmit antenna, and the waveform is normalized so that $\int_{-\infty}^{\infty} |s_l(t)|^2 dt = 1$. We are testing between hypothesis H_0

[2] While we use this approach in our numerical results, other approaches are also possible as long as the ordering is maintained.

[3] See [2] for extensions to most assumptions.

(no target) and hypothesis H_1 (a target moving with velocity (v_x, v_y) located at (x, y) is present). The time delay τ_{kl} and Doppler shift f_{kl} involved in the path from transmitter l to receiver k, via the target reflection, are $\tau_{kl} = (d_l^t + d_k^r)/c$ and $f_{kl} = (v_x(x_l^t - x) + v_y(y_l^t - y))/(\lambda d_l^t) + (v_x(x_k^r - x) + v_y(y_k^r - y))/(\lambda d_k^r)$ respectively, where c denotes the speed of light, $d_l^t = [(x_l^t - x)^2 + (y_l^t - y)^2]^{1/2}$ the distance between the target and the l-th transmitter, $d_k^r = [(x_k^r - x)^2 + (y_k^r - y)^2]^{1/2}$ the distance between the target and the k-th receiver, and λ the wavelength of the carrier. In the noncoherent processing approach, we assume all transmitter and receiver nodes have oscillators which are locked in frequency, possibly due to the use of a beacon. We further assume the transmitted signals are approximately orthogonal and maintain approximate orthogonality after reception for time delays and Doppler shifts of interest [2]. The noise corresponding to the kl-th path $w_{kl}(t)$ is a temporally white, zero-mean complex Gaussian random process with $\mathbb{E}\{w_{kl}(t)w_{kl}^*(u)\} = \sigma_w^2 \delta(t-u)$, where σ_w is a constant, and $\delta(t)$ is a unit impulse function. The noise components are spatially white, such that $\mathbb{E}\{w_{kl}(t)w_{k'l'}^*(u)\} = 0$ if $l \neq l'$ or $k \neq k'$. Since scaling the observations changes nothing, we set $\sigma_w^2 = 1$ without loss of generality.

For the noncoherent MIMO radar, the antennas are separated widely enough such that they are in different target beamwidths and the reflection coefficients for different transmit-receive paths are independent. Denote the complex reflection coefficient for the kl-th path by ζ_{kl}, which is assumed to be a Gaussian random variable with variance σ_{kl}^2 which remains constant over the observation interval. We assume the values of σ_{kl}^2 for various l, k are known and finite, possibly calculated from the known position probed for a target. For simplicity, let us assume $\sigma_{lk}^2 = \sigma_\zeta^2, \forall l, k$. Thus, the received signal can be modeled as

$$H_1: \quad r_{kl}(t) = \sqrt{E}\zeta_{kl}s_l(t - \tau_{kl})e^{j2\pi f_{kl}t} + w_{kl}(t) \tag{3}$$

$$H_0: \quad r_{kl}(t) = w_{kl}(t). \tag{4}$$

The Neyman-Pearson optimum hypothesis test for H_0 (no target) versus H_1 is to compare the overall log-likelihood ratio (LLR) [2]

$$\begin{aligned}\mathcal{L} &= \sum_{k=1}^{N}\sum_{l=1}^{M} \frac{\sigma_\zeta^2 E}{\sigma_\zeta^2 E + 1}\left|\int_{-\infty}^{\infty} \tilde{r}_{kl}(t)s_l^*(t-\tau_{kl})e^{-j2\pi f_{kl}t}dt\right|^2 \\ &= \sum_{k=1}^{N} L_k \end{aligned} \tag{5}$$

to a threshold set to fix the false alarm probability, where $\tilde{r}_{kl}(t)$ represents the actual observed received signal. Since $\mathcal{L}$ is chi-square distributed, we set the threshold as [3]

$$\tau = [E\sigma_\zeta^2/2(E\sigma_\zeta^2 + 1)]F_{\chi^2_{2MN}}^{-1}(1 - P_{\text{FA}}), \tag{6}$$

where $F_{\chi^2_{2MN}}^{-1}$ denotes the inverse cumulative distribution function of a chi-square distribution with $2MN$ degrees of freedom and P_{FA} denotes the desired false alarm probability.

Table 1. Minimum SNR (in dB) for which only one transmission is required under *Algorithm 1*. $P_{\text{FA}} = 10^{-3}$.

$\text{SNR}_{\min}$	N=2	N=10	N=20	N=30	N=40	N=50
M=2	20	13	13	14	15	15
M=3	15	12	13	14	14	15
M=4	13	12	13	14	14	15
M=5	12	11	13	14	14	15
M=6	11	11	12	14	14	15

Next we show that for the noncoherent MIMO radar problem considered, the SNR, defined as SNR$= \sigma_\zeta^2 E/\sigma_w^2 = \sigma_\zeta^2 E$, is a distance measure s of the type described in *Assumption 3*. Under H_1, plugging (3) into (5), we can rewrite $L_k = \sum_{l=1}^{M}[\sigma_\zeta^2 E/(\sigma_\zeta^2 E + 1)]|\sqrt{E}\zeta_{kl} + z_{kl}|^2$, where $z_{kl} = \int_{-\infty}^{\infty} w_{kl}(t)s_l^*(t-\tau_{kl})e^{-j2\pi f_{kl}t}dt$ denotes the output of the matched filter with noise as an input. Since σ_ζ^2 and E are assumed to be known and finite, for any nonzero realization of ζ_{kl} and z_{kl} we have $L_k \to \infty$ as SNR$\to \infty$. Hence, for any finite τ, $\lim_{SNR\to\infty} \Pr(L_k > \tau|H_1) = \Pr(\infty > \tau|H_1) = 1$. That is, $\Pr(L_k < \tau|H_1) \to 1$ as SNR$\to \infty$. This justifies *Assumption 3* for the noncoherent MIMO radar system where SNR is considered as a distance measure.

3.2. Numerical Examples

Assume the target, if present, is located at (150m, 127.5m), moving with velocity (50m/s, 30m/s), and gives $\sigma_\zeta^2 = 1$. The antennas are located along the circumference of a circle of radius 7000m centered at the origin. The transmit antennas are equidistant from each other, and so are the receive antennas. The carrier frequency is set to 1GHz.

Applying *Algorithm 1* to the noncoherent MIMO radar for target detection, *Theorem 2* implies that a single transmission is required to halt transmissions if the SNR is sufficiently large. In Table 1, for various M and N, we provide the minimum SNR ($\text{SNR}_{\min}$), at which only one transmission is required. It is seen that increasing M generally reduces $\text{SNR}_{\min}$ and the reduction is substantial for small N, e.g. for $N = 2$, $\text{SNR}_{\min}$ is almost halved when M is increased from 2 to 6. Increasing N initially decreases $\text{SNR}_{\min}$ but later increases $\text{SNR}_{\min}$, which appears to result from the initial increase in performance due to the increase in diversity. This benefit quickly decays with N and hence only overcomes the increase in τ with N (see (6)) for small N.

Assume the number of receivers is $N = 10$ and the number of transmitters is $M = 2$, 4, or 8. We employ *Algorithm 1* to halt transmissions and make decisions. Assuming H_0 being true, Fig. 1 shows the average percentage saving plotted versus the false alarm probability (P_{FA}) when $N = 10$. The average percentage saving is computed from $\mathbb{E}\{N_s\}/N \times 100\%$. We observe that for $P_{\text{FA}} = 1$, the average percentage saving is 90%, which implies a single transmission as

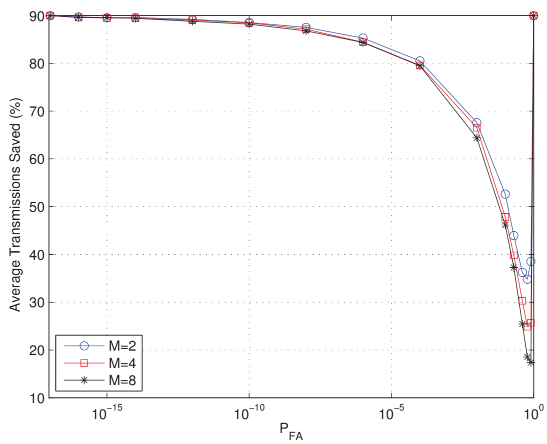

Fig. 1. Average transmissions saved versus P_{FA} for N=10 and M=2, 4, 8 under H_0 hypothesis.

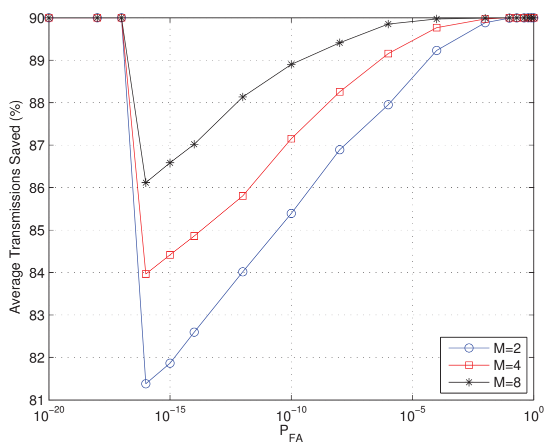

Fig. 2. Average transmissions saved versus P_{FA} for N=10 and M=2, 4, 8 under H_1 hypothesis. SNR=10 dB.

per *Theorem 4*. For $0 < P_{\text{FA}} < 1$, slightly more transmissions are needed. Let $P_{\text{FAC}}(M, N)$ denote the minimum of the corresponding curve for a plot similar to that in Fig. 1 when we employ M transmit antennas and N receive antennas. The difference in the behavior for P_{FA} above and below the minimum $P_{\text{FAC}}(M, N)$ can be explained by *Theorems 3* and *4*. Note that in *Theorem 3*, for small P_{FA}, the stopping condition in (2) applies. On the other hand, in *Theorem 4*, for large P_{FA}, the stopping condition in (1) applies. Thus $P_{\text{FAC}}(M, N)$ denotes the point where we switch from one stopping condition dominating to the other dominating. The results in Fig. 1 also show that $\mathbb{E}\{N_s\}/N \times 100 \rightarrow$90 % [i.e. $\mathbb{E}\{N_s\} \rightarrow (N-1)$] as P_{FA} approaches zero, which agrees with *Theorem 3*. As shown in the figure, the average number of transmissions saved decreases with M for any fixed P_{FA} when H_0 is true. This is is because increasing M leads to an increase in τ for any given P_{FA} as per (6), while no diversity can be exploited in this noise-only scenario. We also notice that $P_{\text{FAC}}(M, N)$ increases as M increases.

In Fig. 2, assuming that H_1 is true and SNR= 10 dB, we repeat the analysis of Fig. 1 and obtain similar findings that can be used to verify the correctness of the results in *Theorem 3* and *4* under H_1 hypothesis. Further, it is seen that in this case, the $P_{\text{FAC}}(M, N)$ of each curve takes on a much smaller value than its counterpart in Fig. 1, and the average percentage saving at the $P_{\text{FAC}}(M, N)$ is bigger than that corresponding value in Fig. 1. We see from Fig. 2, the average number of transmissions saved increases with M for any fixed P_{FA} when H_1 is true. Note that this contrasts with the result in Fig. 1 for H_0 being true, and can be attributed to the extra diversity gain obtained by increasing M. We have tested similar cases with a different N and obtained similar conclusions.

4. CONCLUSION

We study a new method, called ordering, to reduce the number transmissions between a set of sensors and a fusion center in signal detection applications. We focus on the specific case of noncoherent signal detection, where the sensor log-likelihood ratio turns out to be nonnegative, with independent observations form sensor to sensor. While we change the approach in [1] only slightly, the savings appear to be much larger than those demonstrated in [1] for cases where the sensor log-likelihood ratio turns may be negative. In particular, for some example noncoherent MIMO radar systems studied, the savings are found to be much larger than those shown in [1] for most useful false alarm probabilities and signal-to-noise ratios (SNRs). While these savings are demonstrated numerically, we also prove analytically that savings of $(N-1)/N \times 100\%$ are achieved for sufficiently small or large false alarm probabilities and sufficiently large distance measures, a generalization of SNR, for a very large class of signal detection problems which employ N total sensors.

5. REFERENCES

[1] R. S. Blum and B.M. Sadler, “Energy Efficient Signal Detection in Sensor Networks Using Ordered Transmissions,” *IEEE Trans. on Sig. Proc.*, vol. 56, no. 7, pp 3229-3234, Jul. 2008.

[2] Q. He, R. S. Blum, and A. M. Haimovich, “Non-coherent MIMO Radar for location and velocity estimation: More antennas means better performance”, *IEEE Trans on Signal Processing*, vol. 58, no. 7, pp. 3661-3680, Jul. 2010.

[3] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, “Spatial Diversity in Radar - Models and Detection Performance,” *IEEE Trans. on Sig. Proc.*, vol. 54, no. 3, pp. 823-838, Mar. 2006.

[4] H.V. Poor, *An Introduction to Signal Detection and Estimation*, 2nd ed. New York: Springer-Verlag, 1994.