THE POWER GAME BETWEEN A MIMO RADAR AND JAMMER

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ABSTRACT

The interaction between a *smart* target and a *smart* MIMO radar is investigated from a game theory perspective. Since the target and the radar form an adversarial system, their interaction is modeled as a two-person zero-sum game. The mutual information criterion is used to formulate the utility functions. The unilateral, hierarchical, and symmetric games are studied, and the equilibria solutions are derived.

Index Terms— MIMO radar, waveform, jamming, game theory, hierarchical game, Stackelberg equilibrium, Nash equilibrium.

1. INTRODUCTION

A MIMO radar emphasizes spatial complementarity and waveform cooperation, and it may outperform a monostatic one on detection, estimation, and information extraction [1–4]. Current works prefer to investigate the interaction between a *smart* radar and a *dumb* target, where the former has some knowledge of the latter such as radar cross section distribution, while the latter is incapable of interfering with the former. With the development of electronic warfare, many intelligent targets are equipped with countermeasure systems to prevent a radar from operating as well as it might [5]. In this paper, the interaction involves a *smart* target, which carries jamming equipment that could intelligently confuse the radar. If the target always tries to prevent a radar from fulfilling its task, their interaction can be modeled as a *two-person zero-sum* (TPZS) game [6].

The mutual information (MI) criterion [1] is used to formulate the utility functions. The radar controls the waveform matrix to maximize the MI, while the latter has some access to its jamming matrix to minimize it. Based on the information set available for each player, the games fall into one of the three categories: unilateral, hierarchial, and symmetric. For the unilateral case, one player can intercept the other's strategy while the latter *does not* notice that this is happening. The TPZS games are simplified as single person optimizations, and the optimal (water-filling) strategies are derived. For the second case, one player can intercept the other's strategy while the latter *does* notice that. The TPZS game is recast as a conservative minmax or maxmin two-stage optimization, and the Stackelberg equilibria—optimization solutions—are derived. In the last case, no player has the idea of the other's strategy; its secure strategy pair is a Nash equilibrium, of which the existence conditions are analyzed.

Note that a similar idea is recently used in polarimetric MIMO radar detection [7]. This paper focuses on selected parts of [2]; the remaining sections are as follows. Section 2 introduces the MIMO radar signal model, and specifies the game criterion. Section 3 investigates unilateral games, while hierarchical ones are in Section 4. Section 5 focuses on games with symmetric information. Section 6 includes numerical results, and then conclusions are drawn.

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2. MIMO RADAR V.S. JAMMER UNDER MI CRITERION

We are interested in the interaction of a smart MIMO radar system and a smart target: the latter can interfere with the former via waveform independent noise. Let the radar system be comprised of n_t transmitters and n_r receivers. Suppose that the transmitted waveform of the jth transmitter is $\boldsymbol{s}_j$, and then the collection echoes for the MIMO radar is written as

$$\boldsymbol{Y} = \boldsymbol{S}\boldsymbol{H} + \boldsymbol{J} + \boldsymbol{W}, \tag{1}$$

where $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \cdots, \boldsymbol{s}_{n_t}]$ is a $K \times n_t$ transmitted waveform matrix with $K \geq n_t$, K denotes the waveform length, $\boldsymbol{H}$ denote the $n_t \times n_r$ path gain matrix, $\boldsymbol{J}$ denotes the $K \times n_r$ jamming matrix, $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_{n_r}]$ is the $K \times n_r$ noise matrix, and $\boldsymbol{Y}$ denotes the $K \times n_r$ received signal matrix.

The radar-target interaction is investigated under a probability framework, and four assumptions are adopted: ***A1)*** the columns of $\boldsymbol{W}$ are i.i.d. Gaussian vectors with probability density function (pdf) $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_w)$; ***A2)*** the target is comprised of a large number of small i.i.d. random scatterers. With *the central limit theorem*, the columns of $\boldsymbol{H}$ could be considered as i.i.d. Gaussian vectors with distribution $\mathcal{CN}(\boldsymbol{0}, \sigma_h^2 \boldsymbol{I}_{n_t})$ under sufficient antenna separation [4]; ***A3)*** the columns of $\boldsymbol{J}$ are i.i.d. random vectors with distribution $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_b)$, and ***A4)*** $\boldsymbol{H}$, $\boldsymbol{W}$, and $\boldsymbol{J}$ are mutual independent.

Suppose that the MIMO radar wants to extract MI between the received signal $\boldsymbol{Y}$ and the path gain matrix $\boldsymbol{H}$

$$I(\boldsymbol{Y}; \boldsymbol{H}|\boldsymbol{S}) = h(\boldsymbol{Y}|\boldsymbol{S}) - h(\boldsymbol{J} + \boldsymbol{W}) \tag{2}$$

in a contaminated environment, where $h(\cdot)$ denotes the (conditional) differential entropy. As $(\boldsymbol{Y}|\boldsymbol{S}) \sim \mathcal{CN}(\boldsymbol{0}, \sigma_h^2 \boldsymbol{S}\boldsymbol{S}^H + \boldsymbol{R}_b + \boldsymbol{R}_w)$ and $(\boldsymbol{J} + \boldsymbol{W}) \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_b + \boldsymbol{R}_w)$, we have

$$I_b \triangleq I(\boldsymbol{Y}; \boldsymbol{H}|\boldsymbol{S}) = n_r \log \frac{\det(\sigma_h^2 \boldsymbol{S}\boldsymbol{S}^H + \boldsymbol{R}_b + \boldsymbol{R}_w)}{\det(\boldsymbol{R}_b + \boldsymbol{R}_w)}. \tag{3}$$

The MIMO radar controls the waveform matrix $\boldsymbol{S}$ to maximize I_b, while the target tries to minimize it with jamming matrix $\boldsymbol{J}$. Therefore, one player's gain is the other's loss, and this is a TPZS game [6]. I_b is a function of $\boldsymbol{S}$ and $\boldsymbol{R}_b$. The optimal strategy of one player depends on its *inference* of the other's.

The strategy domain of (3) is composed of two Hermitian matrices: $\boldsymbol{R}_b$ and $\boldsymbol{S}\boldsymbol{S}^H$. A direct interaction analysis in matrix domain is rather complex, particularly when one player has no knowledge of the other. As a Hermitian matrix is determined by its eigenvectors and eigenvalues, the TPZS game actually implies two parts: eigenspace selection (where to play) and eigenvalue optimization (how to allocate power). Conservatively, if one player could not precisely infer the other's power allocation, staying at the eigenspace defined by $\boldsymbol{R}_w$ would be a secure choice [2].

Here is another assumption: ***A5)*** the radar and target choose the eigenspace of $\boldsymbol{R}_w$ in game playing. Following this assumption, let the eigendecomposition of $\boldsymbol{R}_w$ be $\boldsymbol{R}_w = \boldsymbol{U}_w\boldsymbol{\Lambda}_w\boldsymbol{U}_w^H$, and then those for $\boldsymbol{SS}^H$ and $\boldsymbol{R}_b$ could be respectively expressed as $\boldsymbol{SS}^H = \boldsymbol{U}_w\boldsymbol{P}_1\boldsymbol{\Gamma}_s\boldsymbol{P}_1^H\boldsymbol{U}_w^H$ and $\boldsymbol{R}_b = \boldsymbol{U}_w\boldsymbol{P}_2\boldsymbol{\Lambda}_b\boldsymbol{P}_2^H\boldsymbol{U}_w^H$, where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are two arbitrary permutation matrices. Since the dimension of signal subspace is n_t, the eigenvalue matrix $\boldsymbol{\Gamma}_s$ can be written as

$$\boldsymbol{\Gamma}_s = \begin{bmatrix} \boldsymbol{\Lambda}_s & \\ & \mathbf{0}_{(K-n_t)\times(K-n_t)} \end{bmatrix}, \tag{4}$$

where $\boldsymbol{\Lambda}_s = \mathrm{diag}([\sigma_1^s, \sigma_2^s, \cdots, \sigma_{n_t}^s])$. Let the diagonal elements of $\boldsymbol{\Lambda}_w = \mathrm{diag}([\sigma_1^w, \sigma_2^w, \cdots, \sigma_K^w])$ be in decreasing order $\sigma_1^w \geq \sigma_2^w \geq \cdots \geq \sigma_K^w$, while those of $\boldsymbol{\Lambda}_b = \mathrm{diag}([\sigma_1^b, \sigma_2^b, \cdots, \sigma_K^b])$ do not have any ordering requirement. Without loss of generality, define the waveform matrix as $\boldsymbol{S} \triangleq \boldsymbol{U}_w\boldsymbol{P}_1\left[\sqrt{\boldsymbol{\Lambda}_s}, \mathbf{0}_{n_t\times(K-n_t)}\right]^T$, and then the MI at the equilibrium is specified as

$$\bar{I}_b = n_r \log\left[\det\left(\sigma_h^2\boldsymbol{\Gamma}_s\boldsymbol{P}_1(\boldsymbol{\Lambda}_b + \boldsymbol{\Lambda}_w)^{-1}\boldsymbol{P}_1^T + \boldsymbol{I}_K\right)\right], \tag{5}$$

where $\boldsymbol{\Gamma}_s$ will reduce the dimension of the game space from K to n_t, and $\boldsymbol{P}_1$ decides which subspace would be selected.

3. UNILATERAL GAMES

3.1. Radar Unilateral Games

If the MIMO radar knows the target's strategy while the latter *does not* notice that, the game degenerates to a unilateral power allocation problem [1], where the radar properly assigns its power into the noise (jamming) space to maximize the MI

$$\max_{\boldsymbol{\Lambda}_s, \boldsymbol{P}_1} \bar{I}_b, \;\; \text{s.t. } \mathrm{Tr}(\boldsymbol{SS}^H) = \mathrm{Tr}(\boldsymbol{\Lambda}_s) \leq P_s, \tag{6}$$

where P_s denotes the available waveform power. Without loss of generality, we assume $\sigma_1^b + \sigma_1^w \geq \sigma_2^b + \sigma_2^w \geq \cdots \geq \sigma_K^b + \sigma_K^w$ and $\sigma_1^s \geq \sigma_2^s \geq \cdots \geq \sigma_{n_t}^s$ in this subsection. With the *Hadamard theory* [1], (6) is maximized if $\boldsymbol{P}_1$ is chosen as $\boldsymbol{P}_1 = \boldsymbol{P}$, where

$$\boldsymbol{P} \triangleq \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix}, \tag{7}$$

and then we have

$$\max_{\sigma_i^s} \sum_{i=1}^{n_t} \log\left(\frac{\sigma_i^s\sigma_h^2}{\sigma_{K+1-i}^b + \sigma_{K+1-i}^w} + 1\right), \text{s.t.} \sum_{i=1}^{n_t} \sigma_i^s \leq P_s. \tag{8}$$

The objective function is concave and monotonically increasing; its optimal solution can be obtained via Lagrange multipliers, and yields a water-filling strategy [1]

$$\sigma_i^s = \left(\lambda_1 - \frac{\sigma_{K+1-i}^b + \sigma_{K+1-i}^w}{\sigma_h^2}\right)^+, \tag{9}$$

where $(x)^+ \triangleq \max\{0, x\}$, and λ_1 is chosen via $\sum_{i=1}^{n_t} \sigma_i^s = P_s$.

3.2. Target Unilateral Games

On the other hand, suppose that the target knows the power allocation strategy of the MIMO radar. The game degenerates to a jamming unilateral optimization, as the radar is not aware of this. In such a circumstance, the target will properly allocate its jamming power to minimize $\bar{I}_b$. Mathematically, this is expressed as

$$\min_{\boldsymbol{\Lambda}_b} \bar{I}_b, \;\; \text{s.t. } \mathrm{Tr}(\boldsymbol{\Lambda}_b) \leq P_b, \tag{10}$$

where P_b bounds the jamming power. For a given radar power allocation strategy, $\bar{\boldsymbol{P}}_1$ plus $\bar{\boldsymbol{\Lambda}}_s = \mathrm{diag}([\bar{\sigma}_1^s, \bar{\sigma}_2^s, \cdots, \bar{\sigma}_{n_t}^s])$, the optimization (10) is specified as

$$\min_{\bar{\sigma}_i^b} \sum_{i=1}^{n_t} \log\left(\frac{\bar{\sigma}_i^s\sigma_h^2}{\bar{\sigma}_i^b + \bar{\sigma}_i^w} + 1\right), \;\; \text{s.t.} \sum_{i=1}^{n_t} \bar{\sigma}_i^b \leq P_b, \tag{11}$$

where $\bar{\sigma}_i^b$ and $\bar{\sigma}_i^w$ correspond to the ith selected jamming-noise subspace, and they are not necessarily identical to σ_i^b and σ_i^w. As the objective function is monotonically decreasing and strictly convex, its optimal solution can be uniquely found with Lagrange multipliers

$$\bar{\sigma}_i^b = \left(\sqrt{\frac{\bar{\sigma}_i^s\sigma_h^2}{\lambda_2} + \frac{(\bar{\sigma}_i^s\sigma_h^2)^2}{4}} - \bar{\sigma}_i^w - \frac{\bar{\sigma}_i^s\sigma_h^2}{2}\right)^+. \tag{12}$$

where $\lambda_2 > 0$ satisfies $\sum_{i=1}^{n_t} \bar{\sigma}_i^b = P_b$. Since the subspace selection privilege belongs to the radar, the target can only optimize its power corresponding to the selected subspace.

4. HIERARCHICAL GAMES

4.1. Target As The Leader

Let the radar system (the follower) possess sufficient interception capacity that it can immediately sense interference. If the target (the leader) knows this and behaves conservatively, the game may converge to a Stackelberg equilibrium (SE) [6], which is defined as the solution of a two-stage optimization [6]

$$\begin{aligned} \min_{\boldsymbol{\Lambda}_b} \max_{\boldsymbol{\Lambda}_s, \boldsymbol{P}_1} \; & \log\left[\det\left(\sigma_h^2\boldsymbol{\Gamma}_s\boldsymbol{P}_1(\boldsymbol{\Lambda}_b + \boldsymbol{\Lambda}_w)^{-1}\boldsymbol{P}_1^T + \boldsymbol{I}_K\right)\right] \\ & \text{s.t. } \mathrm{Tr}(\boldsymbol{\Lambda}_b) \leq P_b, \; \mathrm{Tr}(\boldsymbol{\Lambda}_s) \leq P_s. \end{aligned} \tag{13}$$

The interception capability enables the radar to guarantee an optimal power allocation response for an arbitrary strategy from its opponent, so the results in Subsection 3.1 are still applicable for the first stage. Based on (9), (13) is reduced to

$$\begin{aligned} \min_{\sigma_i^b} & \sum_{i=1}^{n_t} \log\left(\frac{\sigma_i^s\sigma_h^2}{\sigma_{K+1-i}^b + \sigma_{K+1-i}^w} + 1\right) \\ \text{s.t. } & \sigma_i^s = \left(\lambda_1 - \frac{\sigma_{K+1-i}^b + \sigma_{K+1-i}^w}{\sigma_h^2}\right)^+, \\ & \sigma_1^b + \sigma_1^w \geq \sigma_2^b + \sigma_2^w \geq \cdots \geq \sigma_{K-n_t+1}^b + \sigma_{K-n_t+1}^w, \\ & \sum_{i=1}^{n_t} \sigma_i^s = P_s, \quad \sum_{i=1}^{K} \sigma_i^b \leq P_b. \end{aligned} \tag{14}$$

Lemma 1: The power allocation SE for the hierarchical game with the target as the leader is

$$\sigma_i^b = (\lambda_3 - \sigma_i^w)^+, 1 \leq i \leq K \tag{15}$$
$$\sigma_j^s = \left(\min\left\{\lambda_1 - \lambda_3/\sigma_h^2, \lambda_1 - \sigma_{K+1-j}^w/\sigma_h^2\right\}\right)^+, 1 \leq j \leq n_t,$$

where λ_3 and λ_1 are obtained via $\sum_{i=1}^{K}\sigma_i^b = P_b$ and $\sum_{i=j}^{n_t}\sigma_j^s = P_s$.

Proof: Proof can be found in [2].

Intuitively, the SE can be interpreted as a two-step water-filling as shown in Fig. 1 (a): firstly, the target conservatively fills its jamming power to the noise space, and then the radar injects its power to jamming-plus-noise space.

4.2. Radar As The Leader

Let the target be able to sense the radar's power allocation, and let the MIMO radar know that it does. Then a conservative radar system may select its strategy based on

$$\begin{aligned} \max_{\mathbf{\Lambda}_s, \boldsymbol{P}_1} \min_{\mathbf{\Lambda}_b} \log\Big[\det\Big(\sigma_h^2 \mathbf{\Gamma}_s \boldsymbol{P}_1 (\mathbf{\Lambda}_b + \mathbf{\Lambda}_w)^{-1} \boldsymbol{P}_1^T + \boldsymbol{I}_K\Big)\Big] \\ \text{s.t. } \mathrm{Tr}(\mathbf{\Lambda}_b) \le P_b,\ \mathrm{Tr}(\mathbf{\Lambda}_s) \le P_s \end{aligned} \tag{16}$$

in order to optimize the worst case. As for (16), the first stage includes an unknown subspace selection parameter $\boldsymbol{P}_1$; direct optimization is hard. But from rationality considerations we know the radar system will not 'pour' its power to the $(K - n_t)$ subspaces with higher noise levels, because that will make the final result even worse. Let the radar choose the n_t noise subspaces corresponding to eigenvalues σ^w_{K+1-i}'s, where $1 \le i \le n_t$. Without losing generality, one possible choice is $\boldsymbol{P}_1 = \boldsymbol{P}$. Hence, (16) is recast as

$$\begin{aligned} \max_{\sigma_i^s} \min_{\sigma_i^b} \sum_{i=1}^{n_t} \log\left(\frac{\sigma_i^s \sigma_h^2}{\sigma^b_{K+1-i} + \sigma^w_{K+1-i}} + 1\right), \\ \text{s.t. } \sum_{i=1}^{n_t} \sigma^b_{K+1-i} \le P_b,\ \sum_{i=1}^{n_t} \sigma_i^s \le P_s. \end{aligned} \tag{17}$$

In addition to the optimization ordering, there is another significant difference between (14) and (17): the jamming power constraints. In the case of (14), the target moves first. It will conservatively fill its power to the entire noise space; therefore, the power constraint is $\sum_{i=1}^{K} \sigma_K^b \le P_b$. In the case of (17) the radar moves first, so the target can 'see' which subspaces are selected, and then it will pour the jamming energy only to them. Hence, the power constraints are modified to $\sum_{i=1}^{n_t} \sigma^b_{K+1-i} \le P_s$ in (17), and this can be regarded as a game in a reduced space.

Lemma 2*:* The power allocation SE for the hierarchical game with the radar as the leader is

$$\begin{aligned} &\sigma_i^b = 0,\ 1 \le i \le K - n_t \\ &\sigma_i^b = (\lambda_4 - \sigma_i^w)^+,\ K - n_t + 1 \le i \le K \\ &\sigma_j^s = \Big(\min\Big\{\lambda_5 - \lambda_4/\sigma_h^2, \lambda_5 - \sigma^w_{K+1-j}/\sigma_h^2\Big\}\Big)^+,\ 1 \le j \le n_t, \end{aligned} \tag{18}$$

where λ_4 and λ_5 are determined by $\sum_{i=1}^{n_t} \sigma^b_{K+1-i} = P_b$ and $\sum_{i=1}^{n_t} \sigma_i^s = P_s$.

Proof: Proof can be found in [2]

The SE is still a two-step water-filling in a reduced space; an illustration is depicted in Fig. 1 (b).

4.3. Discussion

The equivalence of *Lemma 1* and *2* is straightforward if $K = n_t$. This subsection discusses their relationship for $K > n_t$. Comparing (15) and (18), we see that if and only if (iff) P_b is large enough to activate the noise subspace corresponding to $\sigma^b_{K-n_t}$, say $P_b > P_n \triangleq \sum_{i=1}^{n_t} (\sigma^w_{K-n_t} - \sigma^w_{K+1-i})$, the two lemmas will result

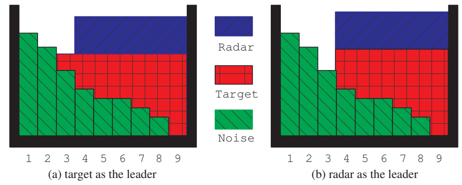


Fig. 1. An intuitive explanation of the SEs for the two hierarchical games: (a) target as the leader, and (b) radar as the leader. The dimension of noise subspace is $K = 9$, while that of the signal subspace is $n_t = 6$. As for case (a), since the target moves first and does not know which amongst the n_t subspaces will the radar choose, it has to allocate its power to the entire noise space (water-filling). As for case (b), target moves late and can observe which subspaces radar selected, so it only (water-filling) allocates its power to the radar-selected-ones: 4-9.

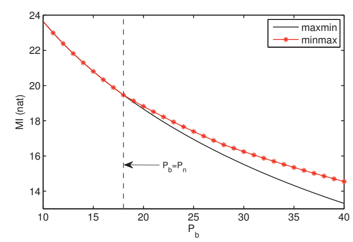


Fig. 2. The MI of at SEs as functions of P_b for the minmax and maxmin games, where $P_s = 40$. If $P_b \le P_n$, the two curves overlap, while the former is always above the latter for $P_b > P_n$.

in different strategy pairs, and it is interesting that the power allocation strategies of the radar are identical in both cases. This can be explained from two perspectives: *1)* if $P_b \le P_n$, the two lemmas are the same, so σ_j^s's are as well; *2)* if $\sigma^b_{K-n_t} > 0$, we must have

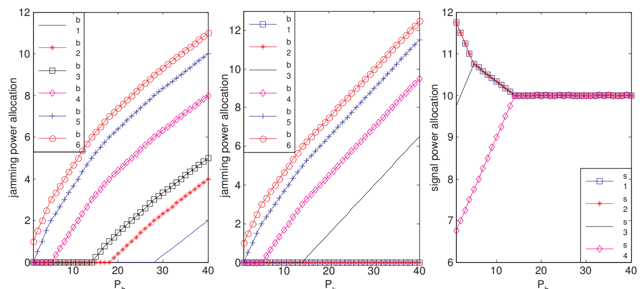
$$\sigma_i^b + \sigma_i^w = \lambda_3 \text{ (or } \lambda_4),\ \text{for } K - n_t + 1 \le i \le K \tag{19}$$

for both of them. Even though $\lambda_3 \ne \lambda_4$, they will both induce a uniform power allocation in the second step, and hence the strategies remain identical. An immediate corollary of this phenomenon is that the power allocation of the MIMO radar becomes uniform with the increase of P_b for both games, because the jamming-plus-noise subspaces, $(\sigma_i^b + \sigma_i^w)$'s, tend toward flat as shown in (19).

In the following, the phrase *minmax (maxmin) game* is used for simplicity to indicate a hierarchical game with radar (target) as leader.

5. GAMES WITH SYMMETRIC INFORMATION

This section studies the cases with symmetric information, where neither player has knowledge of the other's strategy. In such circumstances, the Nash equilibrium (NE) is a good tool to analyze the outcome of the strategic interaction [6]. If a game is competitive and has a unique pure-strategy NE, all the players prefer to stay at NE under the assumptions of conservativeness and rationality. As for a TPZS game with utility function $f(\boldsymbol{a}, \boldsymbol{b})$, where $\boldsymbol{a}$ is a minimizer



(a) target minmax results (b) target maxmin results (c) radar maxmin/maxmin results

Fig. 3. arget and radar power allocation strategies at SEs as functions of P_b for the minmax and maxmin games, where $P_s = 40$.

and $\boldsymbol{b}$ is a maximizer, the pure-strategy NE $(\boldsymbol{a}^*, \boldsymbol{b}^*)$ is defined as [6]

$$f(\boldsymbol{a}, \boldsymbol{b}^*) \geq f(\boldsymbol{a}^*, \boldsymbol{b}^*) \geq f(\boldsymbol{a}^*, \boldsymbol{b}), \text{ for } \forall \boldsymbol{a} \neq \boldsymbol{a}^* \text{ and } \boldsymbol{b} \neq \boldsymbol{b}^*. \tag{20}$$

Informally speaking, the NE of a TPZS game on a continuous space is the saddle point of its utility function, and no player can do better by unilateral deviation. We need the following proposition.

Lemma 3: Let $f(\boldsymbol{a}, \boldsymbol{b})$ be a real valued function for a TPZS game, where $\boldsymbol{a} \in \mathcal{A}$ is a minimizer and $\boldsymbol{b} \in \mathcal{B}$ is a maximizer. Suppose $\bar{\mathcal{A}} \times \bar{\mathcal{B}} \neq \emptyset$ are the solution subspace of

$$(\boldsymbol{a}, \boldsymbol{b}) = \arg \min_{\boldsymbol{a} \in \mathcal{A}} \max_{\boldsymbol{b} \in \mathcal{B}} f(\boldsymbol{a}, \boldsymbol{b}) = \arg \max_{\boldsymbol{b} \in \mathcal{B}} \min_{\boldsymbol{a} \in \mathcal{A}} f(\boldsymbol{a}, \boldsymbol{b}), \tag{21}$$

and then we have that ***1)*** if $(\boldsymbol{a}, \boldsymbol{b}) \in \bar{\mathcal{A}} \times \bar{\mathcal{B}}$, and then $(\boldsymbol{a}, \boldsymbol{b})$ is a NE; ***2)*** if $(\boldsymbol{a}, \boldsymbol{b}) \notin \bar{\mathcal{A}} \times \bar{\mathcal{B}}$, and then $(\boldsymbol{a}, \boldsymbol{b})$ could not be a NE.

Proof: Proof can be found in [6].

Lemma 4*:* The NEs for the MI based TPZS games are that: ***1)*** if $K = n_t$, the NE exists and can be obtained via (15) or (18); ***2)*** if $K > n_t$ and $P_b \leq P_n$, the NE exists and it is the common solution of (15) and (18); ***3)*** if $K > n_t$ and $P_b > P_n$, the NE does not exist.

Proof: Proof can be found in [2]

The existence of a NE depends on K, n_t, P_n, and P_b. As for $K > n_t$, it may not always exist. The behavior of game players is easy to predict if the NE exists; otherwise, it will depend on other factors that are more intricate to formalize. Regarding a *matrix zero-sum game* with finite strategies, one may resort to *mixed-strategy* approach, in which each player chooses a number of strategies with a reasonable probability [6]. Interestingly, the existence of a pure- (or mixed-) strategy NE is guaranteed in theory for a matrix zero-sum game. The games in a continuous space naturally have an infinite number of pure (and mixed) strategies. In the absence of a NE, strategy analysis becomes rather difficult and heuristic. Although the game may not converge to a stationary strategy pair in this case, the players at least can play the minmax or maxmin strategy to avoid the worst case.

6. NUMERICAL RESULTS

This subsection concentrates on the hierarchical games. In simulations, we set $n_t = 4$, $n_r = 6$, and $K = 6$. The noise powers are respectively chosen as $\sigma_1^w = 10$, $\sigma_2^w = 8$, $\sigma_3^w = 7$, $\sigma_4^w = 4$, $\sigma_5^w = 2$, and $\sigma_6^w = 1$. Finally, $\sigma_h = 1$ for simplicity. P_s is fixed at 40, while P_b varies from 1 to 40. The MI of *minmax* and *maxmin* solutions for the hierarchical games are depicted in Fig. 2. Clearly, both of them are decreasing functions of P_b. Moreover, if P_b is below a certain level, $P_n = 18$, the minmax and maxmin solutions are the same, while the minmax curve is always above the maxmin one if $P_b > P_n$. This coincides with the theoretical analysis in Section 4. The dashed threshold line also acts the bound for the existence of pure strategy NE for the games with symmetric information.

Fig. 3 shows their power allocation equilibria. The equilibria perform like a two-step water-filling: firstly, a noise subspace with a low σ_i^w obtains more jamming power; secondly, the subspace with a small $(\sigma_i^b + \sigma_i^w)$ will obtain more waveform energy. From Fig. 3, we observe that all the six σ_i^b's will be sequentially activated with the increase of P_b for the minmax results, while only σ_j^b's, $3 \leq j \leq 6$, will be sequentially activated for the latter. From Fig. 3(c), we know that the waveform power allocation strategy tends toward uniform with an increase of P_b. Here is an explanation. If P_b is sufficiently large, all the selected σ_i^b's will be activated in the first water-filling step; therefore, we have $\sigma_i^b + \sigma_i^w = \lambda$ for $\forall i$. As the σ_i^s's are obtained by a water-filling on $(\sigma_i^b + \sigma_i^w)$'s, the optimal power allocation strategy becomes uniform. Note that since the waveform power allocation strategies are the same for both games, only one plot is shown.

7. CONCLUSIONS

The interaction between a target and a MIMO radar – both smart – is modeled as a two-person zero-sum game under the mutual information criterion. Unilateral, hierarchical, and symmetric games are studied based on the available information set for each player. The optimal strategies for the unilateral games are forms of water-filling, and they can be analytically derived via constrained optimization techniques. Assuming conservativeness and rationality, the optimal strategies for the hierarchical games are Stackelberg equilibria, of which the closed-form expressions can be considered as two-step water-fillings. Nash equilibria are the optimal strategies for the third case; its existence conditions are discussed.

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