EXPLOITING RAUCH-TUNG-STRIEBEL FORMULAE FOR IMM-BASED SMOOTHING OF MARKOVIAN SWITCHING SYSTEMS

Rémy Lopez[†‡⋆] *and Patrick Danès*[†‡]

[†] CNRS; LAAS; 7 avenue du colonel Roche, F-31077 Toulouse Cedex 4, France
[‡] Université de Toulouse; UPS, INSA, INP, ISAE; UT1, UTM, LAAS; F-31077 Toulouse Cedex 4, France
[⋆] Collecte Localisation Satellites; 8-10, rue Hermès, F-31520 Ramonville Saint-Agne, France
{rlopez@cls.fr, patrick.danes@laas.fr}

ABSTRACT

This paper presents a suboptimal fixed-interval smoothing algorithm for nonlinear Markovian switching systems. The posterior smoothed mean and covariance of the system state are approximated by combining in a backward-time recursive process the statistics produced by a forward-time Interacting Multiple Model filter. Each recursion of the backward-time process consists of a smoothing step based on Rauch-Tung-Striebel formulae and of a specific interaction step to allow mode cooperation. A simulated case study in the field of target tracking illustrates the method.

Index Terms— nonlinear Markovian switching systems, Interacting Multiple Model (IMM) filter, multiple-model smoothing, target tracking

Notations are standard. $P(.)$, $p(.)$ and $E[.]$ respectively term a probability, a probability density function (pdf) and an expectation. $\mathcal{N}(\hat{x},P)$ is the Gaussian distribution with mean $\hat{x}$ and covariance P. $\mathcal{N}(.;\hat{x},P)$ is the associated pdf. The shortcut $v_{k:k'} = \{v_k,\ldots,v_{k'}\}$ stands for the values of v between time k and k'. Finally, the notation $\|a\|^2_R = a^T Ra$ is used, with $.^T$ the transpose operator.

1. INTRODUCTION

Many estimation or change detection problems are stated in the context of discrete-time Markovian switching systems [1]. Such systems are described by a bank of state space models, sharing the same state vector and corresponding to admissible modes of operation. These modes constitute the set $\mathcal{M}$ of cardinality M. At each time t_k, $m_k = j$ or m^j_k is the event that mode $j \in \mathcal{M}$ is in effect during the sampling period $(t_{k-1},t_k]$. The global state vector at time t_k is written as ξ_k. It encompasses the base continuous state x_k and the discrete state m_k, so that $\xi_k = (x_k,m_k)$.

At initial time, the distribution of x_0 is characterized by $p(x_0) = \sum_{j\in\mathcal{M}} P(m^j_0)p(x_0|m^j_0)$, where

$$P(m^j_0) = \mu^j_0,\ p(x_0|m^j_0) = \mathcal{N}(x_0;\hat{x}^j_{0|0},P^j_{0|0}), \tag{1}$$

and the statistics $\mu^j_0, \hat{x}^j_{0|0}, P^j_{0|0}$ are given. The sequence of modes follows an homogeneous finite-state Markov chain with given transition probabilities $\pi_{ji} = P(m^i_{k+1}|m^j_k)\ \forall(i, j) \in \mathcal{M}\times\mathcal{M}$.

Conditioned on the fact that mode j is active, the dynamics and measurement equations write as:

$$x_k = f^j_{k-1}(x_{k-1}) + q^j_{k-1} \text{ and } z_k = h^j_k(x_k) + r^j_k \tag{2}$$

where z_k terms the measurement vector, the dynamics and measurement noises $q^j_., r^j_.$ are white mutually independent zero-mean Gaussian sequences independent of the initial state x_0, such that $q^j_{k-1} \sim \mathcal{N}(0,Q^j_{k-1})$ and $r^j_k \sim \mathcal{N}(0,R^j_k)$. The sequences $\{f^j_., Q^j_.\}$ and $\{h^j_., R^j_.\}$ are given.

This paper deals with fixed-interval smoothing. While filtering approximates the posterior density of the state x_k and the posterior mode probabilities $m^1_k,\ldots,m^M_k$ conditioned on the measurements $z_{1:k}$, fixed-interval smoothing uses all the T measurements $z_{1:T}$ in the interval so as to get $p(x_k|z_{1:T})$ and $\{P(m^j_k|z_{1:T})\}_{j\in\mathcal{M}}$. Its exact solution requires to express $p(x_k|z_{1:T})$ as a mixture of the posterior mode-matched densities $\{p(x_k|m_{0:T} = j_{0:T}, z_{1:T})\}_{j_{0:T}\in\mathcal{M}^{T+1}}$ weighted by the posterior probabilities of the mode sequences $\{P(m_{0:T} = j_{0:T}|z_{1:T})\}_{j_{0:T}\in\mathcal{M}^{T+1}}$. The induced complexity being exponential in T, suboptimal techniques must be developed [2][3].

For the filtering problem, the reduction of the exponentially growing tree of mode sequences can be achieved by merging similar sequences as in the celebrated Interacting Multiple Model (IMM) filter [4]. The IMM filter enables the propagation over time of approximations to the modes probabilities $\{\mu^j_{k|k} \approx P(m^j_k|z_{1:k})\}_{j\in\mathcal{M}}$ and of Gaussian approximations to the mode-matched filtering pdfs $\{p(x_k|m^j_k, z_{1:k}) \approx \mathcal{N}(x_k;\hat{x}^j_{k|k},P^j_{k|k})\}_{j\in\mathcal{M}}$, so that $p(x_k|z_{1:k}) \approx \sum_{j\in\mathcal{M}} \mu^j_{k|k}\mathcal{N}(x_k;\hat{x}^j_{k|k},P^j_{k|k})$. Its reasonable complexity comes from its internal computation of the mixing probabilities $\{\mu^{j|i}_{k|k} \approx P(m^j_k|m^i_{k+1}, z_{1:k})\}_{(i,j)\in\mathcal{M}\times\mathcal{M}}$, from which Gaussian approximations to the mode-matched prior pdfs $\{p(x_k|m^i_{k+1}, z_{1:k}) \approx \mathcal{N}(x_k;\bar{x}^i_{k|k},\bar{P}^i_{k|k})\}_{i\in\mathcal{M}}$ are deduced. Starting from these last pdfs, only M independent filters (matched to the modes $\{m_{k+1} = i\}_{i\in\mathcal{M}}$) need to be run between times k and $k+1$ in order to get $\{p(x_{k+1}|m^i_{k+1}, z_{1:k}) \approx \mathcal{N}(x_{k+1};\hat{x}^i_{k+1|k},P^i_{k+1|k})\}_{i\in\mathcal{M}}$, $\{p(x_{k+1}|m^i_{k+1}, z_{1:k+1}) \approx \mathcal{N}(x_{k+1};\hat{x}^i_{k+1|k+1},P^i_{k+1|k+1})\}_{i\in\mathcal{M}}$ and update $\{\mu^i_{k+1|k+1} = P(m^i_{k+1}|z_{1:k+1})\}_{i\in\mathcal{M}}$, leading to $p(x_{k+1}|z_{1:k+1})$. Nonlinear jump Markov systems can be dealt with by using approximate nonlinear filters, e.g. unscented Kalman filters [5][6].

A suboptimal solution to the fixed-interval smoothing problem was proposed in [7]. Following Ref. [8], the fixed-interval smoothed estimates are computed by combining the estimates of a forward-time IMM filter and a custom backward-time IMM filter. This backward-time IMM filter is restricted to jump Markov linear systems with invertible state transition matrix and requires to be initialized at terminal time T with a noninformative prior.

This paper focuses on the implementation of a suboptimal fixed-interval smoother algorithm for Markovian switching systems with possibly non-invertible and/or nonlinear state dynamics. Using the statistics provided by the IMM filtering, the proposed algorithm computes the posterior smoothed mean and covariance of the continuous state with a backward-time recursive process. Each recursion exploits first Rauch-Tung-Striebel formulae, then makes modes cooperate inside an interaction stage. The fixed-interval smoother equations are detailed in Sec. 2. They are illustrated on an example of target tracking in Sec. 3.

2. SMOOTHER FOR JUMP MARKOV SYSTEMS

For jump Markov systems, the global (hybrid) state ξ_k is independent of $z_{k+1:T}$ when conditioned on ξ_{k+1}, so that $p(\xi_k|\xi_{k+1}, z_{1:T}) = p(\xi_k|\xi_{k+1}, z_{1:k})$, or, equivalently, $p(x_k, m_k|x_{k+1}, m_{k+1}, z_{1:T}) = p(x_k, m_k|x_{k+1}, m_{k+1}, z_{1:k})$. By marginalizing over m_k, one gets the equality

$$p(x_k|x_{k+1}, m_{k+1}, z_{1:T}) = p(x_k|x_{k+1}, m_{k+1}, z_{1:k}) \tag{3}$$

which is conditioned only on the active mode over the sampling period ending at t_{k+1}.

All distributions are henceforth approximated by Gaussians. From the statistics $\{\hat{x}^j_{k|k}, P^j_{k|k}, \mu^j_{k|k}\}_{j\in\mathscr{M}}$ produced by an IMM filter at times $k = 0, \ldots, T$, the proposed algorithm recursively determines the smoothing mode-conditioned densities $\{p(x_k|m^j_k, z_{1:T}) \approx \mathscr{N}(x_k; \hat{x}^j_{k|T}, P^j_{k|T})\}_{j\in\mathscr{M}}$ and the smoothing mode probabilities $\{\mu^j_{k|T} = P(m^j_k|z_{1:T})\}_{j\in\mathscr{M}}$ for $k = T-1, \ldots, 0$. Each recursion comes as a two-step process.

2.1. First step: mode-matched smoothing

From the knowledge of $p(x_{k+1}|m^i_{k+1}, z_{1:T}) \approx \mathscr{N}(x_{k+1}; \hat{x}^i_{k+1|T}, P^i_{k+1|T})$ and $\mu^i_{k+1|T} = P(m^i_{k+1}|z_{1:T})$ at time $k+1$, the mean and covariance of the smoothed mixing density $p(x_k|m^i_{k+1}, z_{1:T}) \approx \mathscr{N}(x_k; \bar{x}^i_{k|T}, \bar{P}^i_{k|T})$ are first determined with the Rauch-Tung-Striebel [9] formulae

$$G^i_k = C^i_{k,k+1}\left(P^i_{k+1|k}\right)^{-1} \tag{4}$$

$$\bar{x}^i_{k|T} = \bar{x}^i_{k|k} + G^i_k(\hat{x}^i_{k+1|T} - \hat{x}^i_{k+1|k}) \tag{5}$$

$$\bar{P}^i_{k|T} = \bar{P}^i_{k|k} + G^i_k(P^i_{k+1|T} - P^i_{k+1|k})\left(G^i_k\right)^T \tag{6}$$

where

$$C^i_{k,k+1} = \int \left[x_k - \bar{x}^i_{k|k}\right]\left[f^i_k(x_k) - \hat{x}^i_{k+1|k}\right]^T \mathscr{N}(x_k; \bar{x}^i_{k|k}, \bar{P}^i_{k|k})dx_k. \tag{7}$$

The smoothing equations are completely linear except the evaluation of the integral (7) which can be approximated by using an unscented transform [10]. The equations (4)-(7) can be demonstrated by following exactly the proof of [11, Sec. II.A] with all densities conditioned on m^i_{k+1} and using the property (3).

In contrast to the single model smoother, equations (4) to (6) do not end the recursion cycle because the smoothed densities of x_k is conditioned on m^i_{k+1} instead of m^j_k. The following interaction stage bridges the gap between the mode-conditioned smoothed densities $p(x_k|m^i_{k+1}, z_{1:T})$ and $p(x_k|m^j_k, z_{1:T})$.

2.2. Second step: mode interactions

Using the total probability theorem, the targeted mode-conditioned smoothed density $p(x_k|m^j_k, z_{1:T}) \approx \mathscr{N}(x_k; \hat{x}^j_{k|T}, P^j_{k|T})$ can be expressed as a mixture of densities conditioned on the sequence of modes over two consecutive sampling periods, namely

$$p(x_k|m^j_k, z_{1:T}) = \sum_{i\in\mathscr{M}} p(x_k|m^j_k, m^i_{k+1}, z_{1:T})P(m^i_{k+1}|m^j_k, z_{1:T}). \tag{8}$$

The first two moments of the mode-conditioned densities $p(x_k|m^j_k, m^i_{k+1}, z_{1:T}) \approx \mathscr{N}(x_k; \hat{x}^{ji}_{k|T}, P^{ji}_{k|T})$ will now be computed using the statistics from the above smoothing step and from the data produced by IMM filtering. Following [7], the Bayes formula and the Markov property of the mode sequence lead to

$$p(x_k|m^j_k, m^i_{k+1}, z_{1:T}) \propto p(z_{k+1:T}|x_k, m^i_{k+1})p(x_k|m^j_k, z_{1:k}). \tag{9}$$

Similarly, we have

$$\begin{aligned} &p(x_k|m^i_{k+1}, z_{1:T}) \propto p(z_{k+1:T}|x_k, m^i_{k+1})p(x_k|m^i_{k+1}, z_{1:k}) \\ &\Leftrightarrow p(z_{k+1:T}|x_k, m^i_{k+1}) \propto \frac{p(x_k|m^i_{k+1}, z_{1:T})}{p(x_k|m^i_{k+1}, z_{1:k})} \end{aligned} \tag{10}$$

which yields the final equality

$$p(x_k|m^j_k, m^i_{k+1}, z_{1:T}) \propto \frac{p(x_k|m^i_{k+1}, z_{1:T})}{p(x_k|m^i_{k+1}, z_{1:k})}p(x_k|m^j_k, z_{1:k}). \tag{11}$$

The logarithm of the density $p(x_k|m^j_k, m^i_{k+1}, z_{1:T})$ thus writes as $C - \frac{1}{2}J(x_k)$, with C a constant and

$$J(x_k) = \|x_k - \bar{x}^i_{k|T}\|^2_{\left(\bar{P}^i_{k|T}\right)^{-1}} - \|x_k - \bar{x}^i_{k|k}\|^2_{\left(\bar{P}^i_{k|k}\right)^{-1}} + \|x_k - \hat{x}^j_{k|k}\|^2_{\left(P^j_{k|k}\right)^{-1}}. \tag{12}$$

As $p(x_k|m^j_k, m^i_{k+1}, z_{1:T})$ is approximately Gaussian, its mean $\hat{x}^{ji}_{k|T}$ is also its mode and is computed by minimizing $J(x_k)$. Ones gets

$$\hat{x}^{ji}_{k|T} = P^{ji}_{k|T}\left[\left(\bar{P}^i_{k|T}\right)^{-1}\bar{x}^i_{k|T} - \left(\bar{P}^i_{k|k}\right)^{-1}\bar{x}^i_{k|k} + \left(P^j_{k|k}\right)^{-1}\hat{x}^j_{k|k}\right] \tag{13}$$

with

$$P^{ji}_{k|T} = \left[\left(\bar{P}^i_{k|T}\right)^{-1} - \left(\bar{P}^i_{k|k}\right)^{-1} + \left(P^j_{k|k}\right)^{-1}\right]^{-1}. \tag{14}$$

Interestingly, equation (8) of the interaction step is common with [7, Eq. 73] albeit [7] evaluates the smoothed estimate $\hat{x}^{ji}_{k|T}$ by combining the estimates produced by a conventional IMM filter and a backward-time IMM filter restricted to linear systems with invertible state transition matrix. Moreover, this backward-time IMM filter requires to be initialized at time T with no prior information. Note that the maximum likelihood estimate w.r.t. the density $p(z_{k+1:T}|x_k, m^i_{k+1})$, given by

$$\hat{x}^{b,i}_{k|k+1} = P^{b,i}_{k|k+1}\left[\left(\bar{P}^i_{k|T}\right)^{-1}\bar{x}^i_{k|T} - \left(\bar{P}^i_{k|k}\right)^{-1}\bar{x}^i_{k|k}\right] \tag{15}$$

with

$$P^{b,i}_{k|k+1} = \left[\left(\bar{P}^i_{k|T}\right)^{-1} - \left(\bar{P}^i_{k|k}\right)^{-1}\right]^{-1}, \tag{16}$$

corresponds to the "one-step backward-time predicted estimate and error covariance" computed by the backward-time IMM filter of [7].

The smoothed mixing probabilities $\{\bar{\mu}^{i|j}_{k+1|T}=P(m^i_{k+1}|m^j_k, z_{1:T})\}_{(i,j)\in\mathscr{M}^2}$ involved in (8) are expressed as

$$\bar{\mu}^{i|j}_{k+1|T} = \frac{P(m^i_{k+1}|m^j_k)p(z_{k+1:T}|m^j_k, m^i_{k+1}, z_{1:k})}{p(z_{k+1:T}|m^j_k, z_{1:k})} = \frac{\pi_{ji}\Lambda_{ji}}{d_j}, \quad (17)$$

where Λ_{ji} stands for the likelihood $\Lambda_{ji} = p(z_{k+1:T}|m^j_k, m^i_{k+1}, z_{1:k})$ and $d_j = p(z_{k+1:T}|m^j_k, z_{1:k}) = \sum_{i\in\mathscr{M}} \pi_{ji} p(z_{k+1:T}|m^j_k, m^i_{k+1}, z_{1:k})$ is a normalizing constant. According to [7], the value of Λ_{ji} can be approximated by

$$\Lambda_{ji} \approx \mathscr{N}(\Delta^{ji}_k; 0, D^{ji}_k) \quad (18)$$

where $\Delta^{ji}_k = \hat{x}^{b,i}_{k|k+1} - \hat{x}^j_{k|k}$ and $D^{ji}_k = P^{b,i}_{k|k+1} + P^j_{k|k}$. The posterior smoothed mode probability $\mu^j_{k|T} = P(m^j_k|z_{1:T})$ of mode j at time k is given by

$$\mu^j_{k|T} = \frac{p(z_{k+1:T}|m^j_k, z_{1:k})P(m^j_k|z_{1:k})}{\sum\limits_{j\in\mathscr{M}} p(z_{k+1:T}|m^j_k, z_{1:k})P(m^j_k|z_{1:k})} = \frac{d_j \mu^j_{k|k}}{\sum\limits_{j\in\mathscr{M}} d_j \mu^j_{k|k}}. \quad (19)$$

For detection issues, the MAP mode estimate $\hat{j}_k$ at time t_k writes as

$$\hat{j}_k = \underset{j=1,\ldots,M}{\arg\max}\ \mu^j_{k|T}. \quad (20)$$

Importantly, the covariance $P^{b,i}_{k|k+1}$ may not be defined in the first steps of the backward recursion because $\left(P^{b,i}_{k|k+1}\right)^{-1} = \left(\bar{P}^i_{k|T}\right)^{-1} - \left(\bar{P}^i_{k|k}\right)^{-1}$ may not be invertible. Therefore, the smoothed mixing probabilities and the posterior mode probabilities are set to $\bar{\mu}^{i|j}_{k+1|T} = \frac{1}{M}\ \forall i, j$ and $\mu^j_{k|T} = \mu^j_{k|k}\ \forall j$ until $\left(P^{b,i}_{k|k+1}\right)^{-1}$ becomes nonsingular.

The posterior smoothed estimate $\hat{x}^j_{k|T}$ and covariance $P^j_{k|T}$ are eventually computed via a moment-matching approximation with

$$\hat{x}^j_{k|T} = \sum_{i\in\mathscr{M}} \bar{\mu}^{i|j}_{k+1|T} \hat{x}^{ji}_{k|T} \quad (21)$$

and

$$P^j_{k|T} = \sum_{i\in\mathscr{M}} \bar{\mu}^{i|j}_{k+1|T} \left[P^{ji}_{k|T} + (\hat{x}^{ji}_{k|T} - \hat{x}^j_{k|T})(\hat{x}^{ji}_{k|T} - \hat{x}^j_{k|T})^T\right]. \quad (22)$$

These last equations end the smoother recursion.

For output purposes, the overall smoothed density $p(x_k|z_{1:T}) = \sum_{j\in\mathscr{M}} \mu^j_{k|T} p(x_k|m^j_k, z_{1:T})$ can be approximated to its moment-matched Gaussian pdf $\mathscr{N}(x_k; \hat{x}_{k|T}, P_{k|T})$, where

$$\hat{x}_{k|T} = \sum_{j\in\mathscr{M}} \mu^j_{k|T} \hat{x}^j_{k|T} \quad (23)$$

and

$$P_{k|T} = \sum_{j\in\mathscr{M}} \mu^j_{k|T} \left[P^j_{k|T} + (\hat{x}^j_{k|T} - \hat{x}_{k|T})(\hat{x}^j_{k|T} - \hat{x}_{k|T})^T\right]. \quad (24)$$

3. SIMULATION EXAMPLE

A simulated 2D target tracking example is presented to examine the estimation errors and the posterior mode probabilities produced by the multiple-model smoother. In order to compare the algorithm with [7], an invertible state dynamics is considered.

The system state is defined as $x = [x,\ y,\ \dot{x},\ \dot{y}]^T$ where (x, y) term the Cartesian coordinates of the target and $(\dot{x} = \frac{dx}{dt}, \dot{y} = \frac{dy}{dt})$ stand for its velocities. The mode set contains two discrete-time correlated random walks: a first one with a high diffusion parameter $D_1 = 5^2\,\text{m}^2.\text{s}^{-3}$ (maneuvering mode 1) and a second one with a lower diffusion parameter $D_2 = 0.5^2\,\text{m}^2.\text{s}^{-3}$ (nearly Constant Velocity or CV mode 2). The state space equations write as

$$x_{k+1} = \begin{pmatrix} 1 & 0 & \Delta t_{k+1} & 0 \\ 0 & 1 & 0 & \Delta t_{k+1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x_k + q^j_k \quad (25)$$

and

$$Q^j_k = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2D_j\Delta t_{k+1} & 0 \\ 0 & 0 & 0 & 2D_j\Delta t_{k+1} \end{pmatrix} \text{for } j = 1, 2 \quad (26)$$

with $\Delta t_{k+1} = t_{k+1} - t_k$. The measurement z_k is the noisy position of the target in Cartesian coordinates at time t_k and is sampled for $k = 1, \ldots, T$ with period $\Delta t_{k+1} = 5$ s . Thus, the output equation common to all modes is

$$z_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x_k + r_k \text{ with } R_k = 150^2 \mathbb{I}_2\ \text{m}^2. \quad (27)$$

The probability transition matrix is set to

$$\begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.03 \\ 0.03 & 0.97 \end{pmatrix}. \quad (28)$$

The target is tracked for 90 steps (or 450 s) and the trajectory is randomly generated. It evolves first according the maneuvering mode 1, then the nearly CV mode 2 and finally the maneuvering mode 1 again. The switching times between modes occur at the deterministic values of $k = 30$ and $k = 60$. At the initial time $k = 0$, the mode probabilities are assumed equiprobable and the initial position and velocity are arbitrarily set to 0. The algorithm was evaluated over 50 Monte Carlo simulations and an example of trajectory is displayed in Fig. 1.

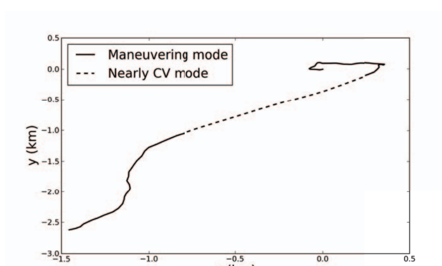

Fig. 1. Example of randomly generated trajectory

The smoother is compared to the IMM filtering solution and the fixed-interval smoothing solution of [7] ("Helmick et al") which requires a backward-time IMM filter initialized with no prior information. As proposed by [7], the initialization is performed by setting at time T the position equal to the final measurement z_T and its covariance; the final velocity is set to 0 with arbitrary great variances of 10^6 m^{2}s^{-2} on $\dot{x}$ and $\dot{y}$; the modes are assumed equiprobable at the terminal time.

The empirical root-mean-square errors (RMSE) over time in position and velocity are shown in Fig. 2(a) and Fig. 2(b) respectively. Fig. 2(c) displays over time the observed average wrong detection probability *i.e.* the average probability of selecting the wrong mode with the MAP of (20). The results show that the smoother is well-behaved with a significant reduction of the RMSE errors in comparison to the filtering solution. The smoother accuracy is similar to the Helmick's smoother when the first mode is active and slightly better when the CV mode is running. The detection of the active model is more efficient than the filter and the Helmick's smoother with less pronounced peaks and a faster reduction of the mode error probability after model jumps.

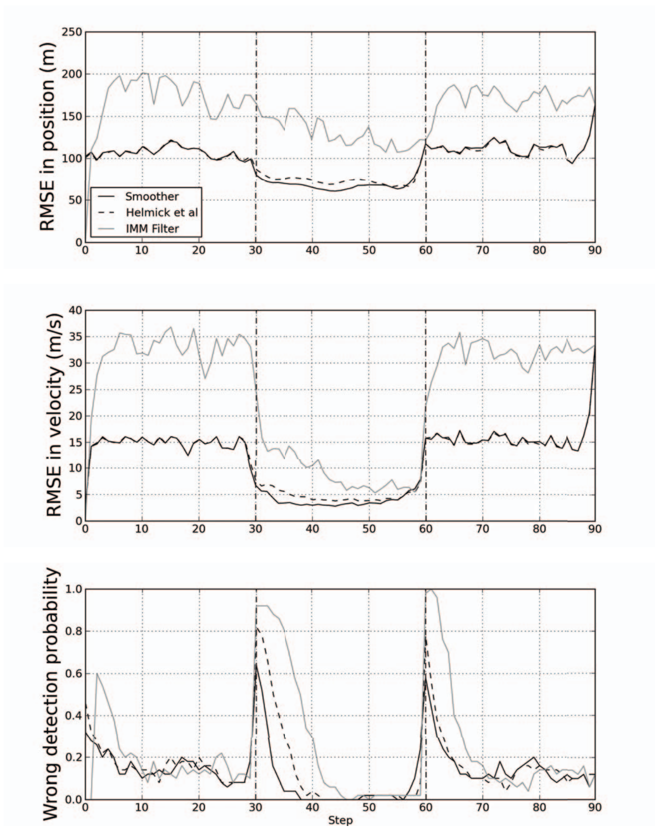

Fig. 2. Comparison of the IMM filter and the smoothers (read top to bottom): (a) RMSE in Position, (b) RMSE in Velocity and (c) Mode error probability.

4. CONCLUSION AND PROSPECTS

This paper investigated a suboptimal fixed-interval smoothing algorithm based on a forward-time IMM filtering and a backward-time recursive process. Each recursion consists of a smoothing step using Rauch-Tung-Striebel equations adapted to jump Markov systems and of a specific interaction step to allow mode cooperation. The first smoothing stage runs M Rauch-Tung-Striebel smoothers in parallel, each one being conditioned on one of the M modes possibly active within the sampling period $(t_k, t_{k+1}]$. The results of the smoothing step are then combined with M^2 interactions related to the M^2 pairs of models of the successive sampling periods $(t_{k-1}, t_k]$ and $(t_k, t_{k+1}]$. The algorithm is suited to nonlinear dynamics and measurement equations. An example of tracking of a maneuvering target shows that the proposed smoother performs significantly better than an IMM filter, and equally well as the two-filter based scheme [7].

Future work will concentrate first on extending the proposed approach to fixed-lag and fixed-point smoothing as suggested in [11] and secondly on adapting it to a bank of heterogeneous-order models, *i.e.* to models which share only parts of their respective state vectors [12].

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