JOINT DETECTION AND TRACKING USING MULTI-STATIC DOPPLER-SHIFT MEASUREMENTS

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ABSTRACT

The problem is to establish the presence and subsequently to track a target using multi-static Doppler shift measurements. The assumption is that in the surveillance volume of interest a single transmitter of known frequency is active with multiple spatially distributed receivers collecting and reporting Doppler-shift frequencies. The measurements are affected by additive noise and also contaminated by false detections. The paper develops a Bernoulli particle filter for this application and analyzes its performance by simulations.

***Index Terms*—** Bayesian estimation, random sets, nonlinear filtering, particle filter

1. INTRODUCTION

The problem of position and velocity estimation of a moving object using measurements of Doppler-shift frequencies at several separate locations has a long history [1, 2, 3]. Renewed interest in this problem is driven by applications, such as passive surveillance, and the technological improvements in wireless sensor networks [4, 5, 6]. The problem can be cast in radar or sonar context. In the radar context, for example, the transmitters (or illuminators) are typically the commercial digital audio/video broadcasters, FM radio transmitters or GSM base stations, whose transmitting frequencies are known. The radar receivers can typically measure the multi-static range, angle and Doppler-shift. The current trend in surveillance, however, is to use many low cost, low power sensors, connected in a network [7]. In line with this trend, this paper investigates the possibility of tracking a moving target using low-cost radars that measure Doppler frequencies only.

Existing literature is mainly focused on *observability* of the target state from the Doppler-shift measurements [4, 5] and geometry-based *localisation algorithms* [8, 3, 6]. In this paper we cast the problem in the nonlinear filtering framework [9]. Moreover, we model target existence by a two-state Markov chain which allows an automatic detection of target presence or absence from the surveillance volume. Finally, we allow for both false detections and miss-detections of the target. The optimal solution for the joint detection and tracking using multi-static Doppler-shifts is thus formulated as a Bernoulli filter in the random set Bayesian estimation framework [10]. This filter is implemented as a particle filter and its performance investigated by a numerical example.

The paper is organised as follows. Section 2 describes the problem. Section 3 presents the Bernoulli filter and explains its particle filter implementation. Section 4 illustrates the Bernoulli-particle filter performance by a numerical example, and finally the conclusions of the study are drawn in Section 5.

2. PROBLEM FORMULATION

The state of the moving object in two-dimensional surveillance area at time t_k is represented by the state vector

$$\mathbf{x}_k = \begin{bmatrix} x_k & \dot{x}_k & y_k & \dot{y}_k \end{bmatrix}^\intercal . \tag{1}$$

where $^\intercal$ denotes matrix transpose. Target position is determined by $\mathbf{p}_k = [x_k \;\; y_k]^\intercal \in \mathbb{R}^2$, while its velocity by $\mathbf{v}_k = [\dot{x}_k \;\; \dot{y}_k]^\intercal \in \mathbb{R}^2$.

Target motion is modelled by a nearly constant velocity (CV) model:

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{u}_k \tag{2}$$

where $\mathbf{F}_k$ is the transition matrix and $\mathbf{u}_k \sim \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{Q}_k)$ is white Gaussian process noise. We adopt:

$$\mathbf{F}_k = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix}, \quad \mathbf{Q}_k = \mathbf{I}_2 \otimes q \begin{bmatrix} \frac{T_k^3}{3} & \frac{T_k^2}{2} \\ \frac{T_k^2}{2} & T_k \end{bmatrix}, \tag{3}$$

where $\otimes$ is the Kroneker product, $T_k = t_{k+1} - t_k$ is the sampling interval and q is the level of power spectral density of the corresponding continuous process noise [11, p.269]. We refer to k as to the discrete-time index.

In order to model target appearance/disappearance we introduce a binary random variable $\epsilon_k \in \{0, 1\}$ referred to as the *target existence* (the convention is that $\epsilon_k = 1$ means that target exists at scan k, and vice versa). Dynamics of ϵ_k is modelled by a two-state Markov chain with transitional probability matrix (TPM) Π whose elements are $[\Pi]_{ij} = P\{\epsilon_{k+1} = j-1 | \epsilon_k = i-1\}$ for $i, j \in \{1, 2\}$. We adopt a TPM as follows:

$$\Pi = \begin{bmatrix} (1-p_b) & p_b \\ (1-p_s) & p_s \end{bmatrix} \tag{4}$$

where $p_b := P\{\epsilon_{k+1} = 1 | \epsilon_k = 0\}$ is the probability of target "birth"and $p_s := P\{\epsilon_{k+1} = 1 | \epsilon_k = 1\}$ the probability of target "survival". These two probabilities together with the initial target existence probability $q_0 = P\{\epsilon_0 = 1\}$ are assumed known.

Target Doppler-shift measurements are collected by spatially distributed sensors (e.g. multi-static Doppler-only radars), as illustrated in Fig.1. A transmitter T at known position $\mathbf{t} = [x_0 \;\; y_0]^\intercal$,

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illuminates the target by a sinusoidal waveform of a known carrier frequency f_c. In Fig.1, the receivers are denoted by R_i, $i = 1, \dots, M = 4$. Target originated Doppler shift frequency,

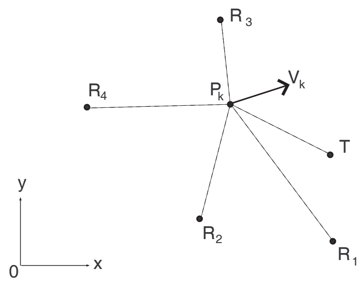


Fig. 1. Multi-static Doppler-only surveillance network in 2D

measured by receiver $i = 1, \dots, M$ placed at known location $\mathbf{r}_i = [x_i \;\; y_i]^\intercal$, is then:

$$z_k^{(i_k)} = h_k^{(i_k)}(\mathbf{x}_k) + w_k^{(i_k)} \tag{5}$$

where

$$h_k^{(i_k)}(\mathbf{x}_k) = -\mathbf{v}_k^\intercal \left[\frac{\mathbf{p}_k - \mathbf{r}_i}{\| \mathbf{p}_k - \mathbf{r}_i \|} + \frac{\mathbf{p}_k - \mathbf{t}}{\| \mathbf{p}_k - \mathbf{t} \|} \right] \frac{f_c}{c} \tag{6}$$

is the Doppler frequency shift, c is the speed of light and $w_k^{(i)}$ is measurement noise in receiver i, modelled by white Gaussian noise with variance $\sigma_w^{(i)}$, and assumed independent of process noise $\mathbf{u}_k$.

The Doppler-shift can be positive or negative. The measurement space is therefore an interval $\mathcal{Z} = [-f_0, +f_0]$, where f_0 is the maximal possible value of Doppler shift, assumed known. Target originated Doppler shifts are collected by all sensors with probability $p_D \leq 1$. False detections can also appear: their distribution over the measurement space $\mathcal{Z}$ is time invariant and denoted by $c(z)$; the number of false detections per scan is assumed to be Poisson distributed, with the constant mean value λ. The measurement set collected by sensor i_k at time t_k is denoted $Z_k^{(i_k)}$. The sensors send their measurements to the fusion center at asynchronous times (whenever available) together with the correct time stamp and the sensor identification.

The problem is to detect when a moving object appears in the surveillance area and then to estimate sequentially its position and velocity vector.

3. BERNOULLI PARTICLE FILTER

3.1. Bernoulli filter

The optimal Bayes filter for the problem described above is the Bernoulli (or JoTT) filter [10, Sec.14.7],[12]. The state of the target is represented by a Bernoulli random finite set $\mathbf{X}$ whose posterior probability density function (PDF) at time k is defined as [10]:

$$f_{k|k}(\mathbf{X}|Z_{1:k}) = \begin{cases} 1 - q_{k|k} & \text{if } \mathbf{X} = \emptyset \\ q_{k|k} \cdot s_{k|k}(\mathbf{x}) & \text{if } \mathbf{X} = \{\mathbf{x}\} \\ 0 & \text{if } |\mathbf{X}| > 1 \end{cases} \tag{7}$$

Here $Z_{1:k} \equiv Z_1^{(i_1)}, \dots, Z_k^{(i_k)}$ is the sequence of measurement sets (originating from various receivers) accumulated up to the current time k; $q_{k|k} := P\{\epsilon_k = 1 | Z_{1:k}\}$ is the posterior probability of target existence and $s_{k|k}(\mathbf{x}) = p(\mathbf{x}_k | Z_{1:k})$ is the posterior spatial PDF of the target.

The Bernoulli filter propagates the posterior $f_{k|k}(\mathbf{X}|Z_{1:k})$ over time in two steps, the *prediction* and *update*. The prediction equations of the Bernoulli filter are [10, Sec.14.7]:

$$q_{k+1|k} = p_b \cdot (1 - q_{k|k}) + p_s \cdot q_{k|k} \tag{8}$$

$$s_{k+1|k}(\mathbf{x}) = \frac{p_b \cdot (1 - q_{k|k}) \cdot b_{k+1|k}(\mathbf{x})}{p_b\,(1 - q_{k|k}) + p_s\, q_{k|k}} + \frac{p_s\, q_{k|k} \int \varphi_{k+1|k}(\mathbf{x}|\mathbf{x}')\, s_{k|k}(\mathbf{x}')\, d\mathbf{x}'}{p_b\,(1 - q_{k|k}) + p_s\, q_{k|k}} \tag{9}$$

The density $\varphi_{k+1|k}(\mathbf{x}|\mathbf{x}')$ in (9) is the target transitional density, which according to (2) is given by $\varphi_{k+1|k}(\mathbf{x}|\mathbf{x}') = \mathcal{N}(\mathbf{x}; \mathbf{F}\mathbf{x}', \mathbf{Q})$. The density $b_{k+1|k}(\mathbf{x})$ is the spatial distribution of predicted "target birth". In the absence of prior knowledge of the state of target birth, this density will have to cover the entire surveillance volume in position and velocity space. Measurements of Doppler-shift will be used to somewhat reduce the uncertainty in the velocity of target birth state.

Upon receiving the measurement set $Z_{k+1} \stackrel{\text{abbr}}{=} Z_{k+1}^{(i_{k+1})}$ from receiver i_{k+1}, and assuming that p_D is state-independent, the Bernoulli filter is updated as follows [10, Sec.14.7]:

$$q_{k+1|k+1} = \frac{1 - \delta_{k+1}}{1 - \delta_{k+1} \cdot q_{k+1|k}} \cdot q_{k+1|k} \tag{10}$$

$$s_{k+1|k+1}(\mathbf{x}) = \frac{1 - p_D + p_D \sum\limits_{z \in Z_{k+1}} \frac{g_{k+1}(z|\mathbf{x})}{\lambda c(z)}}{1 - \delta_{k+1}}\, s_{k+1|k}(\mathbf{x}) \tag{11}$$

where

$$\delta_{k+1} = p_D \left(1 - \sum_{z \in Z_{k+1}} \frac{\int g_{k+1}(z|\mathbf{x})\, s_{k+1|k}(\mathbf{x})\, d\mathbf{x}}{\lambda\, c(z)} \right), \tag{12}$$

and $g_{k+1}(z|\mathbf{x})$ is the measurement likelihood function for a target originated measurement. According to (5) the likelihood for a reporting receiver i, is given by $g_{k+1}(z|\mathbf{x}) = \mathcal{N}\left(z; h_{k+1}^{(i)}(\mathbf{x}), {\sigma_w^{(i)}}^2\right)$.

In this paper we will consider only the state-independent p_D. The update equation of the Bernoulli filter for the state-dependent p_D (which is more realistic, since p_D is a function of the signal-to-noise ratio) are slightly more involved, see [10, Sec.14.7].

3.2. Particle filter implementation

The sequential Monte Carlo method provides a general framework for the implementation of optimal Bayes filters, both in the context of single and multiple targets [13], [10]. When this method is applied to the Bernoulli filter, the resulting Bernoulli particle filter (PF) will approximate the spatial PDF $s_{k|k}(\mathbf{x})$ by a set of weighted random samples or particles. In fact there are two types of particles, corresponding to the persistent and newborn target, distinguished in notation by subscripts p and b, respectively. The pseudo-code of the Bernoulli PF for $k = 1, 2, \dots,$ is given in Algorithm 1. The inputs to the algorithm are: (1) the probability of existence $q_{k|k}$; (2) the set of persistent particles $\mathcal{P}_{k,p} \equiv \{w_{k,p}^n, \mathbf{x}_{k,p}^n\}_{n=1}^{N_p}$, where $\mathbf{x}_{k,p}^n$ is the state of persistent particle n and $w_{k,p}^n$ is its corresponding normalised weight, i.e. $\sum_{n=1}^{N_p} w_{k,p}^n = 1$; the spatial PDF $s_{k|k}(\mathbf{x})$ is approximated as

$$s_{k|k}(\mathbf{x}) \approx \sum_{n=1}^{N_p} w_{k,p}^n \, \delta_{\mathbf{x}_{k,p}^n}(\mathbf{x}) \tag{13}$$

where $\delta_{\mathbf{a}}(\mathbf{x})$ is the Dirac delta function concentrated at $\mathbf{a}$; (3) the set of newborn particles $\mathcal{P}_{k,b} \equiv \{w_{k,b}^n, \mathbf{x}_{k,b}^n\}_{n=1}^{N_{k,b}}$ whose weights are also normalised; (4) the new set of Doppler-shift measurements Z_{k+1} from sensor i_{k+1}.

Algorithm 1 Pseudo-code of the Bernoulli PF at $k = 1, 2, \dots$

1: Input: $q_{k|k}$, $\mathcal{P}_{k,p}$, $\mathcal{P}_{k,b}$ Z_{k+1}
2: Predict existence probability $q_{k+1|k}$ using (8)
3: Predict birth particles $\mathcal{P}_{k+1|k,b}$: for $n = 1, \dots, N_{k,b}$ do $\mathbf{x}_{k+1|k,b}^n \sim \varphi(\mathbf{x}|\mathbf{x}_{k,b}^n)$; $w_{k+1|k,b}^n = p_b(1-q_{k|k})/(p_b(1-q_{k|k}) + p_s q_{k|k})$.
4: Predict persistent particles $\mathcal{P}_{k+1|k,p}$: for $n = 1, \dots, N_p$ do $\mathbf{x}_{k+1|k,p}^n \sim \varphi(\mathbf{x}|\mathbf{x}_{k,p}^n)$; $w_{k+1|k,p}^n = p_s q_{k|k}/(p_b(1-q_{k|k}) + p_s q_{k|k})$.
5: Union: $\{w_{k+1|k}^n, \mathbf{x}_{k+1|k}^n\}_{n=1}^{N_{k+1}} = \mathcal{P}_{k+1|k,b} \cup \mathcal{P}_{k+1|k,p}$
6: Compute $\delta_{k+1} \approx p_D \left(1 - \sum_{z \in Z_{k+1}} \frac{\sum_n g_{k+1}(z|\mathbf{x}_{k+1|k}^n)\, w_{k+1|k}^n}{\lambda\, c(z)}\right)$
7: Update prob. existence $q_{k+1|k+1}$ using (10)
8: Update weights $w_{k+1|k+1}^n$ for $n = 1, \dots, N_{k+1}$ using (11):

$$w_{k+1|k+1}^n = \left[1 - p_D + p_D \sum_{z \in Z_{k+1}} \frac{g_{k+1}(z|\mathbf{x}_{k+1|k}^n)}{\lambda\, c(z)}\right] \frac{w_{k+1|k}^n}{1 - \delta_{k+1}}$$

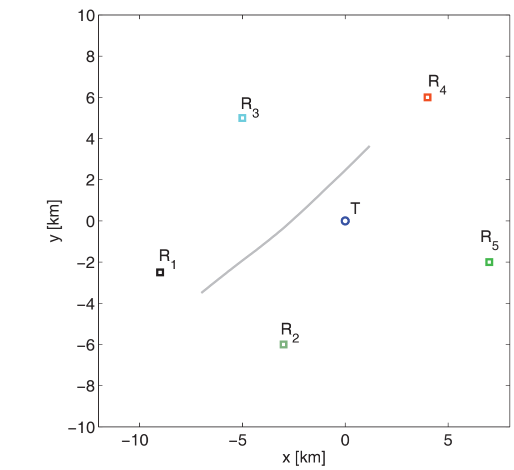
9: Resample N_p times from $\{w_{k+1|k+1}^n, \mathbf{x}_{k+1|k}^n\}_{n=1}^{N_{k+1}}$ to obtain $\mathcal{P}_{k+1,p} \equiv \{w_{k+1,p}^n = \frac{1}{N_p}, \mathbf{x}_{k+1,p}^n\}_{n=1}^{N_p}$
10: Create newborn particles using Z_{k+1}: $\mathcal{P}_{k+1,b}$
11: Report $q_{k+1|k+1}$ and $\mathcal{P}_{k+1,p}$, which approximates $s_{k+1|k+1}(\mathbf{x})$.
12: Output for the next time step: $q_{k+1|k+1}$, $\mathcal{P}_{k+1,p}$, $\mathcal{P}_{k+1,b}$

Steps 3-5 in Algorithm 1 implement eq.(9). After resampling (step 9) we apply MCMC move step [13, p.55] in order to increase the particle diversity. Step 10 in Algorithm 1 creates newborn target particles. This is carried out for each $z \in Z_{k+1}$ by the Accept-Reject method [14]: draw samples from a multivariate Gaussian $\mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \mathbf{C})$ and accept them if their velocity vector is compatible with the Doppler measurement z. For each measurement we create in this way $N_p/2$ newborn particles, that is $N_{k+1,b} = |Z_{k+1}| \times N_p/2$. The weights of newborn particles are uniform. More details about the Bernoulli PF implementation can be found in [15].

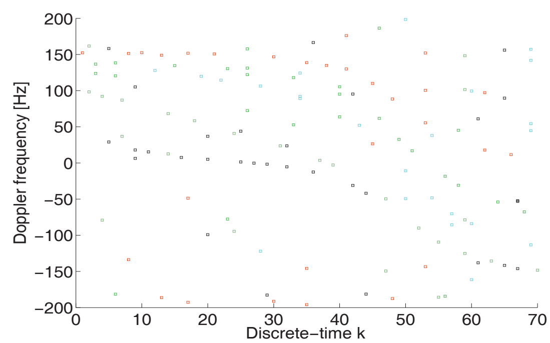
4. NUMERICAL RESULTS

The Bernoulli PF for multi-static Doppler-only radar was tested using the scenario plotted in Fig.2.a. There were five receivers and the target was following the trajectory indicated by the solid line in Fig.2.a. The true target initial state was set to $\mathbf{x}_0 = [-7\text{km} \quad 20\text{m/s} \quad -3.5\text{km} \quad 16\text{m/s}]^\intercal$. The target was present throughout the surveillance interval of 420 seconds.

Every $T = 6$ seconds one of the receivers (chosen at random) reports its set of measurements $Z_k^{(i_k)}$. The measurement noise standard deviation, the probability detection and the clutter parameters for all five receivers were identical: $\sigma_k = 0.5$Hz, $p_D = 0.9$, $\lambda = 1$ and $c(z) = 1/400$Hz for $z \in [-200\text{Hz}, 200\text{Hz}]$ and zero otherwise. The carrier frequency used in the testing setup was $f_c = 900$ MHz. Fig.2.b shows a typical set of Doppler measurements over time. The colour coding corresponds to the colour coding of the receivers in Fig.2.a (e.g. receiver R_4 is in indicated by a *red* square in Fig.2.a and its measurements are shown as *red* dots in Fig.2.b).



(a)



(b)

Fig. 2. Testing scenario: (a) target trajectory and the locations of the transmitter (T) and five receivers (R_1,...,R_5); (b) A typical set of Doppler measurements over time (from all receivers)

The Bernoulli PF was implemented using birth parameters $\boldsymbol{\mu} = [0 \ 0 \ 0 \ 0]^\intercal$, $\mathbf{C} = \text{diag}[(7\text{km})^2 \ (20\text{m/s})^2 \ (7\text{km})^2 \ (20\text{m/s})^2]$, and $N_p = 10000$, $p_b = 0.01$, $p_s = 0.99$ and $q_0 = 0$. A single run of the Bernoulli PF is illustrated in Figs.3–5. The existence of the target is established very quickly, after only a few scans (see the plot of $q_{k|k}$ over time in Fig.3). The probability $q_{k|k}$ remains at the value of 1.0 for most of the time, except when the target detection was missing (e.g. $k = 50$). To estimate the target state accurately it took almost 20 additional scans. This is because the posterior spatial PDF $s_{k|k}(\mathbf{x})$ remains diffuse and multi-modal for a number of scans until it eventually concentrates on the true target state (see Fig.4).

This is all reflected in the optimal sub-pattern assignment (OSPA) error [16] (see Fig.5) which was computed here using parameters $p = 1$ and $c = 10$km (these parameters are chosen so that OSPA error represents the sum of the localisation error and the

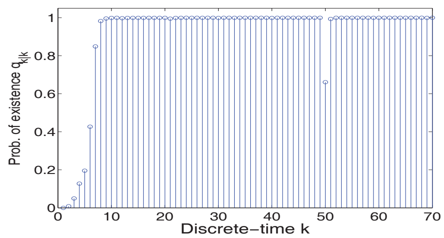


Fig. 3. Probability of existence $q_{k|k}$ over time

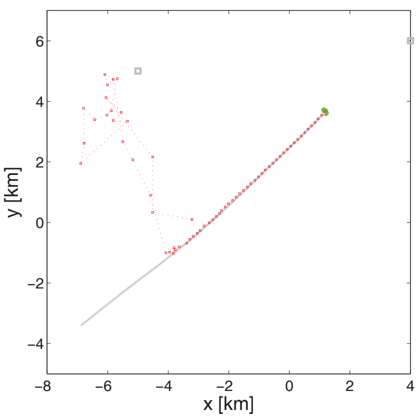


Fig. 4. Estimated (red) and true (gray) tracks; green dots are particles

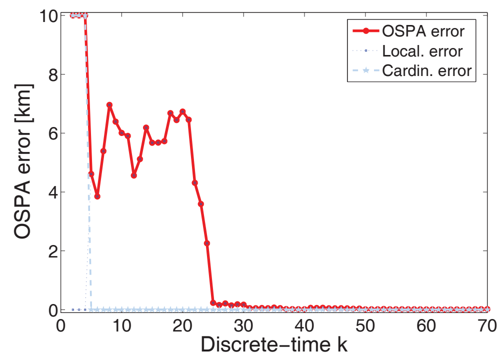


Fig. 5. OSPA error over time

cardinality error; $c = 10$km is the penalty assigned to the cardinality error). Initially, in the first 5 scans, the OSPA error is dominated by the cardinality error (during this period $q_{k|k}$ is below the threshold for track detection/formation, here adopted at 0.5). This is followed by a period of a large localisation error, which eventually (after about 25 scans) drops to almost zero.

5. CONCLUSIONS

The paper developed a Bernoulli particle filter for joint detection and tracking of a target using multi-static Doppler-only radar receivers. Target existence, clutter (false detections) and miss-detections have all been included in the model. Numerical simulations demonstrate that the Bernoulli PF is able to quickly establish the presence of a target. The target spatial posterior density, on the other hand, remains diffuse and multi-modal for a longer period of time, until it eventually focuses on the target. Future work will consider state-dependent probability of detection and sensor scheduling for energy conservation.

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