STEP-EDGE RECONSTRUCTION USING 2D FINITE RATE OF INNOVATION PRINCIPLE

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ABSTRACT

Parametric signals that have a finite number of degrees of freedom per unit of time are defined as signals with Finite Rate of Innovation (FRI). Sampling and reconstruction schemes have been developed based on the 1D FRI principle and applied to reconstructing step edge images on a row by row basis. In this paper, we derive the 2D FRI principle by exploiting the separability of the B-spline sampling kernel. The proposed 2D FRI principle regards the sampling and reconstruction as block by block operations. The step-edge parameters can be retrieved in high accuracy with no post-processing. The performance on synthetic images shows that our proposed technique is more precise than the row by row approaches on Signal-to-Noise Ratio (SNR) levels larger than 4 dB. Experimental results on real images demonstrate that the proposed method can reconstruct the step-edge precisely under noisy and practical sampling conditions.

Index Terms— Two dimensional Finite Rate of Innovation, Step-edge reconstruction, B-spline kernel.

1. INTRODUCTION

The signals with finite rate of innovation (FRI) are defined such that they can be expressed in a parametric form and have a finite number of degrees of freedom per unit of time [1]. New sampling schemes have been developed for perfect reconstruction of the FRI signals by using Sinc or Gaussian sampling kernels. Dragotti *et al.* generalized the sampling scheme to three types of kernels with compact support [2], i.e., polynomial reproducing kernels, exponential reproducing kernels, and rational kernels.

Following [2], Shukla *et al.* extended the sampling schemes to multi-dimension with the polynomial reproducing kernels and proposed three reconstruction schemes [3]. Nevertheless, these reconstruction schemes were developed under the noise free assumption. Baboulaz *et al.* developed a local reconstruction scheme for the step-edge with a polynomial reproducing kernel [4], i.e., the B-spline kernel. The extracted local features can be applied to the registration step in a super-resolution task, and had better performances compared to the traditional approach. Hirabayashi *et al.* proposed reconstruction schemes [5, 6] with the trigonometric and hyperbolic E-spline kernels to achieve better accuracy. Baboulaz's and Hirabayashi's approaches were still based on the 1D FRI principle which treats the images row by row. However, in the image acquisition process the samples are always affected by both horizontal and vertical neighborhoods.

In this paper, the image is regarded on a block by block basis so as to exploit the vertical correlations between different rows. We derive the 2D FRI principle by exploiting the separability of the B-spline kernel. The 2D FRI principle regards the sampling and reconstruction as block by block operations. Reconstruction results on both synthetic and real step-edge images demonstrate that the proposed reconstruction scheme is more precise with Signal-to-Noise Ratio (SNR) levels larger than 4 dB.

This paper is organized as follows: Section 2 briefly reviews the existing step-edge reconstruction approaches. Section 3 presents our proposed method based on the 2D FRI principle. Comparisons on both synthetic and real step-edge images between the proposed method and the existing approaches are given in Section 4. Finally, Section 5 concludes this paper.

2. EXISTING STEP-EDGE RECONSTRUCTION METHODS

The Hough transform [7] is a traditional step-edge reconstruction approach which includes two steps, i.e., an edge detection step and a voting step for parameter estimation. Popovici *et al.* [8] developed a step-edge reconstruction method by the Custom-built moments which use a testing function in an integral to find the edge parameters.

The 1D FRI principle [1] has been developed to retrieve the signal parameters from its sampled version. Recently, Baboulaz *et al.* [4] treated a step-edge as rows of 1D FRI signal and reconstructed it in a sampling framework using the B-spline kernel. First, the 1D moments were obtained by a weighted sum of the differentiated samples in each row. The step-edge parameters in each row were then found in terms of the 1D moments. Finally, the estimation process was iterated row by row along an edge. The estimated step-edge parameters were obtained by averaging over edge points that have similar parameters. However, only the 1D moments from two consecutive rows were considered at a time in the estimation process, and the estimation results from different rows can have large variations under noisy condition. The edge points from the same step-edge cannot be identified precisely by the similarity measure stated in [4]. Even when the estimation results are averaged over multiple rows, the errors are still significant. Such limitation can also be found in Hirabayashi's approaches [5, 6].

3. PROPOSED RECONSTRUCTION SCHEME BASED ON THE 2D FRI PRINCIPLE

3.1. The Sampling Setup

The sampling setup that is considered in this paper can be described by

$$\begin{aligned} g(m,n) =& \frac{1}{T^2}\int\int f(x,y)\beta(x/T-m,y/T-n)dxdy \\ =& \frac{1}{T^2}\Big\langle f(x,y),\beta(x/T-m,y/T-n)\Big\rangle \end{aligned} \tag{1}$$

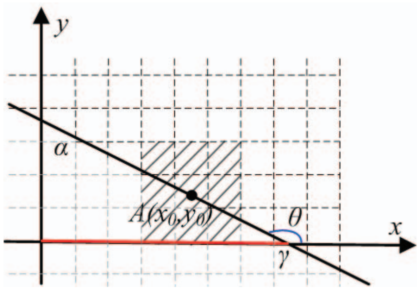

Fig. 1: The illustration for step-edge model with amplitude α, orientation θ and offset γ. The pixels are represented as grids.

where a 2D FRI signal in continuous domain $f(x, y)$ is filtered by the B-spline kernel $\beta(x, y)$ with sampling period T, the filtered version $g(x, y)$ is sampled and discretized to yield the sample values $g(m, n)$, and $\langle \cdot, \cdot \rangle$ is the inner product operation.

Here, we briefly describe the B-spline kernel and its properties to facilitate our further discussion. A 1D zeroth order B-spline sampling kernel $\beta^{(0)}(x)$ [9] is given as:

$$\beta^{(0)}(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ \frac{1}{2}, & |x| = \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} . \tag{2}$$

By successively convolving $P+1$ zeroth order B-spline kernels, the B-spline kernels of order P can be obtained [9]. Note that the B-spline kernel is a polynomial reproducing kernel [2]. For $T = 1$, we have

$$\sum_{m \in \mathbb{Z}} c_m^{(p)} \beta^{(P)}(x - m) = x^p, \ p = 0, 1, \ldots, P \tag{3}$$

where $c_m^{(p)}$ is the the B-spline coefficient. The B-spline kernel of order P can reproduce a polynomial up to order P. The polynomial reproducing property of the B-spline kernel can be extended to two dimensions [9] as follows

$$\begin{aligned} x^p y^q &= \sum_m c_m^{(p)} \beta^{(P)}(x - m) \sum_n c_n^{(q)} \beta^{(Q)}(y - n) \\ &= \sum_m \sum_n c_{m,n}^{(p,q)} \beta^{(P,Q)}(x - m, y - n) \end{aligned} \tag{4}$$

where $p = 0, \ldots, P, q = 0, \ldots, Q, c_{m,n}^{(p,q)} = c_m^{(p)} \cdot c_n^{(q)}, \beta^{(P,Q)}(x, y) = \beta^{(P)}(x) \otimes \beta^{(Q)}(y)$ and '$\otimes$' is the tensor product operation. In this paper, we set $P = Q$.

3.2. The Proposed Reconstruction Scheme

As is shown in Fig. 1, in the sampling and reconstruction process, the value at point $A(x_0, y_0)$ is determined by its neighboring samples covered by the sampling kernel (illustrated by a shaded 3×3 pixels region). We consider all related samples in the parameter estimation step, rather than averaging the estimation results from each row in the post-processing step.

The step-edge in the continuous domain is parameterized by the amplitude α, orientation θ and offset γ [5]

$$h(x, y) = \alpha \cdot H(-x \sin\theta + y \cos\theta + \gamma \sin\theta) \tag{5}$$

where $H(x, y)$ is a unit step function. The differentiated samples are formulated by subsampling the horizontal derivative of the step-edge $h(x, y)$ using a modified 2D B-spline kernel [5], that is,

$$d(m, n) = \frac{1}{T^2} \left\langle \frac{\partial h(x, y)}{\partial x}, \beta^{(P+1)}\left(\frac{x}{T} - m - \frac{1}{2}\right) \otimes \beta^{(P)}\left(\frac{y}{T} - n\right) \right\rangle \tag{6}$$

where $\beta^{(P+1)}(\frac{x}{T} - m - \frac{1}{2}) \otimes \beta^{(P)}(\frac{y}{T} - n)$ is the modified 2D B-spline kernel.

Here, we present our 2D reconstruction scheme for a step-edge $h(x, y)$ with range $x \in \mathcal{X} = [x_0, x_1], y \in \mathcal{Y} = [y_0, y_1]$. Note that the corresponding step edge image is defined over $m \in \mathcal{M} = \{m_0, m_1, \ldots, m_I\}, n \in \mathcal{N} = \{n_0, n_1, \ldots, n_J\}$, where $I, J \in \mathbb{Z}$ and vary according to the image size. Firstly, the 2D moments of the image block are found by extending the 1D FRI principles [2] to two dimensions, that is,

$$\begin{aligned} &\tau_{\mathcal{M},\mathcal{N}}^{(p,q)} \\ &= \int_{\mathcal{Y}} \int_{\mathcal{X}} \frac{\partial h(x, y)}{\partial x} (x - \frac{1}{2})^p y^q dx dy \qquad (7) \\ &\overset{(a)}{=} \frac{1}{T^2} \int_{\mathcal{Y}} \int_{\mathcal{X}} \frac{\partial h(x, y)}{\partial x} \Big[\sum_{m \in \mathcal{M}} c_m^{(p)} \beta^{(P)}(\frac{x}{T} - m - \frac{1}{2}) * \\ &\quad \beta^{(0)}(\frac{x}{T} - m - \frac{1}{2}) \Big] \Big[\sum_{n \in \mathcal{N}} c_n^{(q)} \beta^{(P)}(\frac{y}{T} - n) \Big] dx dy \\ &\overset{(b)}{=} \frac{1}{T^2} \int_{\mathcal{Y}} \int_{\mathcal{X}} \frac{\partial h(x, y)}{\partial x} \Big[\sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} c_{m,n}^{(p,q)} \times \\ &\quad \beta^{(P,P)}(\frac{x}{T} - m - \frac{1}{2}, \frac{y}{T} - n) * \beta^{(0)}(\frac{x}{T} - m - \frac{1}{2}) \Big] dx dy \\ &= \frac{1}{T^2} \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} c_{m,n}^{(p,q)} \int_{\mathcal{Y}} \int_{\mathcal{X}} \frac{\partial h(x, y)}{\partial x} \times \\ &\quad \beta^{(P+1,P)}(\frac{x}{T} - m - \frac{1}{2}, \frac{y}{T} - n) dx dy \\ &\overset{(c)}{=} \sum_{n \in \mathcal{N}} \sum_{m \in S_n} c_{m,n}^{(p,q)} d(m, n) \qquad (8) \end{aligned}$$

where in (a) we have used the polynomial reproducing property of the modified B-spline kernel [2], in (b) we have exploited the separability of the kernel as given in Eq. (4), and in (c) Eq. (6) has been applied and we have identified S_n as the set of pixel indices that are affected by the step-edge in each row. Hence, the 2D moments can be obtained by a linear combination of the B-spline coefficients $c_{m,n}^{(p,q)}$ and the differentiated samples $d(m, n)$ covered by the sampling kernel.

After retrieving the 2D moments, we substitute the step-edge model Eq. (5) into Eq. (7). By considering the sign of $\sin\theta$, a closed form expression for $\tau_{\mathcal{M},\mathcal{N}}^{(p,q)}$ can be obtained

$$\tau_{\mathcal{M},\mathcal{N}}^{(p,q)} = \alpha \cdot \mathrm{sgn}(\sin\theta) \sum_{i=0}^{p} \binom{p}{i} \frac{(\gamma - \frac{1}{2})^i}{(\tan\theta)^{p-i}} \frac{y_0^K - y_1^K}{K} \tag{9}$$

where $\mathrm{sgn}(\cdot)$ takes the sign of a real number, and $K = p + q - i + 1$.

We need at least three sets of (p, q) to solve for α, θ and γ. The smallest possible values of (p, q) should be chosen due to the characteristic of B-spline coefficients [9], i.e., having a horizontal growth rate of order p and a vertical growth rate of order q. For large (p, q), highly different weights will be assigned to the samples which are far from or near the kernel centers. This makes the reconstruction process sensitive to noise [5]. We choose $(p, q) = (0, 0)$ and $(1, 0)$, so that the B-spline coefficients will be constant or increasing linearly.

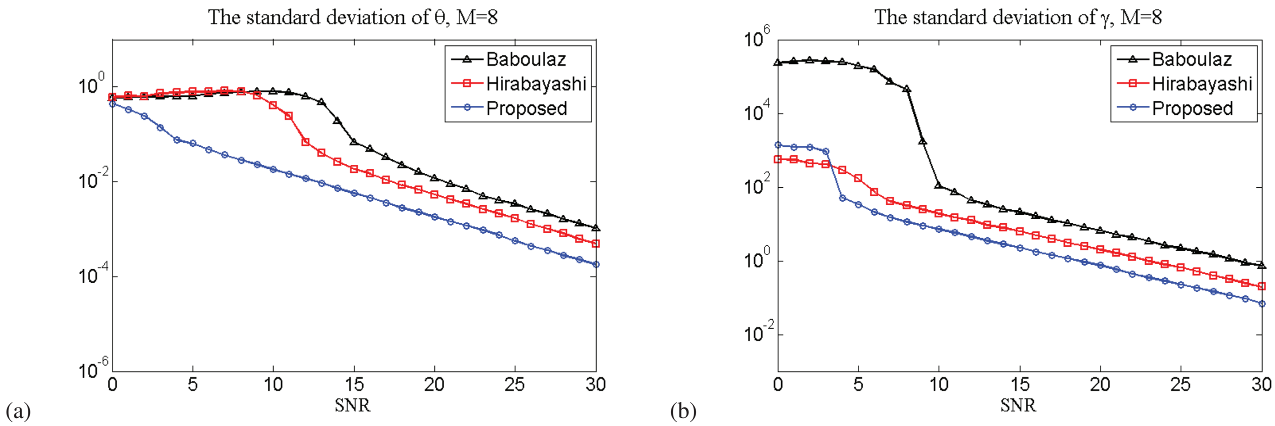

Fig. 2: The experimental results of Baboulaz's approach [4], Hirabayashi's approach [5] and our proposed approach. (a) the standard deviation for θ, (b) the standard deviation for γ.

The image block is of size $N \times M$ which is divided into the upper and lower parts, i.e., $m \in \mathcal{M} = \{m_0, m_1, \ldots, m_I\}$ and $n \in \mathcal{N}_u \cup \mathcal{N}_l = \{n_0, n_1, \ldots, n_0 + N/2 - 1\} \cup \{n_0 + N/2, n_0 + N/2 + 1, \ldots, n_J\}$. It corresponds to $x \in [x_0, x_1]$ and $y \in [y_0, y_0 + N/2] \cup (y_0 + N/2, y_1]$ in the continuous domain. Then, these two parts are substituted into the closed form expression Eq. (9). For $\theta \in [0, \pi]$, we have

$$\begin{aligned}
\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_u} &= \alpha\big[y_0 - (y_0 + \tfrac{N}{2})\big] \\
\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_l} &= \alpha\big[(y_0 + \tfrac{N}{2}) - y_1\big] \\
\tau^{(1,0)}_{\mathcal{M},\mathcal{N}_u} &= \alpha\big[(\gamma - \tfrac{1}{2})(y_0 - (y_0 + \tfrac{N}{2})) + \frac{y_0^2 - (y_0 + \frac{N}{2})^2}{2\tan\theta}\big] \\
\tau^{(1,0)}_{\mathcal{M},\mathcal{N}_l} &= \alpha\big[(\gamma - \tfrac{1}{2})((y_0 + \tfrac{N}{2}) - y_1) + \frac{(y_0 + \frac{N}{2})^2 - y_1^2}{2\tan\theta}\big].
\end{aligned} \tag{10}$$

Similarly, for $\theta \in (\pi, 2\pi]$, we substitute the two image parts into Eq. (9) which leads to another system of equations with $\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_u}, \tau^{(0,0)}_{\mathcal{M},\mathcal{N}_l}, \tau^{(1,0)}_{\mathcal{M},\mathcal{N}_u}$ and $\tau^{(1,0)}_{\mathcal{M},\mathcal{N}_l}$. Together with Eq. (10), we solve the two systems of equations for the step edge parameters as

$$\begin{aligned}
\alpha &= -\operatorname{sgn}(\sin\theta)\frac{\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_u} + \tau^{(0,0)}_{\mathcal{M},\mathcal{N}_l}}{N} \\
\tan\theta &= \frac{N(\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_u} + \tau^{(0,0)}_{\mathcal{M},\mathcal{N}_l})}{4(\tau^{(1,0)}_{\mathcal{M},\mathcal{N}_l} - \tau^{(1,0)}_{\mathcal{M},\mathcal{N}_u})} \\
\gamma &= \frac{3 \cdot \tau^{(1,0)}_{\mathcal{M},\mathcal{N}_u} - \tau^{(1,0)}_{\mathcal{M},\mathcal{N}_l}}{\tau^{(0,0)}_{\mathcal{M},\mathcal{N}_u} + \tau^{(0,0)}_{\mathcal{M},\mathcal{N}_l}} + \frac{1}{2}
\end{aligned} \quad . \tag{11}$$

To this end, we have found the relationship between the 2D moments of the differentiated samples and the step-edge parameters. No averaging step is needed.

Our step-edge reconstruction approach can be briefly summarized as follows:

1. Compute the horizontal differentiated samples $d(m, n)$;
2. Find the edge map of $h(x, y)$ using an edge detector, e.g., the Canny edge detector;
3. Trace the edge pixels along each row to determine the region of interest (ROI), i.e., S_n in each row;
4. Use the horizontal differentiated samples in ROI to compute the 2D moments $\tau^{(p,q)}_{\mathcal{M},\mathcal{N}_u}$ and $\tau^{(p,q)}_{\mathcal{M},\mathcal{N}_l}$ with Eq. (8);
5. Find the set of step-edge parameters $\alpha, \tan\theta$, and γ with the obtained 2D moments and Eq. (11).

4. COMPARISONS WITH THE EXISTING APPROACHES

4.1. Experimental Results on Synthetic Step-Edges

In this section, we compare the results of our proposed method with those obtained using Baboulaz's approach [4] (using the same sampling kernel) and Hirabayashi's approach [5] (which has achieved the best accuracy in the literature) on low resolution (LR) synthetic image blocks. A high resolution (HR) synthetic step-edge image with parameters $\alpha = 1$, $\theta = \pi/4$, $\gamma = 0$ and size 512×512 pixels is created. The LR versions of size 8×8 pixels are obtained by filtering and downsampling the HR step-edge images using the B-spline and E-spline kernels, respectively. The kernel centers are set as the image centers. The Gaussian noise with zero mean and variance σ_n^2 is added to the LR images, and the SNR level varies from 0 to 30 dB. From the noisy and heavily downsampled step-edge image, the ground truth step-edge parameters in the high resolution image are to be estimated. The standard deviations of the estimations are computed over 1000 realizations.

Figure 2 compares the standard deviation for the step-edge parameter orientation θ and offset γ of the proposed technique with the row by row approaches. In Fig. 2 (a), we show the performance of estimation errors for parameter θ with SNR levels from 0-30 dB. The proposed reconstruction scheme can achieve better estimation accuracy by treating the step-edge as a 2D FRI signal. It is precise under an SNR level as low as 4 dB. Meanwhile, the SNR levels that required for the other approaches to be accurate are much higher, i.e., 15 dB for Baboulaz's approach and 13 dB for Hirabayashi's approach. Baboulaz's approach is not able to estimate the step-edge parameters precisely due to the polynomial growth rate of B-spline coefficients [5]. Hirabayashi's approach obtains better estimation accuracy in the moderate noise condition by employing a trigonometric E-spline sampling kernel. For the high SNR scenario, we also obtain a significant improvement in the estimation accuracy. The standard deviations for θ under SNR level of 30 dB are 1.8×10^{-4} for the proposed technique, while 10.5×10^{-4} and 5.0×10^{-4} for Baboulaz's method and Hirabayashi's method, respectively. In Fig. 2 (b), a similar result for the estimation errors for γ under different SNR levels are observed. The proposed approach outperforms the row by row

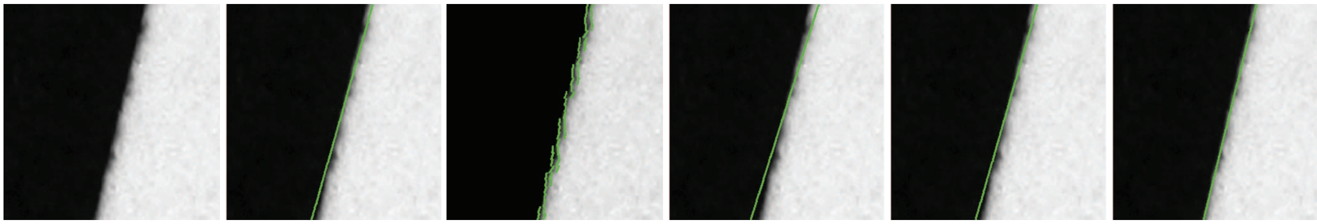

Fig. 3: Comparison of step-edge extraction performances on a real image block. (a) A real step-edge of size 256×256 pixels, (b) the Hough transform, (c) Popovici's method [8], (d) Baboulaz's method [4], (e) Hirabayashi's method [5], (f) the proposed method.

reconstruction schemes in the estimation accuracy under SNR levels from 4-30 dB. Although Hirabayashi's approach shows a better performance under SNR level of 4 dB, the standard deviation of estimation errors are above 100 pixels in a rather small size 512×512 image.

4.2. Experimental Results on Real Step-Edges

In this section, the case of step-edge extraction on a real image is considered. The image is cropped out from a real image that is captured by a Canon EoS 450D DSLR with settings of 1/60s, F5.6, and ISO 800 (see Fig. 3(a)). Thus, the image includes different types of noise from the acquisition process [10].

The traditional Hough transform [7] estimates the step-edge parameters based on the Canny edge detector and followed by a voting procedure. It produces multiple lines for a blurred step-edge due to the presence of several nearby Hough-space peaks [11]. Here, we select the peak with the highest response. However, the performance of the Hough transform is still degraded by the inaccurate edge positions due to the presence of noise as shown in Fig. 3 (b). For Popovici's method [8], the step-edge parameter estimation is done on a block by block basis for the whole image without locating the region of interest. The block size is selected experimentally as 8×8 pixels. With other parameters set to default, the reconstructed step-edge in Fig. 3 (c) is discontinuous and contains spurious responses.

For Baboulaz's and Hirabayashi's approaches, due to large variation of the estimation parameters in each row, no edge points can be merged by the similarity measure [4]. Here, the step-edges are obtained by averaging the parameters from all edge points. As shown in Fig. 3 (d) and (e), the extracted results by Baboulaz's and Hirabayashi's approaches deviate from the real orientation.

For the proposed approach, a B-spline kernel of order 7 as given in [4] is used to simulate the sampling kernel in the camera. A more precise and systematic calibration of the real sampling kernel is to be done. The proposed 2D FRI principle considers all samples from the image at a time. It is shown in Fig. 3 (f) that the edge orientation and location can be retrieved precisely without any postprocessing even with noisy step-edge image.

5. CONCLUSION

In this paper, an approach for extracting step-edge parameters using the 2D FRI principle which treats the images block by block is proposed. Experimental results on both synthetic and real step-edge images show that the proposed approach outperforms existing methods with the 1D FRI principle which treats the images row by row under SNR levels higher than 4 dB. The step-edge extraction method has found its application in local feature extraction for super-resolution technique. With the improvement in the accuracy of step-edge extraction, better performance in the image super-resolution task can be expected. A more detailed version of the paper has been submitted for review [12].

6. REFERENCES

[1] M. Vetterli, P. Marziliano, and T. Blu, "Sampling signals with Finite Rate of Innovation," *IEEE Trans. on Signal Process.*, vol. 50, no. 6, pp. 1417 –1428, Jun. 2002.

[2] P. L. Dragotti, M. Vetterli, and T. Blu, "Sampling moments and reconstructing signals of Finite Rate of Innovation: Shannon meets Strang-Fix," *IEEE Trans. on Signal Process.*, vol. 55, no. 5, Part 1, pp. 1741–1757, May 2007.

[3] P. Shukla and P. L. Dragotti, "Sampling schemes for multidimensional signals with Finite Rate of Innovation," *IEEE Trans. on Signal Process.*, vol. 55, no. 7, pp. 3670 –3686, Jul. 2007.

[4] L. Baboulaz and P. L. Dragotti, "Exact feature extraction using finite rate of innovation principles with an application to image super-resolution," *IEEE Trans. on Image Process.*, vol. 18, no. 2, pp. 281–298, Feb. 2009.

[5] A. Hirabayashi and P. L. Dragotti, "E-spline sampling for precise and robust line-edge extraction," in *Proc. of IEEE Int. Conf. on Image Processing*, Hong Kong, Sep. 2010.

[6] A. Hirabayashi, "Hyperbolic E-spline sampling for precise line-edge extraction," in *Proc. of Int. Conf. on Sampling Theory and Application*, Singapore, May 2011.

[7] R. O. Duda and P. E. Hart, "Use of the Hough transformation to detect lines and curves in pictures," *Communications of the ACM*, vol. 15, pp. 11–15, Jan. 1972.

[8] I. Popovici and W.D. Withers, "Custom-built moments for edge location," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 28, no. 4, pp. 637 –642, Apr. 2006.

[9] M. Unser, A. Aldroubi, and M. Eden, "B-spline signal processing: Part I-theory," *IEEE Trans. on Signal Process.*, vol. 41, no. 2, pp. 821 –833, Feb. 1993.

[10] C. Boncelet, "Image noise models," in *Handbook of Image and Video Processing (Communications, Networking and Multimedia)*, A. C. Bovik, Ed., chapter 4.5, pp. 397–409. Academic Press, Inc., 2005.

[11] R. Fisher, S. Perkins, A. Walker, and E. Wolfart, *Hypermedia Image Processing Reference*, Wiley, New York, 1997.

[12] C. Chen, P. Marziliano, and A. C. Kot, "2D finite rate of innovation reconstruction method for step edge and polygon signals in the presence of noise," *IEEE Trans. on Signal Process.*, to appear 2012.