IMPROVED SNR EVOLUTION FOR OFDM-IDMA SYSTEMS

Jian Dang[1]*, Liuqing Yang*[2]*, and Zaichen Zhang*[1]

[1]National Mobile Communications Research Laboratory,
Southeast University, Nanjing 210096, China
[2]Department of Electrical and Computer Engineering,
Colorado State University, Fort Collins, CO 80523, USA.

ABSTRACT

The bit error rate (BER) performance of interleave division multiple access (IDMA) based systems can be predicted by a semi-analytical method referred to as signal-to-noise ratio (SNR) evolution. SNR evolution tracks the average symbol SNR at each iteration and provides a faster solution than brute-force simulation. In this paper a revised SNR updating formula is proposed for orthogonal frequency division multiplexing interleave division multiple access (OFDM-IDMA) systems in Rayleigh fading channels. By alternating the order of expectations and division of random variables when updating the average symbol SNR, a more accurate approximation of the expected SNR is obtained compared with the existing formula. Hence improved BER prediction performance can be achieved, which is verified by simulations.

Index Terms— OFDM-IDMA, SNR evolution

1. INTRODUCTION

OFDM-IDMA is a promising multiple access scheme for uplink wireless communications thanks to its potential of achieving high spectral efficiency and low decoding complexity [1, 2, 3, 4]. In OFDM-IDMA systems, users are separated by distinct interleavers and the receiver removes the multiple access interference (MAI) for each user using an iterative chip-by-chip detection algorithm. To facilitate the study on the performance of IDMA based systems, the SNR evolution method (see, *e.g.*, [5, 6]) has been proposed to assess the BER performance much more rapidly than brute-force simulations which are time-consuming. The key idea of SNR evolution is to treat the MAI as noise such that the BER performance in a multiuser scenario is approximated by a single user scenario with a specific SNR which is updated at each iteration. The existing SNR updating formula for OFDM-IDMA systems provides a lower bound of the expected average SNR [6]. In this paper, a revised SNR updating formula is proposed. By alternating the order of expectations and division of the random channel frequency responses in the existing updating formula, more accurate approximation of the expected SNR can be obtained. Therefore, SNR evolution with our proposed updating formula is expected to have improved BER prediction performance. Such improvement is verified by simulations.

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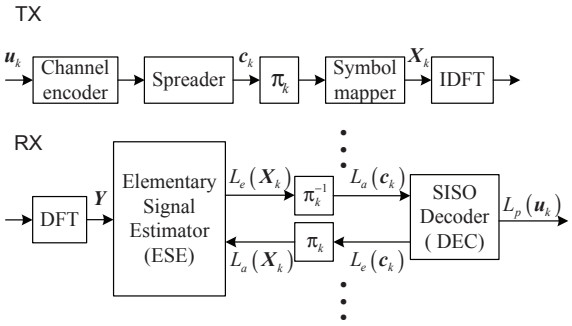

Fig. 1. OFDM-IDMA transmitter for user-k and receiver.

Notations: $\mathrm{E}\{\cdot\}$, $\mathrm{var}\{\cdot\}$ denote the expectation and variance of a random variable. $\Re\{\cdot\}$, $\Im\{\cdot\}$ denote the real and imaginary parts of a complex number.

2. SNR EVOLUTION AND PROBLEM FORMULATION

First a brief introduction to OFDM-IDMA transceiver is given. Consider the uplink of an OFDM-IDMA system with K users and N subcarriers. Fig. 1 shows the block diagram of the transmitter for user-k and the receiver. At the transmitter, information bits $\boldsymbol{u}_k$ are first encoded and then spread by a length-S spreader. The bits $\boldsymbol{c}_k$ (± 1) after spreading are referred to as *chips*. The chips are interleaved by a random *user-specific* interleaver π_k and then modulated using quadrature phase shift keying (QPSK), giving rise to the modulated symbols $\boldsymbol{X}_k$ ($\pm 1 \pm j1$) which are finally transmitted on the N subcarriers via an inverse discrete Fourier transform (IDFT) module. The receiver mainly consists of two modules, *i.e.*, the elementary signal estimator (ESE) for all users and the soft-input soft-output decoders (SISO DECs) for each and every user [5, 2]. ESE and SISO DECs are connected by interleavers and de-interleavers (de-interleaver for user-k is represented by π_k^{-1} in Fig. 1). Both ESE and SISO DEC modules refine the soft estimates of the chips generated by each other, based on the channel input/output (I/O) relationship and the code structure, respectively. $L_a(\cdot)$, $L_p(\cdot)$ and $L_e(\cdot)$ denote the *a priori*, the *a posteriori* and the *extrinsic* log-likelihood ratios (LLRs), respectively. More detailed description of OFDM-IDMA transceiver principles can be found in [1, 2, 3].

Focusing on the ESE module for a particular user at a given iteration, the interference is first generated (from the soft estimates of all other users) and subtracted from the received signal, then with

the channel response of that user, the soft estimates of its chips are obtained. If the interference plus noise term is modeled by an aggregate noise term with correspondingly modified variance, the process of ESE can be approximated by a *single user* system at a particular SNR. This is the basic idea of the SNR evolution method which tracks the symbol SNR of each user at the input of the ESE module at each iteration.

More specifically, the I/O relationship on subcarrier-n is given by:

$$\begin{aligned} Y(n) &= \sum_{k=1}^{K} H_k(n)X_k(n) + Z(n) \\ &= H_k(n)X_k(n) + T_k(n), \end{aligned} \tag{1}$$

where $H_k(n)$ is the channel frequency response of user-k on subcarrier-n, $Z(n)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2, $T_k(n) = \sum_{l=1,\ l\neq k}^{K} H_l(n)X_l(n) + Z(n)$ is the total interference term, compromising of MAI and noise, for user-k on subcarrier-n. At each iteration for user-k, the soft estimate of the interference (mean value of the interference) is reconstructed and subtracted from the received signal. The power of the residual interference for user-k, which is equal to the variance of the random interference term $T_k(n)$, is given by

$$I_k(n) = \sum_{l=1,\ l\neq k}^{K} |H_l(n)|^2 \text{var}\{X_l(n)\} + \sigma^2, \tag{2}$$

where

$$\begin{aligned} \text{var}\{X_l(n)\} &= \text{var}\{\Re\{X_l(n)\}\} + \text{var}\{\Im\{X_l(n)\}\} \\ &= [1 - \tanh^2(L_a(\Re\{X_l(n)\})/2)] \\ &\quad +[1 - \tanh^2(L_a(\Im\{X_l(n)\})/2)], \end{aligned} \tag{3}$$

and $L_a(\Re\{X_l(n)\})$,$L_a(\Im\{X_l(n)\})$ are the *a priori* LLRs of the modulated symbols generated by the SISO DECs at the previous iteration. The values of $L_a(\Re\{X_l(n)\})$ and $L_a(\Im\{X_l(n)\})$ are channel and noise dependent. At the first iteration, $L_a(\Re\{X_l(n)\})$ and $L_a(\Im\{X_l(n)\})$ equal to zero. Therefore, the updated SNR for user-k on subcarrier-n after soft interference cancelation is given by

$$\gamma_k(n) = \frac{2|H_k(n)|^2}{\sum_{l=1,\ l\neq k}^{K} |H_l(n)|^2 \text{var}\{X_l(n)\} + \sigma^2}. \tag{4}$$

In the SNR evolution method, the BER performance of user-k after this iteration is approximated by that of a single user system with zero-mean AWGN at an equivalent average symbol SNR given by:

$$\gamma_k = \text{E}\{\gamma_k(n)\}, \tag{5}$$

where the expectation is taken over all possible channel and noise realizations. Therefore, BER of user-k is a function of the symbol SNR γ, *i.e.*, BER $= g_k(\gamma)$ and $g_k(\gamma)$ can be obtained by simulating a single user system with much lower complexity than the multi-user case.

In general, it is difficult to formulate a closed-form expression of (5), as $H_k(n)$ and var$\{X_k(n)\}$ are random variables for all k and the probability density function (PDF) of var$\{X_k(n)\}$ may not have a closed-form expression. Nevertheless, one can use Monte-Carlo simulation method to numerically evaluate the exact value of (5), *i.e.*, calculating $\gamma_k(n)$ for different n (both $|H_l(n)|^2$ and var$\{X_l(n)\}$ vary with index n) and averaging them out over sufficient number of channel and noise realizations. For each channel and noise realization for user-l, var$\{X_l(n)\}$, $\forall n$, are generated by simulating a single user system with zero-mean AWGN at symbol SNR γ_l which is given by the previous iteration. At the first iteration, var$\{X_l(n)\} = 2, \forall l, n$. We refer to the evolution method with simulated $\gamma_k, \forall k$, as *simulation-based* SNR evolution.

Denote $\gamma_k^{(q)}$ as the symbol SNR of user-k after the q-th iteration and Q the total number of iterations. Given the channel model and $g_k(\gamma)$ of all users, the simulation-based SNR evolution is summarized as follows.

Algorithm: Simulation-based SNR evolution

1) *Initialization*: $q = 1, \gamma_k^{(q)} = \text{E}\{\gamma_k(n)\}, \forall k$, where var$\{X_k(n)\} = 2, \forall k, n$.
2) *SNR updating*: For $q = 2, 3, \ldots, Q$, $\gamma_k^{(q)} = \text{E}\{\gamma_k(n)\}$, $\forall k$, where var$\{X_k(n)\}$ are obtained at SNR $\gamma_k^{(q-1)}$.
3) *Termination*: BER of user-k after Q-th iteration is given by $g_k(\gamma_k^{(Q)})$.

The average SNR obtained by simulation is the expected SNR which is desired. However, numerically evaluating (5) is computationally demanding and, as a consequence, contradicts the purpose of introducing SNR evolution as a fast BER assessment method. In practice, it is more desirable to update the average SNR using *closed-form* expression. Therefore, the problem now is to find good approximation of (5) with closed-form expression.

3. SNR UPDATING FORMULAS

In this section, we briefly review the existing updating formula and propose our revised one.

3.1. Existing Updating Formula

The existing updating formula proposed by J. Tong, *et al.*, is as follows [3]. Denote $\eta_k = \text{E}\{|H_k(n)|^2\}$ as the average channel power for user-k and $f_k(\gamma)$ the average value of var$\{X_k(n)\}$ over index n at symbol SNR γ. $f_k(\gamma)$ is obtained together with $g_k(\gamma)$ by simulating a single user system. At the first iteration, $f_k(\gamma)$ equals 2. The average SNR is approximated by

$$\gamma_k^{(q)} \approx \frac{2\eta_k}{\sum_{l=1,\ l\neq k}^{K} \eta_l f_l\left(\gamma_l^{(q-1)}\right) + \sigma^2}. \tag{6}$$

Note that in this updating formula, the expectations of $|H_l(n)|^2$ and var$\{X_l(n)\}$, $\forall l \neq k$, are taken before the division, which changes the order of calculations in (5). Intuitively, this may cause large approximation error, which motivates us to propose a revised updating formula as detailed in the next subsection.

3.2. Proposed Updating Formula

In this subsection a revised updating formula is proposed. Similar to the existing updating formula, the expectations of variances are taken before the division. However, channel responses are treated in a different manner such that expectations of $|H_l(n)|^2, \forall l$, are taken after the division. Specifically, denote $U_k = |H_k(n)|^2$ and $V_k = \sum_{l=1,\ l\neq k}^{K} |H_l(n)|^2 f_l\left(\gamma_l^{(q-1)}\right)$ as two random variables with PDFs

$p_U(u)$ and $p_V(v)$, respectively. Then at q-th iteration, (5) can be approximated by

$$\gamma_k^{(q)} \approx \int_0^\infty \int_0^\infty \frac{2u}{v+\sigma^2} p_U(u) p_V(v) du dv. \quad (7)$$

It is possible to get a closed-form expression of (7) for commonly used channel models. We consider Rayleigh fading channels where $H_k(n)$ is a complex Gaussian random variable with zero mean and variance η_k.

For the general case in which η_k are different for different k, *i.e.*, different received power levels among users, $f_k\left(\gamma_k^{(q-1)}\right)$ are also different. In this case, U_k is a chi-square random variable with two degrees of freedom and V_k is a generalized chi-square random variable. The PDF of V_k is given by [7]

$$p_V(v) = \sum_{l=1, l\neq k}^{K} c_{k,l} e^{-\frac{v}{d_l}},\ v \geq 0, \quad (8)$$

where

$$c_{k,l} = \left[d_l \prod_{j=1, j\neq l,k}^{K} \left(1 - \frac{d_j}{d_l}\right) \right]^{-1},\ d_l = \eta_l f_l\left(\gamma_l^{(q-1)}\right). \quad (9)$$

After some mathematical manipulations, (7) can be written as

$$\gamma_k^{(q)} \approx 2\eta_k \sum_{l=1, l\neq k}^{K} c_{k,l} e^{\frac{\sigma^2}{d_l}} E_1\left(\frac{\sigma^2}{d_l}\right), \quad (10)$$

where $E_1(x) = \int_x^\infty \frac{1}{t} e^{-t} dt$ is the exponential integral [8].

For the special case where $\eta_k = \eta$, $f_k(\cdot) = f(\cdot)$, $\forall k$, V_k is a chi-square random variable with $2(K-1)$ degrees of freedom. The PDF of V_k is given by [9]

$$p_V(v) = \frac{1}{d^{K-1}\Gamma(K-1)} v^{K-2} e^{-v/d}, \quad (11)$$

where $d = \eta f(\gamma_k^{(q-1)})$ and $\Gamma(x)$ is the Gamma function. Then (7) can be calculated as

$$\gamma_k^{(q)} \approx b \sum_{m=0}^{K-2} \binom{K-2}{m} (-\sigma^2)^{K-2-m} d^m \Gamma\left(m, \frac{\sigma^2}{d}\right), \quad (12)$$

where $\Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete Gamma function [8] and $b = 2\eta e^{\sigma^2/d} / \left(d^{K-1}\Gamma(K-1)\right)$.

By plugging (6) and (10) (or (12)) into step 1) and 2) of the simulation-based SNR evolution process, the predicted BER performance using closed-form updating formulas can be obtained accordingly.

3.3. Discussions

The only difference among the three updating methods lies in the order of the expectations and the division of the random variables. In simulation-based SNR evolution, the expectations of $|H_l(n)|^2$ and var $\{X_l(n)\}$ in (4), $\forall l \neq k$, are taken after the division. On the contrary, in the existing updating formula, the expectations of $|H_l(n)|^2$ and var $\{X_l(n)\}$, $\forall l \neq k$, are taken before the division as shown in (6). In [6] the authors have shown that the SNR obtained using the existing updating formula is a lower bound of the expected SNR given by (5), which is a result of the convexity of $\gamma_k(n)$ in (4) as a function of $|H_l(n)|^2$, $\forall l \neq k$. In our proposed updating formula, the expectations of var $\{X_l(n)\}$, $\forall l \neq k$, are taken before the division to avoid the difficulty of obtaining a closed-form expression of the PDF of var $\{X_l(n)\}$, however, the expectations of $|H_l(n)|^2$, $\forall l \neq k$, are taken after the division, as $|H_l(n)|^2$ usually has an explicit PDF for commonly used wireless channel models. Intuitively, our proposed updating formula will give a SNR value that is in between the existing updating formula and the expected SNR. In fact, in [11] we have shown theoretically that the SNR obtained by our proposed updating formula is indeed a tighter lower bound of the expected SNR compared with the exiting updating formula. Therefore, improved BER prediction performance can be expected and will be verified by simulations in the next section.

4. SIMULATIONS

In this section we present simulations to verify that SNR evolution using our proposed updating formula has improved prediction performance than the existing one. Specifically, we compare the uncoded BER obtained by three means: SNR evolution using existing updating formula; SNR evolution using proposed updating formula; SNR evolution using expected SNR, *i.e.*, simulation-based SNR evolution. For notational simplicity we use BER-1, BER-2 and BER-s to denote those BERs, respectively. System parameters are set as follows: spreading length $S = 8$, number of subcarriers $N = 128$, the channels of all users are assumed to have the same correlation matrix as $\boldsymbol{I}_L/L$ with L denoting the channel length and is set to be 8 here. In this case, $f_k(\cdot) = f(\cdot)$ and $g_k(\cdot) = g(\cdot)$ for all k.

Fig. 2 depicts the average BER performance with $K = 4$ users after $Q = 1, 2$ and 3 iterations. Fig. 3 illustrates the BER performance after $Q = 4$ iterations. Fig. 4 and 5 show the BER performance with $K = 10$ users after $Q = 6$ and 10 iterations, respectively. Several facts can be observed from these simulation results.

First, in all cases as shown in these figures, BER-2 is a tighter upper bound of BER-s, the expected BER, as compared with BER-1. This is consistent with the analysis in 3.3 and therefore verifies that our proposed SNR evolution has improved prediction performance than the existing one.

Second, the difference between BER-1 and BER-2 is significant only for a certain range of Q, depending on the number of users. When $Q = 1$ as shown in Fig. 2, both BERs are very high such that their difference is small. With increased Q, the difference between BER-1 and BER-2 increases first and then decreases. When Q is equal to or larger than 4, both the two curves stick at some certain levels (close to BER-s) and their difference can be ignored (BERs of $Q > 4$ are almost the same as that of $Q = 4$ and are not plotted here). For the case of more active users, *e.g.*, $K = 10$ in Fig. 4 and 5, it takes much more iterations for the two curves to stick at some levels (lower bounded by BER-s).

5. CONCLUSIONS

In this paper a new SNR updating formula in SNR evolution process for OFDM-IDMA systems has been proposed. By alternating the order of expectations and division of the channel frequency responses in the SNR updating formula, a more accurate approximation of the expected SNR has been obtained using our proposed updating formula. Hence improved prediction performance can be expected with the proposed formula, which has been verified by simulation results.

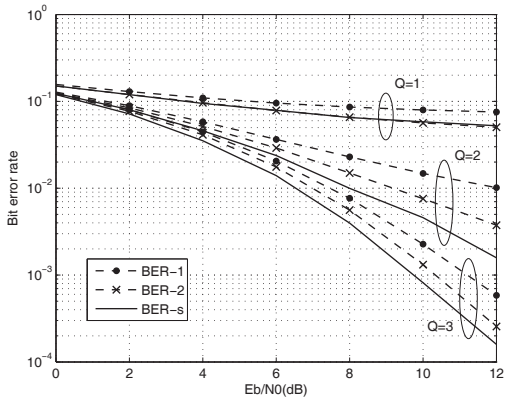

Fig. 2. BER performance comparison with $1-3$ iterations. $K=4$.

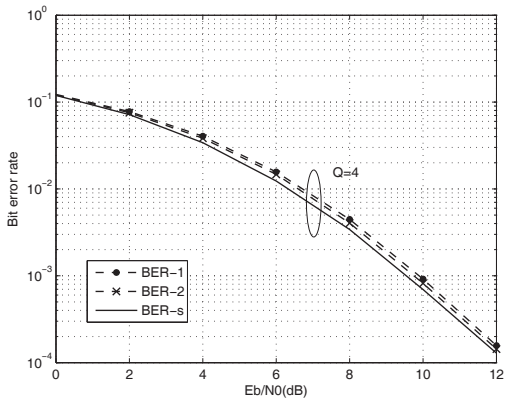

Fig. 3. BER performance comparison with 4 iterations. $K=4$.

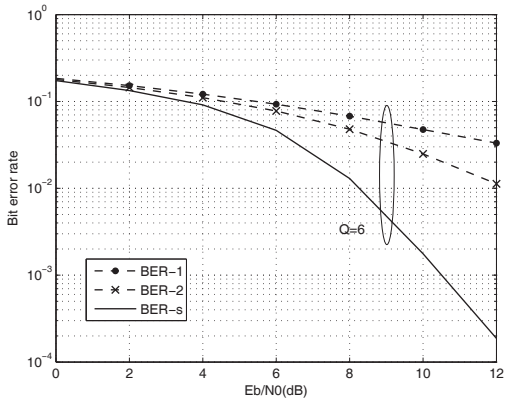

Fig. 4. BER performance comparison with 6 iterations. $K=10$.

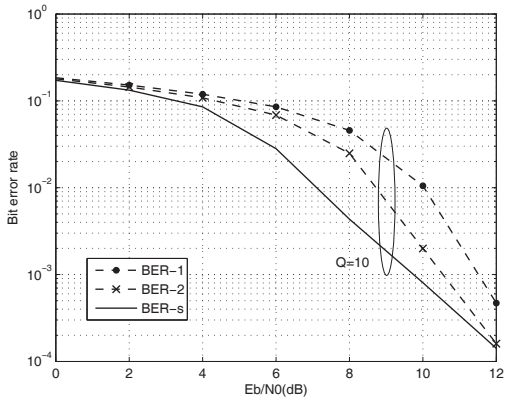

Fig. 5. BER performance comparison with 10 iterations. $K=10$.

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