

ON SCALAR AMBIGUITY IN BLIND CHANNEL ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

Blind channel estimation is a promising technique to reduce the pilot overhead. Unfortunately, most existing algorithms suffer from the scalar ambiguity problem, and hence only achieve semi-blind identification. In this paper, we show that with the information of source constellation, the phase of the ambiguous scalar can be divided into a fractional part and an integer part. Then we propose a multiple-constellation scheme enabling totally blind identification regardless of constellation type for OFDM systems. The necessary and sufficient condition for eliminating the scalar ambiguity is given. An application example shows that our scheme can help other algorithms circumvent the annoying ambiguity.

Index Terms—Totally blind channel estimation, scalar ambiguity, multiple constellations, phase decomposition, OFDM.

1. INTRODUCTION

Blind channel estimation (BCE) is a promising technique to reduce the pilot overhead. Many excellent algorithms have been developed for orthogonal frequency division multiplexing (OFDM) systems, such as subspace method [3], cross-relation method [4] and finite alphabet method [5]. Despite their great success, one problem remains: those algorithms can often only estimate the channel up to an ambiguous scalar. Usually, a few pilots are required to remove the ambiguity [3]. Strictly speaking, they can only be called “semi-blind” in this case.

To the best of our knowledge, only a few papers consider the totally blind channel estimation (TBCE) problem. Necker and Stuber proposed to use dual MPSK constellations for TBCE [7], but the channel can not vary too fast in the frequency domain. Necker and Sanzi tried a particular 8-ary minimum-error (8MiniErr) constellation [8], but no attempt was made for the general case. TBCE for Space-Time Block Coding systems by dual-constellation was rigorously discussed in [9], [10], but still only MPSK constellation was covered. So the feasibility of TBCE for other kinds of constellations, including widely used MQAM, remains unclear. On the other hand, some papers obtained results close to TBCE, though their motivations were not for it. Exploiting the finite alphabet property of input signals, Zhou and Giannakis [5] showed that the channel can be estimated up to a discrete phase

rather than a complex scalar for *any* constellation. Banani and Vaughan developed an interesting blind channel tracking algorithm [11], but *a priori* knowledge of the channel at the beginning of the sequence transmission is required.

In this paper, we study the scalar ambiguity problem in BCE for OFDM systems. With the information of source constellation, we reexamine the identifiability of BCE, and show that TBCE is feasible for a class of constellations. Furthermore, if multiple constellations are used, e.g., adaptive modulation may be adopted to improve the spectrum efficiency, then TBCE can be achieved under mild condition, which allows for any kind of constellations, any number of constellations, and poses no restriction on their relative positions.

2. PROBLEM STATEMENT

In an OFDM system with N subcarriers, the input block $\mathbf{s}(n) \triangleq [s_0(n), s_1(n), \dots, s_{N-1}(n)]$ takes N -point IDFT, and cyclic prefix (CP) is appended to the front of the block, where n is the time index. The resulting sequence is transmitted through the channel. At the receiver, after CP removal and DFT operations, the received signal at the k th subcarrier is

$$y_k(n) = H_k s_k(n) + w_k(n) \quad (1)$$

where $\mathbf{H} = [H_0, H_1, \dots, H_{N-1}]$ is the channel frequency response and $w_k(n)$ is white complex Gaussian noise with variance σ_w^2 .

Many papers discuss the BCE problem for OFDM systems and achieve identifiability within some indeterminacy, e.g., a scalar or a discrete phase [3]-[5]. W.l.o.g., we treat the indeterminacy as a scalar. In this paper we focus on the ambiguity problem, and assume that the channel has already been estimated up to a scalar c independent of subcarrier index k , i.e.,

$$\hat{\mathbf{H}} = \mathbf{H}/c \quad (2)$$

Then in the absence of noise, (1) can be rewritten as

$$r_k(n) \triangleq \frac{y_k(n)}{\hat{H}_k} = c s_k(n) \quad k = 0, 1, \dots, N-1 \quad (3)$$

3. TOTALLY BLIND CHANNEL IDENTIFIABILITY

It's well known that BCE algorithms suffer from the ambiguity problem [3], [4]. If no *a priori* information of input signals is assumed, the ambiguity is an inherent problem [1]. Although those

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algorithms can be applied to any constellation, the resulting ambiguity is unacceptable for signal detection. On the other hand, input signals are drawn from constellations. *Can we design an algorithm, which can totally blindly estimate the channel and still apply to any constellation with this additional information of source constellation?* This question motivates our study on TBCE.

Identifiability plays an important role in BCE. Instead of discussing identifiability for specific algorithms, we are more interested in whether the channel *can be identified* directly based on the observation. Let $\mathbf{s} \in \Theta_s$ be the input signal, $\boldsymbol{\theta} \in \Theta_\theta$ be the deterministic parameter to be estimated, where Θ_s and Θ_θ are their domains respectively; \mathbf{y} be the noiseless observation with $\mathbf{y} = f(\mathbf{s}; \boldsymbol{\theta})$, where $f(\cdot; \boldsymbol{\theta})$ denotes the system transfer function. Suppose that \mathbf{s} is treated as a deterministic vector. Consistent with [2], $\boldsymbol{\theta}$ is said to be identifiable if we have

$$f(\mathbf{s}_1; \boldsymbol{\theta}_1) = f(\mathbf{s}_2; \boldsymbol{\theta}_2) \text{ for some } \mathbf{s}_1, \mathbf{s}_2 \in \Theta_s \Rightarrow \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 \quad (4)$$

Otherwise it is said to be not identifiable. Similar notion was discussed for the statistical model in [1]: $\boldsymbol{\theta}$ is said to be identifiable if it can be identified from the distribution of \mathbf{y} ; otherwise it is said to be not identifiable.

If not identifiable, the parameter can not be identified by any algorithm [2]. If identifiable, at least an algorithm can identify the parameter like exhaustive search (see [2] for more discussions).

In this section, we ignore the noise and investigate the identifiability in BCE with the information of source constellation. We assume that Θ_s is known *a priori* and restricted to constellations, i.e., $\Theta_s = \mathcal{S}_0 \times \mathcal{S}_1 \times \dots \times \mathcal{S}_{L-1}$, where L is the size of \mathbf{s} .

3.1. Illustrative Examples

Here we illustrate our main idea by some examples. For the sake of clarity, we consider only one subcarrier $s_0(n)$. From (3),

$$r_0(n) = cs_0(n) \quad (5)$$

Suppose that $s_0(n)$ is drawn from BPSK $\{-1, 1\}$. With a sample $r_0(0) = 2e^{j\pi/4}$, we can infer that the amplitude of c is 2 and the phase of c is $\pi/4$ or $5\pi/4$, depending on whether $s_0(0)$ is 1 or -1, i.e.,

$$\begin{cases} \text{abs}(c) = 2 \\ \arg(c) = \pi/4 + K\pi \quad K = 0, 1 \end{cases} \quad (6)$$

where $\arg(\bullet)$ denotes the phase of a complex number. So the amplitude is determinable. For the phase, the first term can be determined, while the second term is ambiguous. Moreover, it is easy to verify that the ambiguity in (6) does not vanish no matter how many samples are exploited. The ambiguity can be seen, intuitively stemming from the constellation \mathcal{S} itself: the scalar c amplifies \mathcal{S} by 2, rotates it by $\pi/4$ and then rotates it by $K\pi$. The last step is ambiguous because the shape of received signals in the scatter plot keeps unchanged for any integer K , which implies that the rotation symmetry property of source constellation results in the ambiguity.

The same principle also applies to other constellations. Take 16-QAM for example, although it looks totally different from BPSK, similar symmetry property holds, as illustrated in Fig. 1,

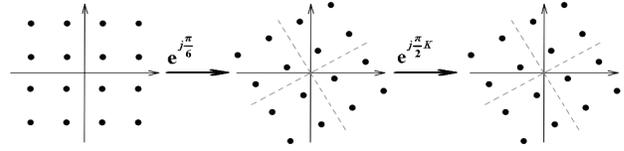


Fig. 1 Ambiguity for 16-QAM

Those examples show that we can determine the amplitude and the first phase term, while the second phase term is ambiguous. Besides, the ambiguous phase term is discrete and is integer multiple of some specific quantity, so it is called integer phase. The determinable phase term is continuous and is smaller than the integer phase, so it is called fractional phase.

3.2. Symmetry Set and Phase Decomposition

Here we quantify the rotation symmetry property. A constellation \mathcal{S} is defined as a finite set of points in the complex plane with more than one element. So $2 \leq |\mathcal{S}| < \infty$, where $|\bullet|$ denotes cardinality. Denote $c\mathcal{S} \triangleq \{cs : s \in \mathcal{S}\}$ the product of c and \mathcal{S} .

Definition 1: A phase α is called *symmetric* to a constellation \mathcal{S} if

$$e^{j\alpha}\mathcal{S} = \mathcal{S} \quad (7)$$

Denote \mathcal{A}_S the set of all the symmetric phases for which $0 \leq \alpha < 2\pi$, called the *symmetry set*.

Intuitively, α is a symmetric phase of \mathcal{S} , if \mathcal{S} is invariant under a rotation of a phase α . The symmetry set is non-empty because $\alpha = 0$ always belongs to the set. Different constellations may have the same symmetry set if they have the same rotation symmetry property. The symmetry sets for some constellations are given as follows: MPAM, $\mathcal{A}_S = \{0, \pi\}$; MPSK, $\mathcal{A}_S = \{2n\pi/M\}_{n=0}^{M-1}$; square MQAM, $\mathcal{A}_S = \{0, \pi/2, \pi, 3\pi/2\}$; and 8MiniErr [8], $\mathcal{A}_S = \{0\}$ since it does not have rotation symmetry property. The following theorem characterizes the structure of \mathcal{A}_S . (Due to space limitation, proofs of three theorems in Section 3 appear in the full paper [12].)

Theorem 1: The symmetry set \mathcal{A}_S of any constellation \mathcal{S} is finite, i.e., $Q_S \triangleq |\mathcal{A}_S| < \infty$. Moreover, \mathcal{A}_S can be expressed in terms of Q_S in the following form,

$$\mathcal{A}_S = \{2n\pi/Q_S\}_{n=0}^{Q_S-1} \quad (8)$$

Theorem 1 shows that all constellations have similar type of rotation symmetry property as that of MPSK constellations, and \mathcal{A}_S is determined by a single parameter Q_S . We call Q_S the *symmetric number* of constellation \mathcal{S} hereafter. From Theorem 1, any phase $\zeta \in [0, 2\pi)$ can be uniquely decomposed into the sum of a fractional part and an integer part in terms of Q_S ,

$$\zeta = \theta + 2K\pi/Q_S \quad (9)$$

where $\theta \in [0, 2\pi/Q_S)$ and $K \in \{0, 1, \dots, Q_S - 1\}$. We can understand (9) as the division of ζ by $2\pi/Q_S$. Hence the values of θ and K are constellation-dependent. Then fractional phase and integer phase can be formally defined as:

Definition 2: Decompose $\arg(c)$ like (9) in terms of Q_S of constellation \mathcal{S} . Then, θ is called the *fractional phase*, and K the *integer phase*, with respect to \mathcal{S} .

3.3. Signal Design for TBCE

Now we discuss the identifiability of the scalar c in BCE and its relationship with the structure of the constellation. In the literature, the input signal is often assumed to be either statistical or deterministic:

A0) $\{\mathbf{s}(n)\}_n$ is treated as a sequence of random vectors. They are drawn independently, identically and equally likely from constellations, and $\{s_0(n), s_1(n), \dots, s_{N-1}(n)\}$ are also independent. Besides, the *distributions* of $y_k(n)$ are known, $k = 0, 1, \dots, N-1$.

A1) $\{\mathbf{s}(n)\}_n$ is treated as a sequence of deterministic vectors. They take values from constellations. Besides, they are *sufficiently long* in the time index n .

Let us first consider the case when one constellation is used.

Theorem 2: Let all $s_k(n)$ use the same constellation \mathcal{S} , $k = 0, 1, \dots, N-1$. Under either A0) or A1), both the amplitude and the fractional phase of c can be uniquely determined, while the integer phase has Q_S possible values. In particular, c is identifiable if and only if

$$Q_S = 1 \quad (10)$$

Theorem 2 not only shows that the rotation symmetry property of source constellation results in the ambiguity of c , but also gives a quantitative description. As a byproduct, we obtain the necessary and sufficient condition for TBCE with one constellation.

Unfortunately, (10) is a stringent constraint, which excludes widely used MPSK, MPAM and square MQAM constellations. On the other hand, adaptive modulation may be employed to improve the spectrum efficiency, where different subcarriers may choose different constellations according to channel conditions [5]. Then an interesting question is: can the channel be totally blindly identified with multiple constellations under a milder condition? We have the following necessary and sufficient condition.

Theorem 3: Let each $s_k(n)$ use constellation \mathcal{S}_k respectively, $k = 0, 1, \dots, N-1$. Under either A0) or A1), c is identifiable if and only if

$$\gcd(Q_0, Q_1, \dots, Q_{N-1}) = 1 \quad (11)$$

where Q_k is the symmetric number of \mathcal{S}_k , and $\gcd(\bullet)$ denotes the greatest common divisor.

The multiple constellations \mathcal{S}_k are not necessary distinct, e.g., if $\mathcal{S}_2 = \dots = \mathcal{S}_{N-1} = \mathcal{S}$ is 16-QAM with $Q_S = 4$, \mathcal{S}_0 is 5-PSK with $Q_0 = 5$, \mathcal{S}_1 is 4-PAM with $Q_1 = 2$, the scalar c is identifiable. Note that none of those constellations satisfies (10), i.e., c becomes identifiable under a mild condition when multiple constellations are used. In the presence of virtual carriers, (11) should be read as that the symmetric numbers of constellations in all data subcarriers are coprime.

Once the scalar c is determined, the ambiguity is eliminated and traditional BCE algorithms become totally blind ones. Our totally blind scheme can be formally summarized as follows:

Proposed Scheme: Select one constellation satisfying (10); or select multiple constellations satisfying (11).

Note that 8MiniErr constellation satisfies (10), which is the case discussed in [8]. Our scheme is motivated by [7] and [9]. But they only dealt with dual constellations, and only MPSK constellations were discussed.

4. ALGORITHM DESIGN

In this section, we develop algorithms to blindly estimate the scalar c . We factorize it into amplitude $|c|$, fractional phase θ and integer phase K with respect to the first constellation \mathcal{S} , i.e.,

$$c = |c| \exp(j\theta) \exp(2\pi K/Q) \quad (12)$$

where Q is the symmetric number of \mathcal{S} , then deal with them respectively. The amplitude can be readily obtained by SOS. Here we focus on the estimation of θ and K .

4.1. Fractional Phase Estimation

According to Theorem 2, the fractional phase can be identified from data using \mathcal{S} solely. Treating input signals as deterministic quantities, we maximize the likelihood function of $y_k(n)$

$$\begin{aligned} & \max_{h, s_k(n)} p(y_k(n) | H_k, s_k(n)) \\ &= \max_{\theta} \max_{K, s_k(n)} \prod_{n=1}^L \frac{1}{\pi \sigma_w^2} \exp\left(-\frac{1}{\sigma_w^2} |y_k(n) - c \hat{H}_k s_k(n)|^2\right) \\ &= \min_{\theta} \sum_{n=1}^L \min_{K, s_k(n)} |y_k(n) - |c| e^{j\theta} e^{jK2\pi/Q} \hat{H}_k s_k(n)|^2 \\ &= \min_{\theta} \sum_{n=1}^L \min_{s_k(n)} |y_k(n) - |c| e^{j\theta} \hat{H}_k s_k(n)|^2 \end{aligned} \quad (13)$$

where L is number of OFDM blocks and $p(\bullet)$ is the likelihood function. The last equality follows from the property of integer phase. If data of multiple subcarriers are available, similarly

$$\begin{aligned} \hat{\theta}_{ML} &= \arg \min_{\theta} \sum_{k \in \mathcal{I}} \sum_{n=1}^L \min_{s_k(n)} |y_k(n) - |c| e^{j\theta} \hat{H}_k s_k(n)|^2 \\ &= \arg \min_{\theta} \sum_{k \in \mathcal{I}} |\hat{H}_k|^2 \sum_{n=1}^L |r_k(n) e^{-j\theta} / |c| - \hat{s}_k(n)|^2 \end{aligned} \quad (14)$$

where $\hat{s}_k(n) \triangleq \arg \min_{s_k(n)} |r_k(n) e^{-j\theta} / |c| - s_k(n)|^2$, and \mathcal{I} is the set of

subcarrier indices on which \mathcal{S} is used. Because the noise is independent in the frequency domain, the search is decoupled and the complexity of (14) is linear with data length, while ML estimations typically require exponential complexity [9], [10]. The summation in (14) is weighted by the channel power, so it is robust to deep fading in some subcarriers. Similar to the proof of Theorem 2, we can show that (14) has a unique solution of θ in the absence of noise if $\mathbf{s}(n)$ is sufficiently long in the time index n .

4.2. Integer Phase Estimation

If one constellation is used and satisfies (10), the integer phase $K \equiv 0$; if multiple constellations are used and satisfy (11), estimation of K is needed. For simplicity, suppose that $s_0(n)$ uses a second constellation \mathcal{T} which is MPSK with $P \triangleq |\mathcal{A}_{\mathcal{T}}| = |\mathcal{T}|$.

(Discussions for the general case are given in [12].) Decompose $\arg(c)$ with respect to \mathcal{S} and \mathcal{T} respectively,

$$\begin{cases} \arg(c) = \theta + 2K\pi/Q \\ \arg(c) = \tilde{\theta} + 2\tilde{K}\pi/P \end{cases} \quad (15)$$

If θ is known or estimated, and the P th-order statistics of $r_0(n)$ is available, K can be obtained with the help of [9, Lemma 4]

$$K = uP^{\varphi(Q)-1} - Q \left\lfloor \frac{uP^{\varphi(Q)-1}}{Q} \right\rfloor \quad (16)$$

where $u \triangleq \frac{PQ}{2\pi} \left(\frac{1}{P} \arg(\mathbb{E}[r_0^P(n)]) - \theta \right)$ is an integer; \mathbb{E} denotes mathematical expectation; $\lfloor m \rfloor$ is the greatest integer not exceeding m ; $\varphi(m)$ is the Euler phi-function defined as the number of positive integers which are coprime to m and not larger than m . Although relying on HOS, we can still expect good performance of (16) because the statistic averaging becomes deterministic for MPSK.

5. APPLICATION TO AN EXISTING ALGORITHM

We have provided a general approach to investigate the scalar ambiguity in BCE, so our TBCE scheme can help lots of existing algorithms with this problem. Due to space limitations, we only illustrate it to the subspace method developed by Muquet *et al* which achieves identifiability within a scalar indeterminacy [3].

It is straightforward to apply our scheme to [3]. We test our scheme in an $N = 64$ subcarriers OFDM systems which essentially follows HIPERLAN/2 and IEEE 802.11a standards. The CP length is 16 samples long. 12 subcarriers are virtual carriers and are distributed evenly at the spectrum edges. Among 52 data subcarriers, 4 subcarriers $\{12, 26, 40, 54\}$ use $\mathcal{T} = 3$ -PSK and other 48 subcarriers use $\mathcal{S} = 16$ -QAM. Note that no pilots or known OFDM blocks are available at the receiver. The scalar c is factorized with respect to 16-QAM. The channel coefficients are drawn from i.i.d. complex Gaussian random variables with unit variance. Results shown are the average of 3000 Monte Carlo trials. With perfect channel knowledge, the best scalar which minimizes the normalized mean square error (NMSE) over all possible values of c can be calculated by the method in [6], and we use it as the reference curve for comparison purpose. Note that no *a priori* knowledge of the channel is required by our scheme.

Fig. 2 shows the NMSE performance of our algorithms, where “SS ref” stands for [3] with [6] and “SS TBCE” denotes [3] with our scheme. It can be seen that the TBCE algorithm suffers very little SNR loss compared with the optimal NMSE when the SNR is higher than 25 dB. With lower SNR, TBCE degrades gracefully.

6. CONCLUSION

In this paper, we discuss the scalar ambiguity problem in BCE for OFDM systems with the information of source constellation. By decomposing the phase of the ambiguous scalar into a fractional phase and an integer phase, we give a quantitative relationship between the ambiguity of the scalar and source constellation when one constellation is used. A multiple-constellation scheme is proposed, in which the channel can be totally blindly identified with no need for any pilots or training sequences under mild condition regardless of constellation type.

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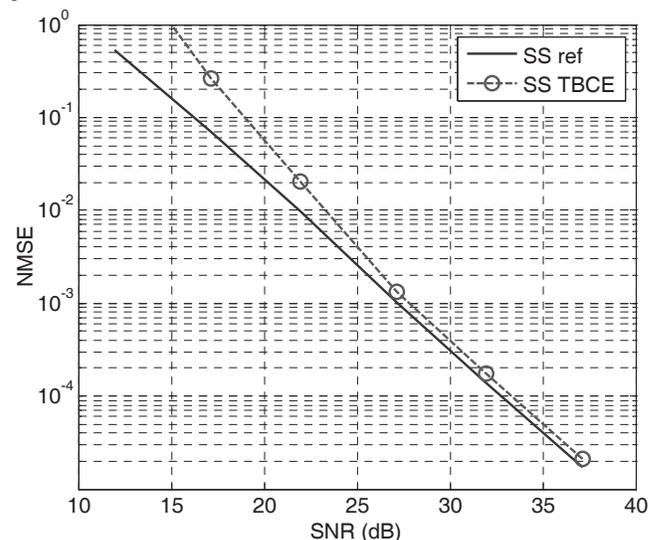


Fig. 2 NMSE of TBCE for subspace method