COOPERATIVE SENSING WITH TERNARY LOCAL DECISIONS

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ABSTRACT

Cognitive radio is gaining increasingly interest as a promising solution to current spectrum resource shortage. Within various tasks of cognitive radio, spectrum sensing is the fundamental one, but is challenged by wireless channel fading. By collecting diversity among different users, cooperative sensing can overcome the fading problem very well. Usually, only the local binary decisions are available for sensing cooperation due to limitation of the channel bandwidth. However, in our previous work [1], we have shown that this strategy will either sacrifice diversity or signal-to-noise ratio (SNR) gain. In this paper, we will study cooperative sensing with ternary local decisions. Compared with the binary cooperative sensing, this strategy will can regain diversity and recover the extra SNR loss by appropriate threshold selection, without increasing the decision forwarding bandwidth.

***Index Terms*—** cognitive radio, cooperative spectrum sensing, decision fusion, diversity gain, local ternary decision

1. INTRODUCTION

Opportunistic spectrum access schemes, a. k. a. cognitive radio system, is proposed to solve the problem of spectrum scarcity by more efficient spectrum utilization [2] compared with the fixed spectrum allocation strategy. There are various issues to realize the cognitive radio system [3]. Among them, detecting the available unused spectrum resources, a. k. a. spectrum sensing, is the first step.

Extensive research has already been conducted to improve the spectrum sensing performance (see e.g. [4, 5, 6, 7, 8]). Among these, cooperative sensing is proposed as an efficient strategy to combat the fading effect. In our previous work [1], we quantified the gain of cooperation by the *cooperative diversities* for missed detection, false alarm and average error probabilities. Using diversity as the performance metric, we designed the sensing threshold strategies for cooperative sensing with both soft information fusion (SCoS) and hard information fusion (HCoS). We found that while SCoS can achieve the maximum diversity, HCoS either loses half of the diversity or achieves the full diversity at the price of signal-to-noise ratio (SNR) reduction.

While the performance of SCoS is desirable, it is unrealistic since it requires infinite bandwidth for the communications between the local sensors and the fusion center. Intuitively, the performance gap between SCoS and HCoS results from the loss of information with the single-bit local decisions in HCoS. It should be possible to improve the performance by providing more information from the local sensors. In this paper, we design the cooperative sensing scheme with local ternary decisions. While developing the optimum strategies is complicated and mathematically intractable, we simplify the detection fusion problem based on our previous work [1] to achieve the diversity and SNR gains. It is shown that with local ternary decisions, it is possible to gain in terms of both diversity and SNR compared with local binary decisions.

The problem formulation, the preliminaries for binary (BD) and ternary (TD) local decisions will be introduced in Section 2. Then, we will determine the detection fusion rules for TD by first finding the relationship between BD and TD in Section 3 and then selecting the detection regions for TD in Section 4 with simulation results in Section 5. Finally, concluding remarks and discussions on future work will be presented in Section 6.

Notation: $x \sim \mathcal{CN}(\mu, \sigma^2)$ denotes a complex Gaussian random variable x with mean μ and variance σ^2. $g(\gamma) \sim f(\gamma)$ means $\lim_{\gamma \to +\infty} \frac{g(\gamma)}{f(\gamma)} = 1$.

2. SYSTEM MODEL

2.1. Signal Model

In the spectrum sensing process, the sensing users observe signals under the following two hypotheses:

$$H_0 : \text{absence of primary user,}$$
$$H_1 : \text{presence of primary user.}$$

We assume that the channels between the primary and the sensing users are Rayleigh fading with additive white Gaussian noise (AWGN). Then after normalization, the signal at each sensing users becomes [1, 5]:

$$\begin{aligned} r_i|H_0 &= n_i \sim \mathcal{CN}(0, 1), \\ r_i|H_1 &= h_i x + n_i \sim \mathcal{CN}(0, \gamma + 1) , \end{aligned} \tag{1}$$

This work is in part supported by Office of Naval Research under grant #N00014-11-1-0667.

where γ is the average SNR at the sensing users. With geographically distributed sensing users, it is reasonable to assume that they experience independent fading channels. With this assumption, the received signals for different sensing users r_is are conditionally independent identically distributed (i.i.d.) under each hypothesis.

2.2. Performance Metrics

For the detection problem introduced in Eq. (1), there are three performance measures, namely false alarm (P_f), missed detection (P_{md}) and average error (P_e) probabilities. As in [1], we will use the diversity defined as $d_* = -\lim_{\gamma\to+\infty} \frac{\log P_*}{\log \gamma}$ to capture those performances.

By definition, diversity only captures the performance at high SNR. Hence, in our analyses, we will aim at achieving better low-SNR performance while maintaining the same diversity gain.

2.3. Binary Local Decision (BD) and HCoS-d_0

For HCoS introduced in [1], the secondary users make local binary decisions $d_i \in \{0, 1\}$ and a fusion center will collect all decisions and make a global decision. The local decisions are:

$$d_i = \begin{cases} 0 & \text{if } 0 \le \|r_i\|^2 < \theta_{l,B} \\ 1 & \text{if } \|r_i\|^2 > \theta_{l,B}\ . \end{cases} \tag{2}$$

If the local decision threshold is $\theta_{l,B} = d_0\theta^o$, where $\theta^o = (1 + \frac{1}{\gamma}) \log(1 + \gamma)$ is the local optimum threshold [1]. Then, $P_{f,l} \sim \gamma^{-d_0}$ and $P_{md,l} \sim d_0\gamma^{-1}$. With the Neyman-Pearson (NP) detector $\sum_{i=1}^{N} d_i \gtrless_{H_0}^{H_1} \theta_{f,B}$, the diversities are $d_{f,B} = d_0\theta_{f,B}$ and $d_{md,B} = N - \theta_{f,B} - 1$. To jointly optimize both diversities, the fusion threshold can be selected as[1]: $\arg\max_{\theta_{f,B}}(\min(d_0\theta_{f,B}, N - \theta_{f,B} - 1)) = \frac{N+1}{d_0+1}$. The optimized diversities can be determined accordingly as $d_e = d_f = d_{md} = \frac{d_0}{d_0+1}(N + 1)$.

This indicates that with larger d_0, HCoS-d_0 can achieve higher diversities. However, in this case, one has larger missed detection probability with $P_{md,B} \sim d_0\gamma^{-1}$. It is highly probable that the SNR loss for missed detection probability will dominate the overall performance at low to medium SNR as shown in [1, Fig. 6].

2.4. Ternary Local Decision (TD)

Similar to BD, the local decisions for TD are:

$$d_i = \begin{cases} 0 & \text{if } 0 \le \|r_i\|^2 < \theta_{l,1} \\ \spadesuit & \text{if } \theta_{l,1} \le \|r_i\|^2 \le \theta_{l,2} \\ 1 & \text{if } \|r_i\|^2 > \theta_{l,2}\ , \end{cases} \tag{3}$$

where $\theta_{l,2} > \theta_{l,1}$ are two local decision thresholds and $\spadesuit$ means "not sure". Then, the "0" or "1" decisions[2] are sent to the fusion center for the global decision with $d \in \{0, 1\}$.

Under this local decisions, the probabilities under each hypothesis are:

$$\begin{aligned} P(d_i = 0|H_0) &= \alpha_1 = 1 - e^{-\theta_{l,1}} \\ P(d_i = \spadesuit|H_0) &= \alpha_2 = e^{-\theta_{l,1}} - e^{-\theta_{l,2}} \\ P(d_i = 1|H_0) &= \alpha_3 = e^{-\theta_{l,2}} \\ P(d_i = 0|H_1) &= \beta_1 = 1 - e^{-\frac{\theta_{l,1}}{\gamma+1}} \\ P(d_i = \spadesuit|H_1) &= \beta_2 = e^{-\frac{\theta_{l,1}}{\gamma+1}} - e^{-\frac{\theta_{l,2}}{\gamma+1}} \\ P(d_i = 1|H_1) &= \beta_3 = e^{-\frac{\theta_{l,2}}{\gamma+1}} \end{aligned} \tag{4}$$

At the fusion center, d_is follow the trinomial distribution as:

$$\begin{aligned} P(d_1, d_2, \ldots, d_N|H_0) &= \alpha_1^{n_0}\alpha_2^{N-n_0-n_1}\alpha_3^{n_1} \\ P(d_1, d_2, \ldots, d_N|H_1) &= \beta_1^{n_0}\beta_2^{N-n_0-n_1}\beta_3^{n_1} \end{aligned} \tag{5}$$

where $n_0 = \{$the number of $d_i = 0\}$, $n_1 = \{$the number of $d_i = 1\}$ and N is the total number of cooperating local detectors. Accordingly, the sufficient statistics is (n_0, n_1). Denoting $\boldsymbol{D}_1$ as the set of (n_0, n_1) to make global decision $d = 1$ and $\boldsymbol{D}_0$ vice versa, we have:

$$\begin{aligned} P_f &= \sum_{(n_0,n_1)\in \boldsymbol{D}_1} P(n_0, n_1|H_0) \\ P_{md} &= \sum_{(n_0,n_1)\in \boldsymbol{D}_0} P(n_0, n_1|H_1) \end{aligned} \tag{6}$$

Based on Eqs. (4), (5) and (6), the optimum fusion rule can be obtained by jointly optimizing $P_e = \frac{1}{2}(P_f + P_{md})$ over $\theta_{l,1}, \theta_{l,2}$ and $\boldsymbol{D}_1$. However, it is very complicated and mathematically intractable and, more importantly, the solution provides no straightforward insights on the diversity gains. As an alternative, we try to first find the relationship between fusions with TD and BD and then develop the fusion rule for TCoS.

3. THE LINK BETWEEN FUSIONS WITH BD AND TD

It is worth noting that at the fusion center, BD has a one-dimensional sufficient statistics set with $n_0 + n_1 = N$ while TD has a two-dimensional set with $n_0 + n_1 \le N$. We find that when the fusion center with TD makes a global decision based on only one of n_0 and n_1, then it is equivalent to the fusion with BD as the following:

Lemma 1 *For cooperative sensing based on local ternary decisions with thresholds $\theta_{l,1}$ and $\theta_{l,2}$:*

[1] It should be noticed that at the fusion center, $\sum d_i$'s can only take integer values. However, to simplify the notations, we ignore the integer restrictions without affection our analysis.

[2] Note that the sensor will remain silent when the local decision is $\spadesuit$.

1. *If $\boldsymbol{D}_1 = \{(n_0, n_1) : n_1 \geq \theta_t\}$, this TD fusion is equivalent to BD fusion with local threshold $\theta_{l,B} = \theta_{l,2}$ and fusion threshold $\theta_{f,B} = \theta_t$;*

2. *If $\boldsymbol{D}_0 = \{(n_0, n_1) : n_0 \geq N - \theta_t + 1\}$, this TD fusion is equivalent to BD fusion with local threshold $\theta_{l,B} = \theta_{l,1}$ and fusion threshold $\theta_{f,B} = \theta_t$;*

Proof 1 : If $\boldsymbol{D}_1 = \{(n_0, n_1) : n_1 \geq \eta_t\}$, then:

$$\begin{aligned} P_{f,t} &= \sum_{n_1=\eta_t}^{N} \sum_{n_0=0}^{N-n_1} \frac{N!}{n_0!(N-n_0-n_1)!n_1!} \alpha_1^{n_0} \alpha_2^{N-n_0-n_1} \alpha_3^{n_1} \\ &= \sum_{n_1=\eta_t}^{N} \frac{N}{n_1!(N-n_1)!} (1-\alpha_3)^{N-n_1} \alpha_3^{n_1} \end{aligned}$$

and

$$\begin{aligned} P_{md,t} &= \sum_{n_1=0}^{\eta_t-1} \sum_{n_0=0}^{N-n_1} \frac{N!}{n_0!(N-n_0-n_1)!n_1!} \beta_1^{n_0} \beta_2^{N-n_0-n_1} \beta_3^{n_1} \\ &= \sum_{n_1=0}^{\eta_t-1} \frac{N}{n_1!(N-n_1)!} (1-\beta_3)^{N-n_1} \beta_3^{n_1} \ , \end{aligned}$$

This is equivalent to BT with $\eta_{l,B} = \eta_{l,2}$ and $\eta_{f,B} = \eta_t$.

If $\boldsymbol{D}_0 = \{(n_0, n_1) : n_0 \geq N + 1 - \eta_t\}$, then:

$$\begin{aligned} P_{f,t} &= \sum_{n_0=0}^{N-\eta_t+1} \sum_{n_1=0}^{N-n_0} \frac{N!}{n_0!(N-n_0-n_1)!n_1!} \alpha_1^{n_0} \alpha_2^{N-n_0-n_1} \alpha_3^{n_1} \\ &= \sum_{n_0=0}^{N-\eta_t+1} \frac{N}{n_0!(N-n_0)!} \alpha_1^{n_0} (1-\alpha_1)^{N-n_0} \end{aligned}$$

and

$$\begin{aligned} P_{md,t} &= \sum_{n_0=N-\eta_t+1}^{N} \sum_{n_1=0}^{N-n_1} \frac{N!}{n_0!(N-n_0-n_1)!n_1!} \beta_1^{n_0} \beta_2^{N-n_0-n_1} \beta_3^{n_1} \\ &= \sum_{n_0=N-\eta_t+1}^{N} \frac{N}{n_0!(N-n_0)!} \beta_1^{n_0} (1-\beta_1)^{N-n_0} \end{aligned}$$

This is equivalent to BT with $\eta_{l,B} = \eta_{l,1}$ and $\eta_{f,B} = \eta_t$. ■

4. FUSION RULE FOR TCOS

With the relationship between BD and TD illustrated in Lemma 1, we will next develop the fusion rule for TCoS based on HCoS. We know that with $\theta_{l,B} = \theta^o$, HCoS has the local optimum sensing strategy and the best SNR gain. Thus, we select the lower threshold of TCoS as $\theta_{l,1} = \theta^o$. Then, for the higher threshold, we select $\theta_{l,2} = d_0 \theta^o$ which can achieve higher diversity for HCoS. This sensing strategy is termed as TCoS-1-d_0.

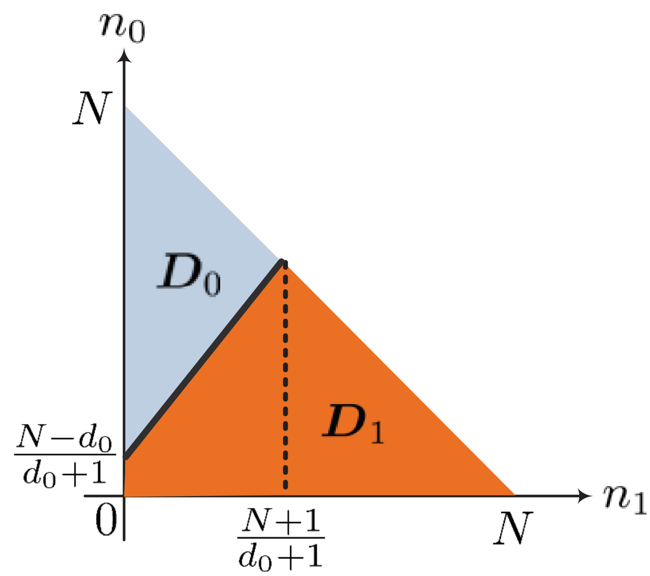

Fig. 1. The decision region for TCoS-1-d_0 with the points at the boundary belonging to $\boldsymbol{D}_1$.

To achieve the maximum diversity, we start with the decision rule as $\boldsymbol{D}_1 = \{(n_0, n_1) : n_1 \leq \frac{N+1}{d_0+1}\}$ which is equivalent to HCoS-d_0 in Section 2.3. Then, since the problem with HCoS-d_0 is the SNR loss for missed detection, here we try to increase the missed detection performance by assigning some terms in $\boldsymbol{D}_0$ to $\boldsymbol{D}_1$ without affecting the false alarm diversity.

With TCoS-1-d_0, the probabilities for local decisions are: $\alpha_1 \sim 1 - \gamma^{-1}, \alpha_2 \sim \gamma^{-1}, \alpha_3 \sim \gamma^{-d_0}$ and $\beta_1 \sim \gamma^{-1}, \beta_2 \sim (d_0 - 1)\gamma^{-1}, \beta_3 \sim 1 - d_0\gamma^{-1}$. Accordingly,

$$\begin{aligned} P(n_0, n_1|H_0) &\sim \alpha_2^{N-n_0-n_1} \alpha_3^{n_1} \sim \gamma^{-(N-n_0+(d_0-1)n_1)} \\ P(n_0, n_1|H_1) &\sim \beta_1^{n_0} \beta_2^{N-n_0-n_1} \sim \gamma^{-(N-n_1)} \end{aligned} \tag{7}$$

Recall that for HCoS-d_0, $d_{f,\mathrm{HCoS}-d_0} = \frac{d_0}{d_0+1}(N+1)$. Then, for the terms in $\boldsymbol{D}_0 = \{(n_0, n_1) : n_1 < \frac{N+1}{d_0+1}\}$, as long as $N - n_0 + (d_0 - 1)n_1 \geq \frac{d_0}{d_0+1}(N+1)$, i.e., $n_0 \leq \frac{N-d_0}{d_0+1} + (d_0 - 1)n_1$, they can be moved into $\boldsymbol{D}_1$ without affecting the diversity of false alarm. The boundary between $\boldsymbol{D}_1$ and $\boldsymbol{D}_0$ of the resultant fusion rule is a line $n_0 = \frac{N-d_0}{d_0+1} + (d_0 - 1)n_1$, which starts from $(n_0, n_1) = \left(\frac{N-d_0}{d_0+1}, 0\right)$ and ends at $(n_0, n_1) = \left(\frac{d_0 N-1}{d_0+1}, \frac{N+1}{d_0+1}\right)$ where $\frac{d_0 N-1}{d_0+1} = N - \frac{N+1}{d_0+1}$.

The decision region for TCoS-1-d_0 is illustrated in Fig. 1. The bold line is the boundary between $\boldsymbol{D}_0$ and $\boldsymbol{D}_1$ for TCoS-1-d_0 while the dashed line is the boundary for TCoS to be equivalent to HCoS-d_0. The improvement for the missed detection probability is:

$$\Delta P_{md} = \sum_{0 \leq n_1 < \frac{N+1}{d_0+1},\ 0 \leq n_0 \leq \frac{N-d_0}{d_0+1} + (d_0-1)n_1} \beta_1^{n_0} \beta_2^{N-n_0-n_1} \beta_3^{n_1} \tag{8}$$

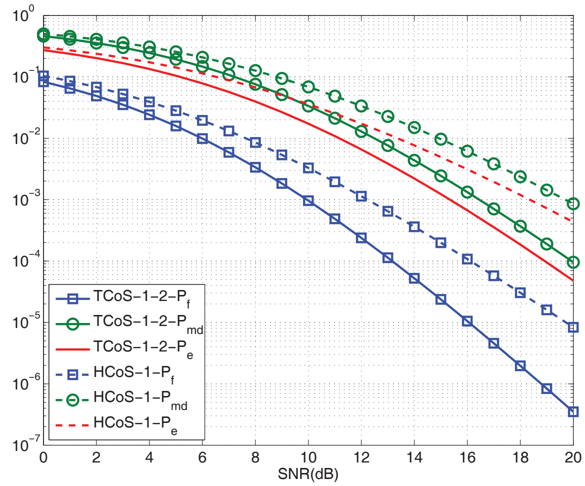

Fig. 2. TCoS-1-2 vs. HCoS-1.

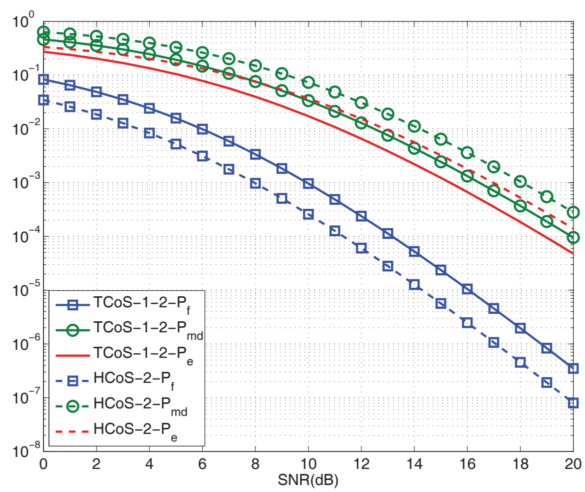

Fig. 3. TCoS-1-2 vs. HCoS-2.

5. SIMULATION RESULTS

To illustrate the performance gain of TCoS-1-d_0 over HCoS, we simulated TCoS-1-2 with 5 cooperating users.

In Fig. 2, the performance of TCoS-1-2 is compared with that of HCoS-1. It can be shown that TCoS has a higher diversity ($d_{\text{TCoS}-1-2} = 4$ and $d_{\text{HCoS}-1} = 3$) and the advantage of diversity shows up very early at the low SNR.

In Fig. 3, the performance of TCoS-1-2 is compared with that of HCoS-2. It can be shown that TCoS achieve the same diversity with HCoS ($d_{\text{TCoS}-1-2} = d_{\text{HCoS}-2} = 4$), but has about 2dB SNR gain.

From these comparisons, we see that TCoS can achieve both diversity and SNR gains over HCoS.

6. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we proposed the cooperative sensing with ternary local decisions (TCoS) to improve the cooperative sensing with binary hard decisions (HCoS-d_0) by pursuing more SNR gain while maintaining the same diversity. The link between the fusion with BD and TD was established and it was used to determine the fusion rule for TCoS. The simulations showed that, as a middle ground between HCoS and SCoS, TCoS-1-d_0 achieves both diversity and SNR gain compared with HCoS schemes. In our future work, we will study the TCoS with arbitrary thresholds, i.e., TCoS-d_1-d_2. In addition, it will also be interesting to study the cooperative sensing with multi-level local decisions to further improve the sensing performance.

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