BLIND VOLTERRA SYSTEM LINEARIZATION WITH APPLICATIONS TO POST COMPENSATION OF ADC NONLINEARITIES

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ABSTRACT

Blind Volterra system linearization is a challenging problem with a variety of applications where nonlinearities are present and training signals are not. This paper focuses on digital post compensation of nonlinear analog-to-digital converters (ADCs) and proposes a blind linearization approach based on the Volterra series. Assuming that the input signal does not occupy the full Nyquist bandwidth, i.e., there exists an energy-free band, the Volterra series coefficients are estimated by minimizing the linearized signal power in the energy-free band. Simulation results are presented which demonstrate the effectiveness of the proposed method.

Index Terms— Volterra, analog-to-digital converter (ADC), energy-free band, post compensation, linearization

1. INTRODUCTION

Shrinking integrated circuit feature sizes allows for more complex digital circuits and algorithms to enhance analog design, frequently via digital correction and calibration. Post compensation of ADCs is one of the examples that has gained increasing interest.

A number of post compensation techniques have been proposed in the literature [1], [2]. However, most methods address static errors in the converter cores and are not effective at removing dynamic nonlinearities, which refer to the input signal dependent distortion. Dynamic specifications for ADCs are important in high-speed applications such as digital communication. The key element for the dynamic performance of an ADC is the track-and-hold (TH) circuit.

From an analog circuit design perspective, 2 major effects that degrade the dynamic performance of ADCs are signal-dependent switch on-resistance and sampling instant errors, both of which introduce frequency dependent distortions and are performance limiting factors in high-frequency applications. A number of digital compensation methods have been proposed to correct for frequency dependent nonlinearities [3].

The basic concept of these methods is to invert the ADC nonlinearity with another nonlinearity (e.g., a Volterra system) at the digital output of the ADC. The Volterra series kernel coefficients are obtained by applying least squares to a set of input and output training signals.

A drawback of this approach is that the ADC input signal is an analog voltage. As such, in order to know its sampled value, the ADC input needs to be generated in the digital domain and converted to the analog domain. This typically requires a very accurate digital-to-analog converter (DAC) with greater linearity than that desired in the ADC, which is not easy to achieve in practice. This motivates the interest in blind methods which only depend on the ADC output signal.

In this paper, a Volterra series is used to model the nonlinear ADC [4], [5]. With a memory length L and nonlinearity order P, the input $x(n)$ and output $y(n)$ are related by

$$y(n) = \sum_{p=1}^{P} y_p(n), \tag{1}$$

where $y_p(n)$ is the pth-order kernel output

$$y_p(n) = \sum_{l_1=0}^{L-1} \cdots \sum_{l_p=0}^{L-1} g_p(l_1, ..., l_p) \prod_{i=1}^{p} x(n - l_i), \tag{2}$$

and $g_p(l_1, ..., l_p)$ is the pth-order kernel coefficient.

A post compensation structure is then proposed in which the nonlinear distortion is blindly estimated and subtracted from the ADC output. The distortion model is parameterized and an adaptive estimation structure is presented. The estimation method adopts the concept of the energy-free method in [6].

The key observation of energy-free methods is that bandwidth expansion is a hallmark of nonlinear systems. As such, if the input signal does not occupy the full system bandwidth, the unoccupied portion of the input signal bandwidth can be used at the system output as an error signal for parameter adaptation as energy in this portion of the band is a result of nonlinearities.

2. COMPENSATION STRUCTURE

In this section, a post compensation structure is proposed based on a Volterra series representation of a nonlinear ADC. Without loss of generality, the ADC output can be written in 2

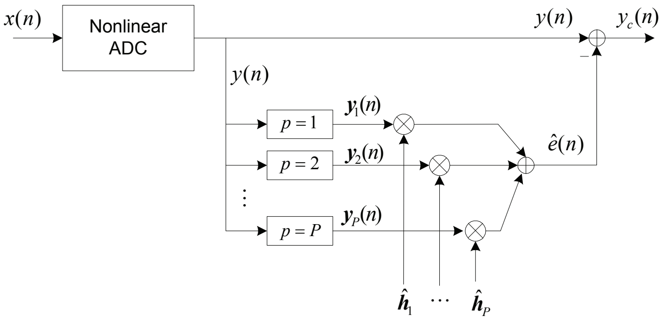

Fig. 1. Post compensation for dynamic distortion in an ADC.

parts: the ideal output (scaled input) and the nonlinear distortion

$$y(n) = Gx(n) + e(n), \tag{3}$$

where G is the gain of the ADC and $e(n)$ denotes the error generated by nonlinearity. Combining (1) and (3), the distortion $e(n)$ can be expressed as a Volterra series in terms of the ADC input

$$e(n) = \sum_{p=1}^{P} \sum_{l_1=0}^{L-1} \cdots \sum_{l_p=0}^{L-1} h_p(l_1, ..., l_p) \prod_{i=1}^{p} x(n - l_i), \tag{4}$$

where

$$h_p(l_1, ..., l_p) = \begin{cases} g_p(l_1, ..., l_p) - G, \ p = 1, \ l_p = 0 \\ g_p(l_1, ..., l_p), \qquad \text{elsewhere} \end{cases}. \tag{5}$$

Define vectors

$$\boldsymbol{x}(n) = \left[\boldsymbol{x}_1^T(n), ..., \boldsymbol{x}_P^T(n)\right]^T \tag{6}$$

$$\boldsymbol{h} = \left[\boldsymbol{h}_1^T, ..., \boldsymbol{h}_P^T\right]^T \tag{7}$$

where the pth-order input vector $\boldsymbol{x}_p(n)$ contains all the p-tuples $\prod_{i=1}^{p} x(n - l_i), \ l_i = 0, ..., L-1$ and $\boldsymbol{h}_p$ contains all the kernel coefficients $h_p(l_1, ..., l_p)$ arranged in order according to $\boldsymbol{x}_p(n)$. The nonlinear distortion $e(n)$ can now be cast in a matrix form which is linear in the kernel coefficients

$$e(n) = \boldsymbol{h}^T \boldsymbol{x}(n) = \sum_{p=1}^{P} \boldsymbol{h}_p^T \boldsymbol{x}_p(n). \tag{8}$$

In order to remove nonlinear distortion the error signal $e(n)$ needs to be estimated. Although (8) provides an explicit model of $e(n)$, the ADC input signal $x(n)$ is not available for calibration. The alternative proposed here is to estimate the distortion using the same model as in (4) and (8), with the ADC input replaced by its output

$$\begin{aligned} \hat{e}(n) &= \sum_{p=1}^{P} \sum_{l_1=0}^{L-1} \cdots \sum_{l_p=0}^{L-1} \hat{h}_p(l_1, ..., l_p) \prod_{i=1}^{p} y(n - l_i) \\ &= \hat{\boldsymbol{h}}^T \boldsymbol{y}(n) = \sum_{p=1}^{P} \hat{\boldsymbol{h}}_p^T \boldsymbol{y}_p(n), \end{aligned} \tag{9}$$

where $\hat{h}_p(l_1, ..., l_p)$ denotes the estimated kernel coefficients. Since the distortion is relatively small compared to the output signal, the replacement of input signal with output signal results in a sufficiently accurate estimate.

Once $\hat{e}(n)$ is obtained, the ADC is linearized by subtracting the distortion from its output

$$y_c(n) = y(n) - \hat{e}(n), \tag{10}$$

where $y_c(n)$ denotes the linearized ADC output. The corresponding post-compensation structure is depicted in Fig. 1.

3. OPTIMAL COMPENSATOR COEFFICIENTS

In this section, the optimal compensator coefficients based on the structure shown in Fig. 1 are analyzed. For a typical ADC, the output signal power is significantly larger than that of the distortion power. Neglecting $e(n)$ and substituting (3) into (9) results in

$$\hat{e}(n) \approx \sum_{p=1}^{P} G^p \hat{\boldsymbol{h}}_p^T \boldsymbol{x}_p(n). \tag{11}$$

Next, combining (4) and (11), (10) can be rewritten as

$$y_c(n) = Gx(n) + \sum_{p=1}^{P} \left(\boldsymbol{h}_p - G^p \hat{\boldsymbol{h}}_p\right). \tag{12}$$

It is seen that the compensated output $y_c(n)$ is equal to $Gx(n)$ and the nonlinear distortion is removed if the estimated coefficients satisfy

$$\hat{\boldsymbol{h}}_p = \frac{1}{G^p} \boldsymbol{h}_p, p = 1, ..., P, \tag{13}$$

which indicates the optimal solution of the compensator coefficients. In the following section, a blind adaptive estimation method is developed to estimate (13).

4. ADAPTIVE COEFFICIENT ESTIMATION

Assume that the ADC input signal $x(n)$ only occupies the lower frequency portion of the entire Nyquist band, leaving a energy-free band in the higher frequency portion. The ADC output $y(n)$ is corrupted by nonlinear distortion and it's frequency components spread into the energy-free band.

Passing the ADC output through a high-pass filter results in

$$\tilde{y}(n) = y(n) * f_{HP}(n), \tag{14}$$

where $*$ denotes the linear convolution and $f_{HP}(n)$ denotes a high-pass filter impulse response. $\tilde{y}(n)$ contains the distortion in the energy-free band if the cut-off frequency of $f_{HP}(n)$ is equal or greater than the bandwidth of the input signal.

Applying the same high-pass filter to the estimated distortion signal $\hat{e}(n)$ and combining with (9) yields

$$\tilde{\hat{e}}(n) = \hat{e}(n) * f_{HP}(n) = \sum_{p=1}^{P} \hat{\boldsymbol{h}}_p^T \tilde{\boldsymbol{y}}_p(n), \quad (15)$$

where $\tilde{\boldsymbol{y}}_p(n)$ denotes the high-pass filtered version of the pth-order Volterra kernel

$$\tilde{\boldsymbol{y}}_p(n) = \boldsymbol{y}_p(n) * f_{HP}(n). \quad (16)$$

Note that $\tilde{\hat{e}}(n)$ is equal to $\tilde{y}(n)$ if the coefficients $\boldsymbol{h}_p$ are correctly estimated. By defining the error signal

$$\epsilon(n) = \tilde{y}(n) - \tilde{\hat{e}}(n) = \tilde{y}(n) - \sum_{p=1}^{P} \hat{\boldsymbol{h}}_p^T \tilde{\boldsymbol{y}}_p(n), \quad (17)$$

the coefficients $\boldsymbol{h}_p$ can be obtained by minimizing $E\left[\epsilon^2(n)\right]$.

As a result of the linear relationship between the kernel coefficients and error signal, least squares (LS) method can be applied to carry out the estimation. Usually, the ADC nonlinearity drifts due to the temperature variation. Thus, adaptive algorithms can be employed. For example, the normalized least mean square (NLMS) algorithm leads to the coefficient update [7]

$$\hat{\boldsymbol{h}}_p(n) = \hat{\boldsymbol{h}}_p(n-1) + \mu_p \frac{\tilde{\boldsymbol{y}}_p(n)\epsilon(n)}{\sum_{p=1}^{P} \|\tilde{\boldsymbol{y}}_p(n)\|_2^2}, p = 1, ..., P, \quad (18)$$

where μ_p is the step size and $\|\cdot\|_2$ denotes the l_2 norm. This estimation structure is shown in Fig. 2.

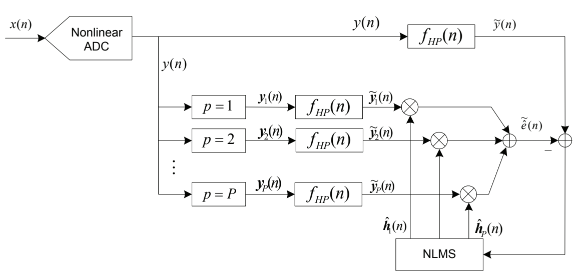

Fig. 2. Adaptive coefficient estimation.

Using (1), (11), (14) and (15), (17) can be rewritten as

$$\epsilon(n) = \sum_{p=1}^{P} \left(\boldsymbol{h}_p - G^p \hat{\boldsymbol{h}}_p\right)^T \tilde{\boldsymbol{x}}_p(n), \quad (19)$$

where $\tilde{\boldsymbol{x}}_p(n) = \boldsymbol{x}_p(n) * f_{HP}(c)$. It is seen that $\epsilon(n)$ is zero if (13) is satisfied. However, note that $\tilde{\boldsymbol{x}}_1(n) = 0$ because the input signal does not contain any frequency components in the energy-free band.

Thus, the NLMS algorithm does not guarantee the convergence of $\hat{\boldsymbol{h}}_1$ and instead of updating $\hat{\boldsymbol{h}}_1(n)$ it is set to zero. Consequently, once $\boldsymbol{h}_p(n), p = 2, ..., P$ converges, the ADC output after compensation is

$$\bar{y}_c(n) = y_1(n) = \sum_{l=0}^{L-1} g_1(l) x(n-l), \quad (20)$$

rather than $Gx(n)$, which indicates that the proposed method only removes nonlinear distortion and leaves the linear response of the ADC intact. Typically, an ADC has a flat linear frequency response over the Nyquist bandwidth. Therefore, the proposed method is effective for ADC nonlinearity post compensation.

5. RESULTS

The performance of the proposed method was assessed via computer simulations. A nonlinear ADC was modeled by a Wiener system consisting of a FIR filter with response [0.9, 0.02] and a memoryless nonlinearity represented by the polynomial function $u(x) = 0.005x^3 - 0.003x^2 + x$. Thus, the equivalent Volterra model has $L = 2$ and $P = 3$. The ADC input was generated by passing a zero-mean and unit-variance i.i.d. Gaussian signal through a low-pass filter with cut-off frequency 0.6π and limiting the peak-to-peak value to 2. An i.i.d uniformly distributed random signal was added to the ADC output to mimic quantization noise of the ADC. For post compensation, the high-pass filter $f_{HP}(n)$ was an Elliptic filter with a cut-off frequency of 0.7π and order 15. The step size of the NLMS algorithm was chosen as 0.001.

First, compensation performance was evaluated by signal power spectral densities (PSDs). Fig. 3 shows the PSDs of the ADC input, output and compensated output. It is seen that in the energy-free region (from 0.6π to π) the uncompensated TH output has distortion energy around -80 dB. After the compensation, the distortion is reduced to -120 dB, indicating an effective correction of the nonlinear distortion.

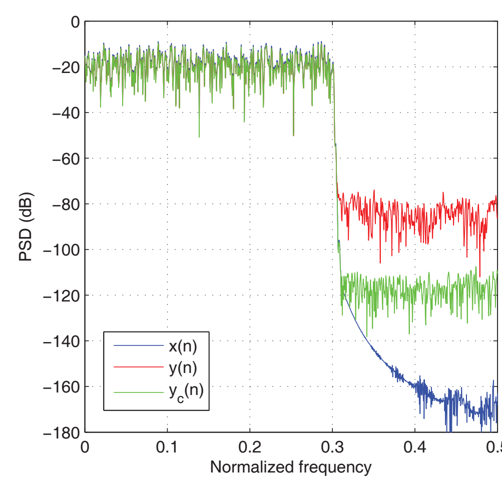

Fig. 3. Signal PSD.

Next, the convergence of the proposed algorithm was evaluated. Based on (13), the mean square deviation (MSD) is used as the figure of merit, which is defined for the pth-order kernel coefficients as

$$\mathrm{MSD}_p(n)(\mathrm{dB}) = 10\log_{10}\|\boldsymbol{h}_p - G^p\hat{\boldsymbol{h}}_p(n)\|_2^2. \tag{21}$$

The MSDs for $p = 2, 3$ are shown in Fig. 4. It can be seen that both the 2nd and 3rd-order model coefficients converge.

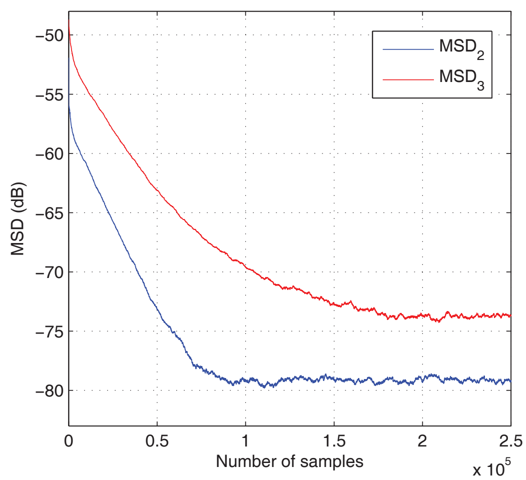

Fig. 4. MSDs of the 2nd and 3rd-order kernel coefficients.

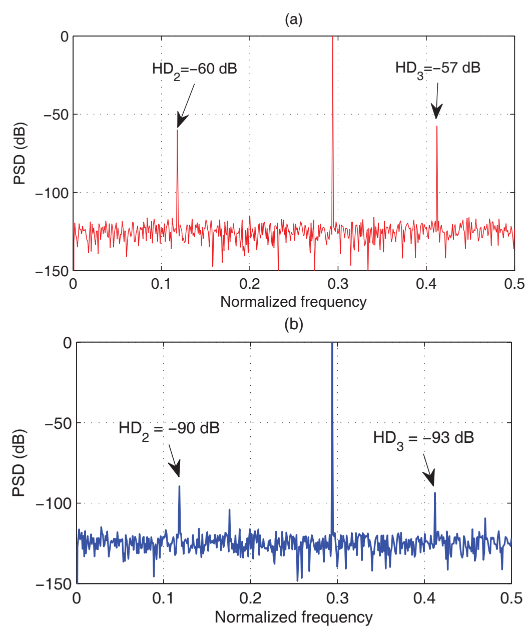

Fig. 5. FFT of a sinusoidal signal: (a) before calibration; (b) after calibration.

Finally, the dynamic performance of the ADC was evaluated. Two typical ADC measures are spurious free dynamic range (SFDR) and harmonic distortion (HD), both of which are obtained via a tone test and measured using a fast Fourier transform (FFT) on the ADC output signal. A sinusoid signal with a normalized frequency of 0.29π was used. The FFT of the ADC output before and after compensation is shown in Fig. 5. After post compensation, HD_2 is reduced from -57 dB to -92 dB and the HD_3 is reduced from -60 dB to -90 dB. Corresponding, the SFDR is improved around 33 dB.

6. CONCLUSIONS

A digital post correction scheme was proposed to compensate for dynamic errors in ADCs. Based on a Volterra series model of nonlinear ADC behavior, a compensation structure was proposed and the optimal compensator coefficients were analyzed. A blind adaptive method was developed for estimating the coefficients based on an energy-free method. Improvements in ADC linearity were demonstrated through the computer simulations.

7. REFERENCES

[1] E. Balistrieri, P. Daponte, and S. Rapuano, “A state of the art on ADC error compensation methods,” *in Proceedings IEEE Instrumentation And Measurement Technology Conference*, pp. 711–716, Como, Italy, May 2004.

[2] H. F. Lundin, *Characterization and Correction of Analog-to-Digital Converters*, Doctoral Thesis in Signal Processing, Stockholm, Sweden 2005

[3] P. Nikaeen and B. Murmann, “Digital compensation of dynamic acquistion errors at the front-end of high-performance A/D converters,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, pp. 499–508, Jun. 2009.

[4] W. Yu, S. Sen, and B. Leung, “Distortion analysis of MOS track-andhold sampling mixers using time-varying Volterra series,” *IEEE Transactions on Circuits and Systems II*, vol. 46, pp. 101–113, Feb. 1999.

[5] A. Gothenberg and H. Tenhunen, “Performance analysis of sampling switches in voltage and frequency domains using Volterra series,” *in Proc. IEEE International Symposium on Circuits and Systems*, pp. 765–768, Vancouver, BC, Canada, May 2004.

[6] P. Satarzadeh, B. C. Levy, and P. J. Hurst, “Digital calibration of a nonlinear T/H,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 3, pp. 454–471, Jun. 2009.

[7] A. H. Sayed, *Fundamentals of Adaptive Filtering*, New York, Wiely, 2003.