ROBUST POSITIONING IN NLOS ENVIRONMENTS USING NONPARAMETRIC ADAPTIVE KERNEL DENSITY ESTIMATION

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ABSTRACT

The problem of locating a mobile station in a wireless network has been extensively investigated due to the growing need for reliable location-based services. In non-line-of-sight environments, the positioning accuracy of classical least-squares based solutions is inaccurate. In order to mitigate the effect induced by non-line-of-sight errors, we address a novel robust nonparametric approach. Herein, we first estimate the range error distribution using nonparametric adaptive kernel density estimation and then optimize the approximate log-likelihood function via a quasi-Newton method. In simulations, the proposed approach shows improved positioning accuracy when the non-line-of-sight contamination is high as compared to several other competitors. Furthermore, it requires only a small amount of computational time.

Index Terms— Non-line-of-sight (NLOS) mitigation, nonparametric, adaptive kernel density estimation, robust positioning.

1. INTRODUCTION

Wireless location refers to the problem of finding the position of a mobile station in cellular or wireless local area network environments [1]. Due to its expanding applications, various positioning approaches based on different measurements, such as received-signal-strength (RSS), time-of-arrival (TOA), time-difference-of-arrival (TDOA) and angle-of-arrival (AOA), have been proposed and can be found in the literature of [2]. We consider here only the range-based positioning using TOA measurements.

In urban areas and indoor environments, the errors induced by the non-line-of-sight (NLOS) propagation may result in degraded location accuracy using traditional positioning approaches, which are developed under the line-of-sight (LOS) circumstance. In order to mitigate the effect of the NLOS errors, various robust estimation approaches exist, including the hypothesis testing based approach [3], minimum entropy based approach [4], M-estimation approach [5] and the semi-parametric approach [6]. In order to conquer the deficiencies involved in the semi-parametric approach [6], we proposed a robust nonparametric estimation approach with the key steps summarized as follows. First, the range error distribution is estimated using the nonparametric adaptive kernel density estimation. Thereafter, the approximate log-likelihood function is constructed and optimized using a traditional quasi-Newton method.

The rest of this paper is organized as follows. In Section 2, we introduce the signal model. In Section 3, the semi-parametric estimation approach is first briefly overviewed and the deficiencies therein are then analyzed. Section 4 introduces the nonparametric adaptive kernel density estimation, followed by the new proposed robust nonparametric estimation approach in Section 5. Simulation results are shown in Section 6 and finally Section 7 concludes the paper.

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2. SIGNAL MODEL

Consider the scenario that a mobile station (MS) is surrounded by N base stations (BSs). The positions of all the BSs are assumed to be known *a priori* and the coordinates of the ith BS are denoted as $[x_i, y_i]^T$ for $i = 1, 2, ..., N$. Besides, let $\boldsymbol{\theta} = [x, y]^T$ be the coordinates of a stationary MS to be determined. The subscript T stands for transpose. At each BS, a total number of K range measurements are obtained. The kth range measurement $r_i(k)$ between the MS and the ith BS in NLOS environments is given by

$$r_i(k) = \underbrace{\sqrt{(x - x_i)^2 + (y - y_i)^2}}_{h_i(\boldsymbol{\theta})} + v_i(k), \tag{1}$$

for $i = 1, 2, ..., N$ and $k = 1, 2, ..., K$. In Eq. (1), $h_i(\boldsymbol{\theta})$ represents the Euclidean distance between the MS and the ith BS. The range errors $v_i(k)$s for $i = 1, 2, ..., N$ and $k = 1, 2, ..., K$ are assumed to be independent and identically distributed (i.i.d) random variables with probability density function (pdf)

$$f_V(v) = (1 - \varepsilon)\mathcal{N}(v; 0, \sigma^2_{\text{LOS}}) + \varepsilon\mathcal{H}(v), \tag{2}$$

with NLOS contamination degree ε quantifying the probability that NLOS errors occur and satisfying $0 \leq \varepsilon \leq 1$. In Eq. (2), $\mathcal{N}(v; 0, \sigma^2_{\text{LOS}})$ stands for the real Gaussian distribution with zero mean and variance σ^2_{LOS} and models the errors in LOS only situation. $\mathcal{H}(v)$ stands for the statistical distribution of the errors due to the NLOS propagation. In general, $\mathcal{H}(v)$ has a positive bias and a larger variance [2] thus is frequently modeled as a shifted Gaussian distribution $\mathcal{N}(v; \mu_{\text{NLOS}}, \sigma^2_{\text{NLOS}})$ or a Rayleigh distribution $\mathcal{R}(v; \sigma_{\text{NLOS}})$. It is noted that, both ε and $f_V(v)$ in Eq. (2) are assumed to be unknown throughout this paper.

3. SEMI-PARAMETRIC ESTIMATION APPROACH

In this section, we give a brief overview of the robust semi-parametric estimation approach proposed in [6], which outperforms a plenty of other salient robust estimation approaches as shown in [5, Chap. 3], [6] and [7]. The core steps of this approach are summarized below. First of all, the nonlinear model introduced in Section 2 is linearized by squaring both sides of Eq. (1) and introducing an

auxiliary variable R that satisfies $R = x^2 + y^2$. Subsequently, a linear regression model

$$\tilde{\mathbf{r}} = \mathbf{S}\tilde{\boldsymbol{\theta}} + \tilde{\mathbf{v}}, \tag{3}$$

is formulated by introducing the vector notations

$$\tilde{\mathbf{r}} = \begin{bmatrix} r_1^2(1) - R_1 \\ \vdots \\ r_1^2(K) - R_1 \\ \vdots \\ r_N^2(1) - R_N \\ \vdots \\ r_N^2(K) - R_N \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ & \vdots & \\ -2x_1 & -2y_1 & 1 \\ & \vdots & \\ -2x_N & -2y_N & 1 \\ & \vdots & \\ -2x_N & -2y_N & 1 \end{bmatrix},$$

and $\tilde{\mathbf{v}} = [\tilde{v}_1(1), ..., \tilde{v}_1(K), ..., \tilde{v}_N(1), ..., \tilde{v}_N(K)]^T$, where $R_i = x_i^2 + y_i^2$ for $i = 1, 2, ..., N$. Vectors $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{v}}$ are both of dimension $NK \times 1$ and matrix $\mathbf{S}$ is of dimension $NK \times 3$. The parameters to be determined here are stacked into the vector parameter $\tilde{\boldsymbol{\theta}} = [x, y, R]^T$. In the next stage, the elements in the vector $\tilde{\mathbf{v}}$ are assumed to be i.i.d random variables with pdf $f_{\tilde{V}}(\tilde{v})$, which is further utilized to calculate the score function $\hat{\varphi}(\tilde{v})$. Next, the score function $\hat{\varphi}(\tilde{v})$ and the vector parameter $\tilde{\boldsymbol{\theta}}$ are estimated jointly in an iterative process.

Although the semi-parametric approach gives considerable improved estimation performance, some deficiencies still remain unsolved. First, an auxiliary parameter R is introduced, but the constraint condition $R = x^2 + y^2$ is not incorporated into the optimization process, which may lead to a suboptimal solution. Secondly, $\tilde{v}_i(k)$s for $k = 1, 2, ..., K$ and $i = 1, 2, ..., N$ strongly violate the i.i.d assumption after the linearization process. Therefore, it is not plausible to utilize the residuals $\hat{\tilde{\mathbf{v}}}$ obtained from the second step of [6, Table1] for the estimation of $f_{\tilde{V}}(\tilde{v})$. Thirdly, an estimate of $f_{\tilde{V}}(\tilde{v})$ is obtained by using the transformation kernel density estimation (TKDE), in which a tuning parameter λ has to be selected by maximizing a log-likelihood function in terms of λ. But the problem exists in how to properly choose an interval $[\lambda_L, \lambda_U]$, in which the global optimum λ resides, automatically according to different impact factors such as network topologies, range error distributions and so on. A wrongly selected interval can lead to severe performance degradation, which has been illustrated in [6].

4. ADAPTIVE KERNEL DENSITY ESTIMATION

Before proceeding with the new positioing approach, we introduce the nonparametric adaptive kernel density estimation (AKDE) [8]. Assume we have M i.i.d observations $x_1, x_2, ..., x_M$ from a continuous univariate distribution with probability density function $f(x)$, AKDE can provide an accurate estimate of $f(x)$ through the following steps.

- Step 1. Find a pilot density estimator $\hat{f}_0(x)$ by

$$\hat{f}_0(x) = \frac{1}{M}\sum_{i=1}^{M}\frac{1}{h_0}K(\frac{x - x_i}{h_0}), \tag{4}$$

where

$$K(x) = \frac{1}{\sqrt{2\pi}}\exp\left[-x^2/2\right] \tag{5}$$

is a Gaussian kernel and $h_0 = 0.79RM^{-1/5}$ with R denoting the interquartile range of the M observations.

- Step 2. Define the local bandwidth factors λ_is by

$$\lambda_i = \left(\hat{f}_0(x_i) \Big/ \left[\prod_{i=1}^{M}\hat{f}_0(x_i)\right]^{\frac{1}{M}}\right)^{-\alpha}, \tag{6}$$

for $i = 1, 2, ..., M$, with α denoting the sensitivity parameter and satisfying $0 \le \alpha \le 1$. In this paper, we choose $\alpha = 0.5$ as suggested in [8].

- Step 3. Construct the adaptive kernel estimator (AKE) $\hat{f}(x)$ by

$$\hat{f}(x) = \frac{1}{M}\sum_{i=1}^{M}\frac{1}{h\lambda_i}K(\frac{x - x_i}{h\lambda_i}). \tag{7}$$

For fixed local bandwidth factors, the smoothing parameter h is set to $h = 0.79RM^{-1/5}$ in [9]. In this paper, we choose h automatically using the least-square cross-validation [8], which provides better performance both in the main body and in the tail of the density function as illustrated in [8, Sec 5.3.5]. The basic principle of the least-square cross-validation is to minimize the score function $M_0(h)$ in terms of h. The score function $M_0(h)$ is defined by

$$M_0(h) = \int_{-\infty}^{\infty}\hat{f}^2(x)\mathrm{d}x - \frac{2}{M}\sum_{i=1}^{M}\hat{f}_{-i}(x_i), \tag{8}$$

where

$$\hat{f}_{-i}(x) = \frac{1}{M-1}\sum_{j=1, j\neq i}^{M}\frac{1}{h\lambda_j}K(\frac{x - x_j}{h\lambda_j}) \tag{9}$$

is also an estimator of the density, which is constructed from all i.i.d observations except x_i. Since $K(x)$ is a Gaussian kernel, it is shown in [8] that

$$\int_{-\infty}^{\infty}\hat{f}^2(x)\mathrm{d}x = \frac{1}{M^2}\sum_{i=1}^{M}\sum_{j=1}^{M}\frac{1}{\sqrt{2\pi(h^2\lambda_i^2 + h^2\lambda_j^2)}} \times \exp\left[-\frac{(x_i - x_j)^2}{2(h^2\lambda_i^2 + h^2\lambda_j^2)}\right]. \tag{10}$$

In [8], minimizing $M_0(h)$ defined in Eq. (8) is supposed to produce a smoothing parameter h that also approximately minimizes the mean integrated square error (MISE) given by

$$E\left[\int_{-\infty}^{\infty}\left(\hat{f}(x) - f(x)\right)^2\mathrm{d}x\right], \tag{11}$$

where the symbol $E[\cdot]$ stands for the expectation. The numerical method suggested at the end of [8, Sec. 3.5] is performed to calculate the optimum h.

5. ROBUST NONPARAMETRIC ESTIMATION APPROACH

Based on the signal model introduced in Section 2, the following robust nonparametric approach is proposed to give improved positioning accuracy. The key steps of it are summarized below.

- Step 1. Calculate the least square solution of Eq. (3) by $\hat{\tilde{\boldsymbol{\theta}}} = [\tilde{x}_{\mathrm{LS}}, \tilde{y}_{\mathrm{LS}}, \tilde{R}_{\mathrm{LS}}]^T = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\tilde{\mathbf{r}}$, and obtain an initial estimate $\boldsymbol{\theta}_{\mathrm{LS}} = [\tilde{x}_{\mathrm{LS}}, \tilde{y}_{\mathrm{LS}}]^T$.

- Step 2. Calculate the residual vector $\hat{\mathbf{v}}$ by

$$\hat{\mathbf{v}} = \mathbf{r} - \hat{\mathbf{D}}, \tag{12}$$

where $\mathbf{r} = [r_1(1), ..., r_1(K), ..., r_N(1), ..., r_N(K)]^T$ and $\hat{\mathbf{D}} = [\underbrace{h_1(\boldsymbol{\theta}_{\text{LS}}), ..., h_1(\boldsymbol{\theta}_{\text{LS}})}_{K}, ..., \underbrace{h_N(\boldsymbol{\theta}_{\text{LS}}), ..., h_N(\boldsymbol{\theta}_{\text{LS}})}_{K}]^T$. Vectors $\hat{\mathbf{v}}$, $\mathbf{r}$ and $\hat{\mathbf{D}}$ are all of dimension $NK \times 1$.

- Step 3. Construct an estimate $\hat{f}_V(v)$ of the true pdf $f_V(v)$ from the NK approximate i.i.d entries of $\hat{\mathbf{v}}$ using the technique introduced in Section 4. When $\boldsymbol{\theta}_{\text{LS}}$ is close to the true vector parameter $\boldsymbol{\theta}$, $\hat{f}_V(v)$ will surely approach to $f_V(v)$.

- Step 4. Suppose $\hat{f}_V(v)$ is close to the true pdf $f_V(v)$, we can approximate the log-likelihood function by

$$ll(\boldsymbol{\theta}) = \sum_{i=1}^{N}\sum_{k=1}^{K} \log \hat{f}_V(r_i(k) - h_i(\boldsymbol{\theta})). \tag{13}$$

From Section 4, we know that,

$$\hat{f}_V(v) = \frac{1}{NK}\sum_{j=1}^{NK} \frac{1}{\sqrt{2\pi}h\lambda_j} \exp\left[-\frac{(v-\hat{v}_j)^2}{2h^2\lambda_j^2}\right], \tag{14}$$

where $\hat{v}_j$ is the jth entry of the residual vector $\hat{\mathbf{v}}$.

- Step 5. The maximum-likelihood estimator is obtained by minimizing the negative of the log-likelihood function, i.e.

$$\hat{\boldsymbol{\theta}}_{\text{MLE}} = \arg\min_{\boldsymbol{\theta}} \; g(\boldsymbol{\theta}) = -ll(\boldsymbol{\theta}). \tag{15}$$

From Eq. (14), we have

$$g(\boldsymbol{\theta}) = -\sum_{i=1}^{N}\sum_{k=1}^{K} \log\{\frac{1}{NK}\sum_{j=1}^{NK} \frac{1}{\sqrt{2\pi}h\lambda_j} \times \exp[-\frac{(r_i(k) - h_i(\boldsymbol{\theta}) - \hat{v}_j)^2}{2h^2\lambda_j^2}]\}. \tag{16}$$

Many numerical methods can be utilized to solve Eq. (16), e.g. Newton-Raphson method, scoring approach and expectation maximization algorithm. As compared to them, the safest way to find the global minimum is to perform a 2-dimensional grid search over a large area [10]. But the drawback of the grid search exists in the higher computational load. In this paper, we choose the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton method [11], since it guarantees downhill progress towards the minimum in each Newton step [12]. Details of this method are shown in Table 1.

In the first step, the initial guess of $\boldsymbol{\theta}$ is set to $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_{\text{LS}} = [\tilde{x}_{\text{LS}}, \tilde{y}_{\text{LS}}]^T$, which is an adequate initialization [2]. Besides, the initial approximate Hessian matrix is given by $\mathbf{H}_1 = \mathbf{I}_2$, where $\mathbf{I}_2$ is an identity matrix of dimension 2×2. Due to space limitations, the analytical expression of $\bigtriangledown g(\boldsymbol{\theta})$ is not shown here.

6. NUMERICAL EXPERIMENT

As in [6], we consider here a wireless network with $N = 10$ BSs and one MS. The BSs are locating in a two dimensional plane with fixed coordinates $(x_1 = 2.5, y_1 = 5)$, $(x_2 = 1, y_2 = 3.5)$, $(x_3 = 4.5, y_3 = 1.75)$, $(x_4 = 1.5, y_4 = 4)$, $(x_5 = 3, y_5 = 4.5)$, $(x_6 = 1.75, y_6 = 1)$, $(x_7 = 4, y_7 = 0.75)$, $(x_8 = 5, y_8 = 1.25)$, $(x_9 = 0.5, y_9 = 2)$, $(x_{10} = 3, y_{10} = 0.25)$ in kilometer (km) respectively.

Table 1. BFGS quasi-Newton method with a cubic line search procedure.

Step 1. Obtain a search direction $\mathbf{s}_k = -\mathbf{H}_k \bigtriangledown g(\boldsymbol{\theta}_k)$, where

$$\bigtriangledown g(\boldsymbol{\theta}_k) = \left[\frac{\partial g(\boldsymbol{\theta})}{\partial x}|_{\boldsymbol{\theta}=\boldsymbol{\theta}_k}, \frac{\partial g(\boldsymbol{\theta})}{\partial y}|_{\boldsymbol{\theta}=\boldsymbol{\theta}_k}\right]^T \tag{17}$$

is the gradient of the objective function $g(\boldsymbol{\theta})$ evaluated at $\boldsymbol{\theta}_k$.
Step 2. Find the step size α_k along the direction $\mathbf{s}_k$ via cubic line search introduced in [13, Algorithm 3.5 and 3.6].
Step 3. Update the estimate of the vector parameter by $\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k + \alpha_k \mathbf{s}_k$.
Step 4. Set $\boldsymbol{\delta}_k = \alpha_k \mathbf{s}_k$ and $\boldsymbol{\gamma}_k = \bigtriangledown g(\boldsymbol{\theta}_{k+1}) - \bigtriangledown g(\boldsymbol{\theta}_k)$.
Step 5. Update the approximate Hessian matrix by

$$\mathbf{H}_{k+1} = \mathbf{H}_k + (1 + \frac{\boldsymbol{\gamma}_k^T \mathbf{H}_k \boldsymbol{\gamma}_k}{\boldsymbol{\delta}_k^T \boldsymbol{\gamma}_k})\frac{\boldsymbol{\delta}_k \boldsymbol{\delta}_k^T}{\boldsymbol{\delta}_k^T \boldsymbol{\gamma}_k} - (\frac{\boldsymbol{\delta}_k \boldsymbol{\gamma}_k^T \mathbf{H}_k + \mathbf{H}_k \boldsymbol{\gamma}_k \boldsymbol{\delta}_k^T}{\boldsymbol{\delta}_k^T \boldsymbol{\gamma}_k}), \tag{18}$$

which is well known as BFGS formula.
Step 6. If $\|\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k\| < \Delta$, then stop, otherwise repeat the whole procedure. Threshold Δ takes a small value.

In the Monte-Carlo simulation, the true MS coordinates (x, y) are both uniformly generated from the interval $[2\,\text{km}, 3\,\text{km}]$. At each BS, a number of $K = 15$ range measurements are obtained.

In our simulations, we focus on two important aspects, namely the positioning accuracy as well as the computational time, which are crucial to the positioning algorithm design. We test here four different estimation approaches, namely the traditional least square approach, robust M-estimation approach, semi-parametric approach and the new proposed robust nonparametric approach. Note that, the first three approaches are all developed under the linear regression model in Eq. (3). For the robust M-estimation approach in [5], Huber's score function with a clipping point $c = 0.6/(1.483\text{mad}(\hat{\tilde{\mathbf{v}}}))$ is empirically selected so as to achieve a tradeoff between efficiency and robustness. Here, mad$(\cdot)$ denotes the median absolute deviation. In the semi-parametric approach, the bounds of the parameter λ tuning the parametric transformation function are set to 0.9 and 1 as suggested in [6]. On the contrary, no parameters need to be provided in our approach.

We start with the positioning accuracy by demonstrating the mean error distance (MED) as a function of the NLOS contamination degree ε ranging from 0 to 1 at an increment 0.1. For every ε, 1500 Monte-Carlo runs have been done to calculate the MED. Two different statistical distributions of the NLOS errors are considered, namely $\mathcal{H}(v) = \mathcal{N}(\mu_{\text{NLOS}}, \sigma^2_{\text{NLOS}})$ with $\mu_{\text{NLOS}} = 650$ and $\sigma_{\text{NLOS}} = 450$ in the first case and $\mathcal{H}(v) = \mathcal{R}(\sigma_{\text{NLOS}})$ with parameter $\sigma_{\text{NLOS}} = 500$ in the second case. In both cases, σ_{LOS} of the underlying zero mean Gaussian distribution is set to 150. Fig.1 and Fig.2 illustrate the MED versus the NLOS contamination degree ε under the two cases, from which we have observed the following phenomena. First, the robust M-estimation approach breaks down for a contamination degree higher than 0.5 due to the fact that the median has a breakdown point of 0.5 according to the robust statistics [14]. Furthermore, the estimation performance of the semi-parametric approach degrades more rapidly as compared to that of the robust nonparametric approach when the contamination degree

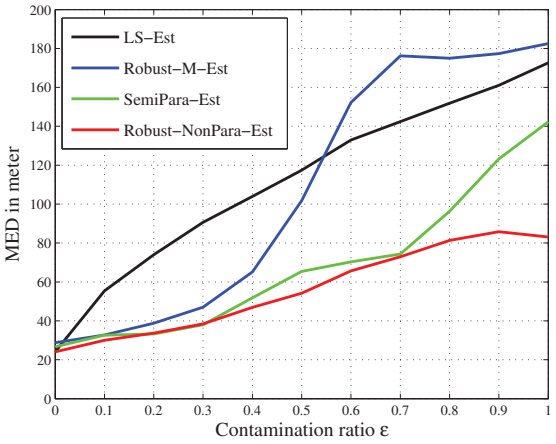

Fig. 1. MED versus NLOS contamination degree. NLOS errors take shifted Gaussian distribution $\mathcal{N}(\mu_{\text{NLOS}}, \sigma^2_{\text{NLOS}})$ with $\mu_{\text{NLOS}} = 650$ and $\sigma_{\text{NLOS}} = 450$.

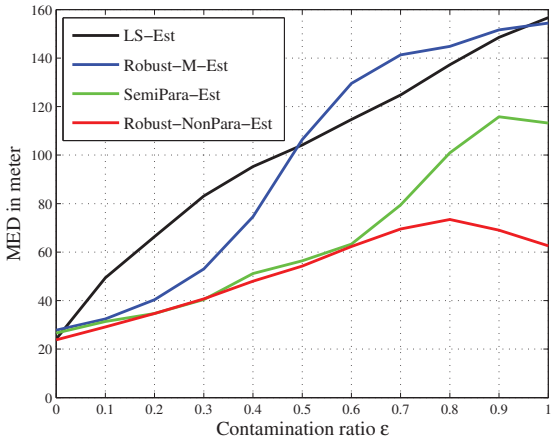

Fig. 2. MED versus NLOS contamination degree. NLOS errors take Rayleigh distribution $\mathcal{R}(\sigma_{\text{NLOS}})$ with parameter $\sigma_{\text{NLOS}} = 500$.

Table 2. The average computational time

	Computational time
Least square approach	0.00005 sec.
Robust M-estimation approach	0.01612 sec.
Semi-parametric approach	1.09378 sec.
Robust nonparametric approach	2.06402 sec.

is high. A probable reason for this result is that the i.i.d assumption made in the semi-parametric approach is vastly violated in this case. Next, the average computational time of one Monte-Carlo run is calculated for the previous simulations. The results are listed in Table 2. Our approach is also tested in various other situations and achieves well improved positioning accuracy as compared to the competitors. The results are not shown here due to space limitations.

7. CONCLUSION

In order to mitigate the effect induced by the NLOS propagation, we addressed here a robust nonparametric estimation approach, which chooses all the parameters automatically and adaptively. In comparison with several other salient robust approaches, the new approach has shown extensively enhanced positioning accuracy in different scenarios when the NLOS contamination degree is high. However, the improved performance of our approach comes at the expense of an increased computational time.

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