ON CAPACITY OF MISO BEAMFORMING WITH CODEBOOK BASED SCALAR QUANTIZATION

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ABSTRACT

Three scalar quantization strategies for practical codebook generation are proposed in this paper. All of these three strategies are implemented in MISO beamforming systems. Achievable capacities of these scenarios are derived and discussed. The numerical results show that our proposed codebook generation strategies have lower computational burden compared with other strategies based on higher dimensional vector quantization, and provide quite a small capacity loss only. Furthermore, our on-going work shows that the proposed strategies work well for MIMO precoding systems also.

Index Terms— MIMO, scalar quantization, codebook, limited feedback, capacity

1. INTRODUCTION

Nowadays, higher and higher data rate is desired to supply high quality multi-media services. Multiple-input multiple-output (MIMO) technology has attracted much more attentions since MIMO wireless channels, created by exploiting antenna arrays at both the transmitter and receiver, promises high capacity and high quality wireless communication links [1]. It is well-known that with full channel state information (CSI) at the transmitter (CSIT), employment of precoding techniques in the form of water-filling across the eigen states of a MIMO channel can improve its capacity [2], [3]. This implies that the transmitter requires some form of knowledge on the wireless channel conditions. In time division duplexing (TDD) systems, reciprocity can be employed to investigate CSI and make CSI available at transmitter. However, employing reciprocity in frequency division duplexing (FDD) systems has been impossible since the forward and reverse links in FDD generally have highly uncorrelated channels.

The adaptation of feedback makes instantaneous CSIT possible. In practical systems, the receiver estimates the channel conditions based on the pre-defined reference signals known by both transmitter and receiver. After channel estimation, the receiver sends the estimated channel information back to the transmitter side, and the transmitter uses this information to adapt the forward link transmission. The adoption of feedback can improve the system performance, such as increasing capacity and reducing the failure rate of data transmission, especially when the channel introduces some form of disturbance (such as spatial interference, inter-symbol interference, and multiuser interference, etc.) that cannot be handled by the receiver alone. However on the other hand, the feedback information itself occupies some frequency resource and decreases the spectrum efficiency of system. That's why limited feedback (finite rate feedback) is necessary in practical systems over limited bandwidth feedback channels.

There are many recent works dealing with limited feedback [4]-[6]. One of them is generating limited feedback based on the pre-defined codebook, which is known by both transmitter and receiver. Once the receiver obtained the CSI by channel estimation, it checks the codebook to figure out the quantization partitions, and represent all the channel information located in these partitions by the corresponding codewords. After finding those codewords from codebook, the receiver sends the codebook index corresponding to these codewords back to the transmitter instead of sending full CSI back. At the transmitter side, the transmitter chooses the codewords from the identical codebook based on the feedback information from receiver. Usually, the systems suffer lower burden to send codebook index rather than feedback full CSI. In this situation, the strategies of generating codebook become the key issues.

In this paper, we propose several codebook generation strategies for MISO beamforming systems. Generally, our strategies are based on scalar quantization, which has lower computational complexity compared to high dimensional (more than 2 dimension) vector quantization strategies. The achievable capacities of systems employing the proposed strategies are derived and discussed. It is found that though our proposed codebook generation strategies are simple, they provide only a small ergodic capacity lag compared with optimal beamforming.

The remaining part of this paper is organized as follows. Codebook generation strategies using scalar quantization for multiple-input single-output (MISO) beamforming are described in Section 2. In Section 3, the achievable capacities using various codebook generation strategies are derived, and numerical results are provided and analyzed in Section 4. Finally, we conclude the paper in Section 5.

2. CODEBOOK DESIGN STRATEGIES USING SCALAR QUANTIZATION

In this paper, we concentrate on the scalar quantization of MIMO channel matrix for codebook generation. Our proposed scalar quantization strategies work on each channel element of channel matrix. Assume that it is a multiuser MIMO (MU-MIMO) downlink, which is typical in cellular systems [7]. As shown in Figure 1, there are N_t transmit antennas at base station (BS) side, and N_r mobile stations (MS) communicate with BS simultaneously. Each MS is equipped with a single antenna, and the relationship of

$N_r \le N_t$ holds in this system. The whole channel matrix of this system is an $N_r \times N_t$ complex matrix with the channel element $h_{m,n} = \left|h_{m,n}\right| e^{j\theta}$, where *m*, *n* represent the *m*-th row and the *n*-th column, $\left|h_{m,n}\right|$ is the channel amplitude, θ is the phase rotation, and $j = \sqrt{-1}$ is the imaginary unit. Since there are two variables for each channel element (channel amplitude and phase rotation), we design separate scalar quantizers for them.

2.1. Amplitude Quantization of Channel Elements

In the case of channel amplitude quantization, two different kinds of quantizers are proposed. The simplest amplitude one is linear amplitude scalar quantizer (LASQ), where the amplitude of channel gain is linearly quantized. The other quantizer is named as non-linear amplitude scalar quantizer (NLASQ), where the channel amplitude is quantized linearly from the cumulative distribution function (CDF) perspective. Therefore, this quantization is non-linear quantization under the consideration of channel amplitude dimension.

Assume that the MIMO channel is narrow band flat Rayleigh distributed. This channel model can be generated by standard Gaussian distribution with variance $\sigma^2 = 1$.

$$\text{Rayleigh} = \sqrt{\text{Gaussian}^2 + \text{Gaussian}^2}\,. \tag{1}$$

For a Rayleigh distributed variable *x*, the probability density function (pdf) is given as

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \qquad x \ge 0\,, \tag{2}$$

and CDF function is

$$F(x) = 1 - e^{-x^2/2\sigma^2}, \qquad \text{for } x \in [0,\infty)\,. \tag{3}$$

If the desired codebook size is $N_{amplitude}$, then after scalar quantization, there are $N_{amplitude}$ partitions and $N_{amplitude}$ codewords. The number of feedback bits for quantization index is $B_{amplitude} = \log_2 N_{amplitude}$. In order to get a general formula, we define each quantization partition can be represented as a region $[a,b)$. Therefore, the expectation of variable *x* located in this region is given as

$$\begin{aligned} E\left[x\right] &= \int_a^b x \cdot f(x) dx, \\ &= (ae^{-\frac{a^2}{2}} - be^{-\frac{b^2}{2}}) + \frac{\sqrt{2\pi}}{2} \cdot [erfc(a) - erfc(b)], \end{aligned} \tag{4}$$

where *erfc*(.) is the complementary error function.

The difference between LASQ and NLASQ is how to figure out the quantization partitions. In the case of LASQ, quantization partitions are calculated by uniformly dividing the whole available region $[0,\infty)$. However, it is impossible to generate a limited size codebook for this variable *x*. In order to guarantee the high accuracy of quantization, we make the codebook based on a truncated Rayleigh distribution,

$$F(x) = 1 - e^{-x^2/2\sigma^2} = 0.999999\,. \tag{5}$$

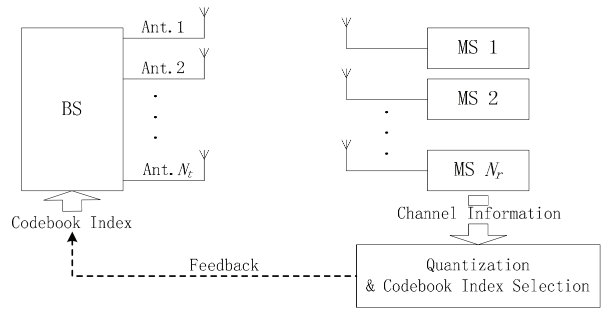

Fig. 1. Block diagram of MU-MIMO system with limited feedback.

Equation (5) indicates that the truncated Rayleigh distribution is 99.9999% approximation of normal Rayleigh distribution. In this case, the available region of variable *x* is $x \in [0, 5.2565)$. The general solutions of partition boundaries of LASQ for the *n*-th partition are

$$a = \frac{(n-1)\times 5.2565}{N_{amplitude}}, \text{ and } b = \frac{n \times 5.2565}{N_{amplitude}}, \tag{6}$$

where *n* is the codeword index or partition index.

For NLASQ, the partition calculation is different from LASQ, since the partitions are calculated by uniformly dividing the distribution probability. For the *n*-th partition, we calculate

$$F(x) = 1 - e^{-x_n^2/2\sigma^2} = \frac{n}{N_{amplitude}}\,. \tag{7}$$

Equation (7) indicates that the probabilities of variable *x* located in different partitions are equal. The partition boundaries of NLASQ for the *n*-th partition are

$$a = x_{n-1}, \text{ and } b = x_n\,. \tag{8}$$

2.2. Phase Quantization of Channel Elements

The phase rotations of Rayleigh distributed channel elements are uniform distributed in region $[0, 2\pi)$. Assuming the codebook size for phase quantization is N_{phase}, the number of bits for codebook index is $B_{phase} = \log_2 N_{phase}$.

3. ACHIEVABLE CAPACITY OF CODEBOOK BASED SCALAR QUANTIZATION

In this section, achievable capacities for MISO beamforming systems are derived under various scenarios. Assume that CSIT is fully available at transmitter side, the fixed total transmit power $P_{Tx} = N_t$, and the reveived signal to noise ratio (SNR) is ρ under the assumption that noise power equals 1. In a MISO system employing transmit beamforming, the achievable capacity is

$$C_{MISO-BF} = \log_2(1 + \rho\left|\mathbf{hv}\right|^2)\,, \tag{9}$$

where $\mathbf{h}$ is the $1 \times N_t$ channel vector, and $\mathbf{v}$ is beamforming vector. The capacity employing optimal beamforming is

$$C_{MISO-optimalBF} = \log_2(1+\rho\left|\mathbf{h}\mathbf{v}_{opt}\right|^2) = \log_2(1+\rho N_t), \tag{10}$$

where $\mathbf{v}_{opt} = \mathbf{h}^H / \left\|\mathbf{h}\right\|$. Equation (10) is given for the purpose of setting capacity upper bound. When scalar quantizers for amplitude and phase are adopted, the capacity formula can be re-written as

$$\widehat{C}_{MISO-quantBF} = \log_2(1+\rho\left|\mathbf{h}\widehat{\mathbf{v}}\right|^2), \tag{11}$$

where the quantized beamforming vector is

$$\widehat{\mathbf{v}} = \begin{bmatrix} \left|\hat{v}_1\right| e^{-j\hat{\theta}_1} \\ \left|\hat{v}_2\right| e^{-j\hat{\theta}_2} \\ \vdots \\ \left|\hat{v}_i\right| e^{-j\hat{\theta}_i} \end{bmatrix} = \begin{bmatrix} \frac{1}{\left|\hat{h}_1\right|} e^{-j\hat{\theta}_1} \\ \frac{1}{\left|\hat{h}_2\right|} e^{-j\hat{\theta}_2} \\ \vdots \\ \frac{1}{\left|\hat{h}_i\right|} e^{-j\hat{\theta}_i} \end{bmatrix}. \tag{12}$$

Scenario 1] Uniform Phase Quantization (UPQ): We start from the simplest scenario, where assuming the channel amplitude can be perfectly compensated and only phase quantization error exists in the MISO beamforming systems. Under this assumption, the channel capacity is given as

$$\begin{aligned} \widehat{C}_{MISO-quantBF} &= \log_2(1+\rho\left|\mathbf{h}\widehat{\mathbf{v}}\right|^2) \\ &= \log_2(1+\tfrac{\rho}{N_t}\left|\sum_{i=1}^{N_t} h_i \widehat{v}_i\right|^2) \\ &= \log_2(1+\tfrac{\rho}{N_t}\left|\sum_{i=1}^{N_t} \left|h_i\right| e^{j\theta_i} \cdot \frac{1}{\left|\hat{h}_i\right|} e^{-j\hat{\theta}_i}\right|^2) \\ &= \log_2(1+\tfrac{\rho}{N_t}\left|\sum_{i=1}^{N_t} \left|\Delta h_i\right| e^{j\Delta\theta_i}\right|^2). \end{aligned} \tag{13}$$

In (13), $\left|\Delta h_i\right|$ and $\Delta\theta_i$ indicate the quantization error after amplitude and phase compensation at transmitter, where $\Delta\theta_i$ is uniform distributed in the region $[-\frac{\pi}{N_{phase}}, \frac{\pi}{N_{phase}})$. The ergodic capacity of this system is shown as

$$\bar{C}_{MISO-quantBF} = \log_2(1+\tfrac{\rho}{N_t}\left|\sum_{i=1}^{N_t} E[\left|\Delta h_i\right|] \cdot E[e^{j\Delta\theta_i}]\right|^2). \tag{14}$$

Since channel amplitudes are perfectly compensated and phase quantization errors are uniform distributed, then the expectation of channel amplitude $E[\left|\Delta h_i\right|] = 1$, and

$$E[e^{j\Delta\theta}] = E[\cos(\Delta\theta)] + jE[\sin(\Delta\theta)] = \mathrm{sinc}(\frac{\pi}{N_{phase}}). \tag{15}$$

By combination of (14) and (15), the ergodic capacity is finally given as

$$\bar{C}_{MISO-quantBF} = \log_2(1+\rho N_t \,\mathrm{sinc}(\frac{\pi}{N_{phase}})). \tag{16}$$

Scenario 2] Linear Amplitude Scalar Quantizer with Uniform Phase Quantization (LASQ + UPQ): In this subsection, we consider more general scenario that both channel amplitude and channel phase are quantized. Under these assumptions, ergodic capacity employing amplitude and phase quantizers can be represented as

$$\bar{C}_{MISO\_LASQ+UPQ} = \log_2(1+\tfrac{\rho}{N_t}\left|\sum_{i=1}^{N_t} E\left[\frac{\left|h_i\right|}{\left|\hat{h}_i\right|}\right] \cdot E[e^{j\Delta\theta_i}]\right|^2). \tag{17}$$

In (17), the expectation value for phase error is the same as in (15). By using (4), the expectation of amplitude error is given as

$$\begin{aligned} E\left[\frac{\left|h_i\right|}{\left|\hat{h}_i\right|}\right] &= \sum_{n=1}^{N_{amplitude}} \left\{\frac{1}{\left|\hat{h}_{n,i}\right|} E\left[\left|h_{n,i}\right|\right]\right\} \\ &= \sum_{n=1}^{N_{amplitude}} \left\{\frac{1}{\left|\hat{h}_{n,i}\right|} \cdot \{(ae^{-\frac{a^2}{2}} - be^{-\frac{b^2}{2}}) + \frac{\sqrt{2\pi}}{2} \cdot [erfc(a) - erfc(b)]\}\right\}. \end{aligned} \tag{18}$$

Note that the partition boundaries *a* and *b* should follow (6). By combining (15), (17), and (18), the ergodic capacity of scenario 2 is

$$\bar{C}_{MISO\_LASQ+UPQ} = \log_2(1+\frac{\rho}{N_t} \cdot E\left[\frac{\left|h_i\right|}{\left|\hat{h}_i\right|}\right] \mathrm{sinc}(\frac{\pi}{N_{phase}})). \tag{19}$$

Scenario 3] Non-linear Amplitude Scalar Quantizer with Uniform Phase Quantization (NLASQ + UPQ): The ergodic capacity formula in this scenario is exactly the same as in (19). As aforementioned, the only difference is the partition calculation, and the partition boundaries *a* and *b* should follow (7) and (8).

4. NUMERICAL RESULTS AND ANALYSIS

The achievable capacities of the previous three MISO beamforming scenarios are provided and discussed in this section. MISO system and error-free feedback are assumed. Figure 2 shows the numerical results of scenario 1. All the curves are plotted based on (16). From the numerical results, we can easily find out that 4-bit feedback shows good enough performance since the capacity gap between “4 bits” and “8 bits” is too small (0.05 bps/Hz) and almost negligible. 8 bits UPQ curve shows exactly the same performance as that of optimal beamforming. When less number of phase quantization bits (4 bits and 2 bits) are adopted, there is always a capacity gap among those curves and optimal beamforming curve due to quantization error.

Figure 3 shows the capacity curves for scenario 2 and 3. The optimal beamforming curve is added in this figure as an upper bound. Solid curves are for scenario 2 (LASQ+UPQ) and dashed curves are for scenario 3 (NLASQ+UPQ). It is obvious that the adoption of amplitude quantization introduces additional quantization error. From Figure 3, we can observe that the capacity of scenario 3 performs much better than those of scenario 2 in the case of 2-bit and 4-bit amplitude scalar quantization. Particularly, in the case of 2-bit quantization, the achievable capacity of scenario 3 is nearly doubled compared with that of scenario 2. However, in the case of 8-bit amplitude scalar quantization, the achievable capacities of scenario 2 and 3 are almost the same.

5. CONCLUSIONS

In order to overcome the exponential computational complexity of unstructured precoding codebooks, three simple scalar quantization strategies are proposed for practical codebook generation. The proposed strategies are evaluated in MISO beamforming systems, and the general solutions of achievable ergodic capacity formulas are derived. The numerical results show that our proposed simple codebook generation strategies based on scalar quantization are capacity efficient, and they require smaller computational burden than other codebook generation algorithms based on higher dimensional vector quantization. Additionally, our current on-going work shows that these simple scalar quantization strategies proposed in this paper are also efficient for MIMO systems employing precoding techniques.

6. ACKNOWLEDGEMENT

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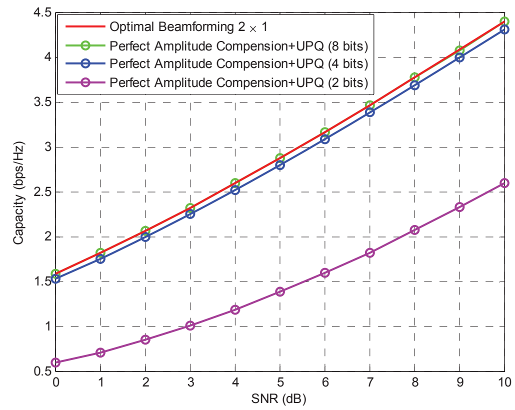

Fig. 2. Capacity of scenario 1 for MISO beamforming systems.

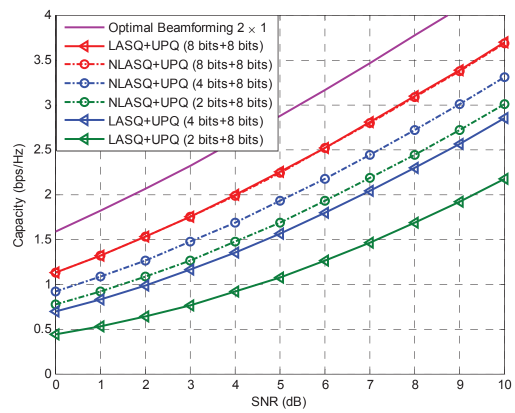

Fig. 3. Capacity of scenario 2 & 3 for MISO beamforming systems.