GENERALISED PHASE SYNCHRONY WITHIN MULTIVARIATE SIGNALS: AN EMERGING CONCEPT IN TIME-FREQUENCY ANALYSIS

Amir Omidvarnia, Boualem Boashash, Ghasem Azemi, Paul Colditz and Sampsa Vanhatalo

The University of Queensland, UQ Centre for Clinical Research, QLD 4029, Australia

a.omidvarnia@uq.edu.au, boualem@qu.edu.qa, g.azemi@uq.edu.au, p.colditz@uq.edu.au, sampsa.vanhatalo@helsinki.fi

ABSTRACT

This paper introduces the notion of the instantaneous frequency (IF) based generalized phase synchrony in time-frequency analysis based on the concept of cointegration. This phase synchrony is then quantified by investigating the linear relationships between IF laws of nonstationary multivariate signals. The proposed approach is applied to a multichannel newborn EEG signal and the results are compared with that of a bivariate phase synchrony measure.

***Index Terms*—** Phase Synchrony, Instantaneous Frequency, Cointegration, EEG, Time-Frequency Analysis

1. INTRODUCTION

Phase synchrony plays an important role in the investigation of the dynamics of complex systems. In many practical cases, this dynamic is due to time-varying interactions of several subsystems. Human brain is an example of such complex systems where different components interact with each other dynamically. Bivariate measures have been used for measuring pair-wise synchrony in multivariate systems. These measures are, however, unable to capture global synchrony observed in nonstationary multivariate signals.

Instantaneous phase of a real-valued signal can be obtained using the concept of analytic signals [1, 2] or complex Gabor wavelet filtering [3]. The resulting phase information is then employed to extract a measure of phase synchrony between the signals [3]. Several approaches including *Mean Phase Coherence* (also, sometimes referred to as *Phase Locking Value*) [2, 3], *Evolution Map Approach (EMA)* [4], *Instantaneous Period Approach (IPA)* [5], *Mutual Prediction Approach (MPA)* [5], *General Field Synchronization (GFS)* [6], EMD-based methods [7, 8] and *frequency flows analysis (FFA)* [9] have been proposed for the evaluation of phase synchrony within bivariate and multivariate data.

The assumption shared by all these methods is that the phase-locking ratio between signals is always rational. Recently, the concept of *cointegration* [10], initially introduced in econometrics, has been used to extend the classical definition of phase synchrony to include generalized phase synchrony where the phase locking ratio can be irrational.

This paper proposes a new interpretation of generalized phase synchrony in the time-frequency domain based on the linear relationships between IF ridges of a multivariate nonstationary signal. This interpretation applies to the classical definition of phase synchrony as well. Then, we suggest an approach to extract generalized phase synchronous intervals within nonstationary multivariate signals. Finally, we apply the approach to a multichannel newborn EEG signal exhibiting inter-hemispheric asynchronous burst patterns and compare the results with the performance of *mean phase coherence* [2], a well-known bivariate phase synchrony measure.

The contributions of this paper can be summarized as: a new interpretation of generalized phase synchrony in the time-frequency domain, a new approach for computing the multivariate phase synchrony for nonstationary signals and the application of generalized phase synchrony to newborn EEG. This measure may be useful for assessing inter-hemispheric asynchrony in neonatal EEG, which is known to be the hallmark of EEG abnormality in many newborn neurological disorders.

2. METHODS

2.1 Classical definition of phase synchrony

Let $z_x(t)$ and $z_y(t)$ be the analytic associates of two one-dimensional stochastic real-valued signals $x(t)$ and $y(t)$, respectively; that is:

$$z_x(t) = x(t) + j\tilde{x}(t) = a_x(t)e^{j\varphi_x(t)} \quad (1)$$

$$z_y(t) = y(t) + j\tilde{y}(t) = a_y(t)e^{j\varphi_y(t)} \quad (2)$$

where $\tilde{x}(t)$ and $\tilde{y}(t)$ are the Hilbert transforms of $x(t)$ and $y(t)$, respectively. The original signals are assumed to be asymptotic signals [1]. The two signals $x(t)$ and $y(t)$ are said to be *phase-locked of order* $m{:}n$ if [2]:

$$\Delta\varphi_{x,y}(t) = m\varphi_x(t) - n\varphi_y(t) = const. \quad (3)$$

Such a strict condition is rarely satisfied for real-life signals. Therefore, this condition is replaced with a more relaxed condition called *phase entrainment* condition expressed by [2]:

$$|m\varphi_x(t) - n\varphi_y(t)| < const. \quad (4)$$

The ratio m/n is assumed to be rational. In the case of discrete signals and for the case $m = n = 1$ (phase-locking of order 1:1), the phase synchrony measure is given by Eq. (5) [2]:

$$R = \left|\frac{1}{N}\sum_{k=1}^{N-1} e^{j(\varphi_x(k\Delta t)-\varphi_y(k\Delta t))}\right| \quad (5)$$

where Δt is the sampling period and N the length of the two signals in samples. R is often referred to as *mean phase coherence* (*MPC*) or *phase locking value* (*PLV*) [2, 3].

There are two major limitations of the classical phase synchrony measure. The first one refers to the fact that bivariate measures are only able to detect pair-wise phase synchrony. Such measure is not able to capture global synchrony in multivariate signals.

The second limitation is related to the restricted assumption of rational order $m{:}n$. In the case where $\Delta\varphi_{x,y}(t)$ given by Eq. (3) is not a zero-mean stationary noise, the two signals x and y are asynchronous in conventional terms, while they may actually be connected together in terms of a more general definition of synchrony.

2.2 Multivariate phase synchronization based on the cointegration concept

A one-dimensional stochastic process is said to be *integrated of order d* ($I(d)$) if the reverse characteristic polynomial of its fitted multivariate autoregressive (MVAR) model has *d* roots on the unit circle in the complex plane [10]. The $I(d)$ process is unstable, but it can be converted into a stable one ($I(0)$) by *d* times differentiation [10]. Two or more integrated signals can be in a long-term relation with each other if there is a linear combination of these signals that results in a stationary process [10]. In such case, the underlying signals are called *cointegrated* signals. Unit root tests such as the *Augmented Dickey-Fuller (ADF)* test are employed to determine if a signal is integrated of order 1(random walk) [11].

Two signals $x_1(t)$ and $x_2(t)$ are said to be in a *generalized phase synchronous relationship* if the following equation is satisfied [10]:

$$\exists\, c_1, c_2 :\ c_1\varphi_1(t) + c_2\varphi_2(t) = z(t) \quad (6)$$

where $z(t) \sim N(m, \sigma)$ is a normally distributed stationary stochastic process and c_1 and c_2 are real-valued numbers. Note that in Eq. (6), c_1 and c_2 are not necessarily integer. Eq. (6) reflects a cointegrating relationship between two phase signals $\varphi_1(t)$ and $\varphi_2(t)$. Such relationship is generalized to a multivariate cointegrating relationship among *K* phase signals as:

$$\exists\, c_1, \dots, c_k : c_1\varphi_1(t) + c_2\varphi_2(t) + \cdots + c_K\varphi_K(t) = z(t). \quad (7)$$

Let $\boldsymbol{x}(t) = [x_1(t), \ \dots \ , x_K(t)]^T$ be a multivariate real-valued signal with K variables and N samples ($t = 1, \dots, N$). The Hilbert transform is used to obtain the phase of each signal component $x_i(t)$ separately. Cointegrating relationships can then be formulated for $\boldsymbol{x}(t)$ by using the multivariate Johansen test, a statistical approach based on the MVAR models [10].

Given $\boldsymbol{\varphi}_x(t) = [\varphi_1(t), \ \dots \ , \varphi_K(t)]$ as the multivariate phase signal extracted from $\boldsymbol{x}(t)$ and assuming $\varphi_i(t)$ are integrated processes of either order zero (stationary) or order one (random walk), cointegrating coefficients $c_1, \dots, c_k$ and the cointegration rank r ($0 \le r \le K$) are estimated using the multivariate Johansen test [10]. If the multivariate instantaneous phase signal $\boldsymbol{\varphi}_x(t)$ is integrated of order r, there are r stationary linear relationships within $\boldsymbol{\varphi}_x(t)$ and the signal $\boldsymbol{x}(t)$ is said to be *generalized phase synchronous with rank* r [10]. A higher rank implies a larger number of interconnected phase signals and therefore, higher synchrony within channels.

2.3 Interpretation of generalized phase synchrony in the time-frequency domain

Suppose $x(t)$ and $y(t)$ are two periodic signals, phase-locked of order $m{:}n$ where both *m* and *n* are integer. Let the two signals start from a similar point on the time axis. If the phase-locking ratio is rational, it implies that the two signals will cross each other periodically at the same initial common value and this period is related to the least common multiple (LCM) between *m* and *n*. Therefore, the rational phase-locking order is associated with an intuitive physical meaning for periodic signals. In contrast, the two periodic signals never reach the same point by passing time in the case of irrational n/m.

Explanation of phase synchrony for non-periodic signals is not such straightforward. It becomes even more difficult for nonstationary signals which by definition cannot be periodic. In this case, the concept of frequency flows [9] in the time-frequency domain may help to clarify the issue. The notion of phase synchrony in equation (4) is strictly equivalent to the concept of frequency synchrony through the following formulation [9]:

$$\Delta\varphi_{x,y}(t) = m\varphi_x(t) - n\varphi_y(t) \approx const \quad (8)$$

if and only if (iff)

$$\frac{1}{2\pi}\frac{d\Delta\varphi_{x,y}(t)}{dt} = \frac{m}{2\pi}\frac{d\varphi_x(t)}{dt} - \frac{n}{2\pi}\frac{d\varphi_y(t)}{dt} = mf_x(t) - nf_y(t) \approx 0 \quad (9)$$

where $f(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} > 0$. In other words, variations of instantaneous phases $\varphi_x(t)$ and $\varphi_y(t)$ over time results in the instantaneous frequency $f(t)$ expressed in Hertz. Studying the IFs is more convenient than IPs, as the issue of phase unwrapping can be bypassed. From this perspective, the concepts of phase synchrony and IF are connected together [1, 9]. If two signals have similar IF laws during a time interval, they are phase-locked of order 1:1 over that time period [9]. Consequently, a linear relationship between two IF laws with rational slope (m/n) over time implies phase-locking of order $m{:}n$. Such definition is not able to explain generalized phase synchrony where the linear relationships between phase signals can be irrational. Therefore, we propose the following interpretation for generalized phase synchronization based on the concept of cointegration:

For a multichannel nonstationary signal, if there is a linear relationship between the IF laws of a subgroup of channels during a reasonably long time period, they are said to be generally phase synchronized over that time period.

The linear coefficient shouldn't be necessarily rational. Figure 1 illustrates an example of generalized phase synchrony within the IF laws of three channels. As the figure shows, there is *a linear expansion* (see the shadowed area) for all three IF laws during the shadowed time interval. Such linear expansion defines a generalized phase-locking between channels. Note that the new explanation covers the classical definition of phase synchrony with rational phase-locking orders.

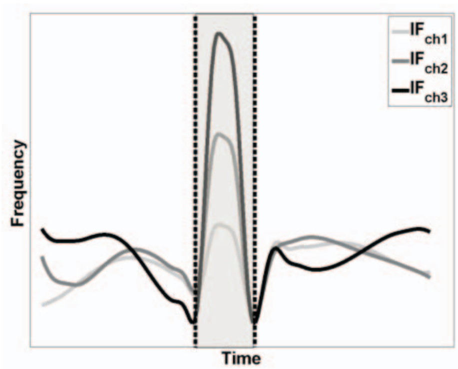


Figure 1: An example of generalized phase synchrony within a three-channel signal. Curves show IF ridges in the time-frequency domain. Shadowed area illustrates the phase-locking time period.

2.4 Numerical detection of generalized phase synchrony

The proposed procedure of generalized phase synchrony assessment for the nonstationary multichannel discrete signal $\boldsymbol{x}[n]$ with K channels is described in the following steps:

A) The IF $f_i[n],\ i = 1, \dots, K$ is extracted for each channel using IF estimates such as the real base-band delay demodulator approach [12]. The dynamics of $f_i[n]$ is then slowed down artificially by a smoothing procedure (e.g., moving average) to magnify slow drift of the mean frequency.

B) IF laws are divided into non-overlapping time segments with adequate length. The minimum window length is determined based on the requirement of the MVAR parameter estimation where the length should be significantly larger than K^2p (p is the MVAR model order in the Johansen test) [13].

C) The Johansen method (maximum eigenvalues test) [10] is applied on each multivariate segment at 99% confidence level and the linear relationships between IFs are extracted as follows:

$$\begin{cases} c_{11}f_1^i[n] + c_{12}f_2^i[n] + \dots + c_{1K}f_K^i[n] = z_1^i[n] \\ c_{21}f_1^i[n] + c_{22}f_2^i[n] + \dots + c_{2K}f_K^i[n] = z_2^i[n] \\ \vdots \\ c_{r1}f_1^i[n] + c_{r2}f_2^i[n] + \dots + c_{rK}f_K^i[n] = z_r^i[n] \end{cases} \quad (10)$$

where $f_j^i[n]$ represents the j^{th} IF $(j = 1, \dots, K)$ of i^{th} segment, r $(0 \le r \le K)$ is the number of cointegrating relationships within the segment, c_{kj} $(k = 1, \dots, r,\ j = 1, \dots, K)$ is the k^{th} cointegrating coefficient of the j^{th} IF and $z_j^i[n]$ is the stationary residual noise of the k^{th} cointegrating relationship of i^{th} segment.

D) The phase synchrony measure for the underlying segment is defined as the ratio of the number of cointegrating relationships r over the number of channels K. The measure always takes values within 0 and 1 where 0 means no cointegrating relationship within phase signals and 1 implies complete phase cointegration within the multivariate segment.

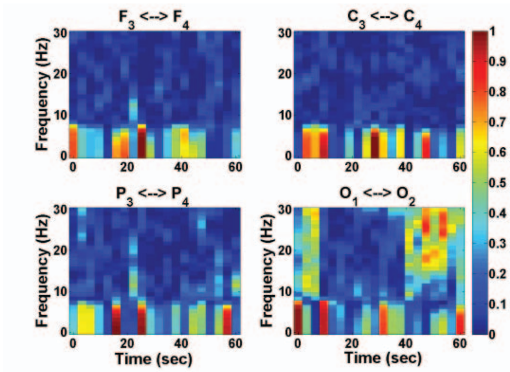
E) Finally, a binary mask for significant values is obtained by thresholding the time-varying measure using a surrogate data method in which, all segments are shuffled over time.

3. RESULTS

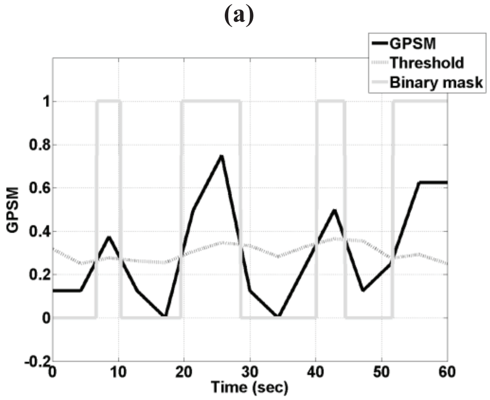
The newborn EEG data was recorded using a NicOne EEG system (Cardinal Healthcare, Madison, USA) with 256 Hz sampling rate (data obtained from the Helsinki University Hospital, Helsinki, Finland). We used eight Laplacian derivations (F_4, C_4, P_4, O_2, F_3, C_3, P_3, O_1) for all further analysis. One minute of EEG was segmented into 4 second windows with no overlapping. The window length (1024 samples) was chosen to be significantly larger than K^2p (here, 320) where p is the MVAR model order in the Johansen test (here, $p = 5$) and K is the number of channels (here, $K = 8$).

The generalized phase synchrony measure was computed for eight derivations (4 in left and 4 in right hemispheres) and a binary mask was obtained by generating 50 surrogates (shuffling over segments) and applying a time-varying threshold on the measure. For the mask, synchronous intervals (values above the threshold) were represented by 1. A Bivariate MPC measure was also calculated using Eq. (5) for each left-right pair of electrodes in the time-frequency domain after 1-Hz width bandpass filtering of each channel from 0.1 Hz to 50 Hz. Figure 2 illustrates the results of bivariate (MPC) and multivariate phase synchrony measures.

As the figure shows, the proposed measure (panel **b**) covers the synchronous intervals highlighted in some pair-wise comparisons (panel **a**). High values of the pair-wise assessments in panel **a** are concentrated around t≈10s (two pairs), t=20~30s (three pairs), t=40~50s (three pairs) and t=50~60s (two to three pairs). The binary mask in panel **b** marks these time intervals as synchronous. According to panel **b**, the number of cointegrating relationships within the multivariate phase of the underlying newborn EEG signal varies from 0 (t=17s, 34s) to 6 (t=26s) out of 8.



(a)



(b)

Figure 2: a) Bivariate phase synchrony measure (MPC) of four left-right pairs of electrodes. b) Generalized Phase Synchrony Measure (GPSM) and the time-varying threshold.

Also, the average cointegration rank for the surrogates varies between 2 and 3 (average of the dashed line in panel **b**).

4. CONCLUSION

The results obtained from the newborn EEG signal validate the performance of the proposed measure for quantifying generalized phase synchrony. They are also consistent with the results provided by the pair-wise (bivariate) comparisons using the MPC measure. Unlike MPC, the proposed measure deals with the generalized phase synchronization where phase-locking ratio is not assumed to be rational. This allows a more comprehensive view of synchronous cycles within the nonstationary multichannel signal. Ratio of the synchronous periods detected by the proposed measure compared to the asynchronous ones in the multichannel EEG might be useful for quantifying the inter-hemispheric asynchrony in newborn EEG.

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REFRENCES

[1] B. Boashash, Time frequency signal analysis and processing: a comprehensive reference Amsterdam; Boston: Elsevier, 2003.

[2] F. Mormann, K. Lehnertz, P. David et al., “Mean phase coherence as a measure for phase synchronization and its application to the EEG of epilepsy patients,” Physica D: Nonlinear Phenomena, vol. 144, no. 3-4, pp. 358-369, 2000.

[3] J.-P. Lachaux, E. Rodriguez, J. Martinerie et al., “Measuring phase synchrony in brain signals,” Human Brain Mapping, vol. 8, no. 4, pp. 194-208, 1999.

[4] M. G. Rosenblum, and A. S. Pikovsky, “Detecting direction of coupling in interacting oscillators,” Physical Review E, vol. 64, no. 4, pp. 045202, 2001.

[5] M. G. Rosenblum, L. Cimponeriu, A. Bezerianos et al., “Identification of coupling direction: Application to cardiorespiratory interaction,” Physical Review E, vol. 65, no. 4, pp. 041909, 2002.

[6] T. Koenig, D. Lehmann, N. Saito et al., “Decreased functional connectivity of EEG theta-frequency activity in first-episode, neuroleptic-naïve patients with schizophrenia: preliminary results,” Schizophrenia Research, vol. 50, no. 1-2, pp. 55-60, 2001.

[7] D. Looney, C. Park, P. Kidmose et al., “Measuring phase synchrony using complex extensions of EMD,” in Statistical Signal Processing, IEEE 5th Workshop on, 2009, pp. 49-52.

[8] A. Y. Mutlu, and S. Aviyente, “Multivariate Empirical Mode Decomposition for Quantifying Multivariate Phase Synchronization,” EURASIP Journal on Advances in Signal Processing, 2011.

[9] D. Rudrauf, A. Douiri, C. Kovach et al., “Frequency flows and the time-frequency dynamics of multivariate phase synchronization in brain signals,” Neuroimage, vol. 31, no. 1, pp. 209-227, 2006.

[10] W. Chaovalitwongse, P. M. Pardalos, P. Xanthopoulos et al., "Analysis of Multichannel EEG Recordings Based on Generalized Phase Synchronization and Cointegrated VAR," Computational Neuroscience, Springer Optimization and Its Applications, pp. 317-339: Springer New York, 2010.

[11] S. E. Said, and D. A. Dickey, “Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order,” Biometrika, vol. 71, no. 3, pp. 599-607, 1984.

[12] E. Zandi, and G. Azemi, “Estimating the Scattering Distribution of the Received Signal in Micro-Cellular Systems,” in 16th European Signal Processing Conference (EUSIPCO), Lausanne, Switzerland, 2008.

[13] H. Hytti, R. Takalo, and H. Ihalainen, “Tutorial on Multivariate Autoregressive Modelling,” Journal of Clinical Monitoring and Computing, vol. 20, no. 2, pp. 101-108, 2006.