LOW-COMPLEXITY VARIABLE FORGETTING FACTOR MECHANISM FOR BLIND ADAPTIVE CONSTRAINED CONSTANT MODULUS ALGORITHMS

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ABSTRACT

In this work, we propose a low-complexity variable forgetting factor (VFF) mechanism for blind adaptive constrained constant modulus (CCM) recursive least square (RLS) algorithms applied to linear interference suppression in direct-sequence code division multiple access (DS-CDMA) systems. The proposed VFF mechanism employs an updated component relating to the time average of the constant modulus (CM) cost function to automatically adjust the forgetting factor in order to ensure good tracking of the interference and the channel. Analytical expressions for predicting the mean-squared error of the proposed adaptation technique are obtained. Simulation results show that the proposed VFF mechanism achieves superior performance to existing methods at a reduced complexity.

***Index Terms*—** Blind multiuser detection, adaptive filtering, variable forgetting factor mechanisms.

1. INTRODUCTION

Blind algorithms with adaptive implementation have attracted considerable interest and found applications in beamforming, multiuser detection and source separation [1]-[6]. They operate without knowledge of the channel input, and lead to a solution comparable to that obtained from the minimization of the mean squared error (MSE) [1, 3]. The constrained minimum variance (CMV) based algorithm is designed in such a way that it attempts to minimize the filter output power while maintaining a constant response in the direction of a signal of interest [1, 3, 4].

The constrained constant modulus (CCM) based algorithms are based on a criterion that penalizes deviations of the modulus of the received signal away from a fixed value and forced to satisfy one or a set of linear constraints such that signals from the desired user are detected [5, 6]. In particular, the work in [3], [5] and [6] shows the robustness of the blind adaptive techniques with the CCM criterion against nonstationary environments, which can be implemented with a stochastic gradient or a recursive least squares (RLS) algorithm.

The RLS algorithm is considered as one of the fastest and most effective methods for adaptive implementation [10]. However, in nonstationary wireless environments in which users often enter and exit the system, it is impractical to compute a predetermined value for the forgetting factor. Adjusting the forgetting factor of blind RLS algorithms based on some adaptive rules has received very little attention. The most common method is the gradient-based variable forgetting factor (GVFF) algorithm proposed in [10], where a GVFF scheme with the MSE criterion is investigated. In this work, we propose a novel low-complexity VFF mechanism for blind linear receivers for DS-CDMA systems using the CCM criterion and RLS algorithms. The proposed VFF mechanism employs an updated component related to the time average of the CM cost function to automatically adjust the forgetting factor in order to ensure good tracking of the interference and the channel. We refer to the proposed VFF scheme as time-averaged VFF (TAVFF). A convergence analysis of the proposed adaptation technique is carried out and analytical expressions to predict the MSE are obtained. Simulation results are presented for nonstationary environments, showing that the new mechanism achieves superior performance to previously reported methods at a reduced complexity.

2. SYSTEM MODEL

Let us consider the downlink of an uncoded synchronous binary phase-shift keying (BPSK) DS-CDMA system with K users, N chips per symbol and L_p propagation paths. The delays are multiples of the chip duration and the receiver is synchronized with the main path. The M-dimensional received vector is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} \big(A_k b_k(i) \mathbf{C}_k \mathbf{h}(i) + \boldsymbol{\eta}_k(i)\big) + \mathbf{n}(i), \qquad (1)$$

where $M = N + L_p - 1$, $b_k(i) \in \{\pm 1\}$ is the i-th symbol for user k, and the amplitude associated with user k is A_k. The $M \times L_p$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of the spreading code of user k:

$$\mathbf{C}_k = \begin{pmatrix} a_k(1) & & \mathbf{0} \\ \vdots & \ddots & a_k(1) \\ a_k(N) & & \vdots \\ \mathbf{0} & \ddots & a_k(N) \end{pmatrix},$$

where $a_k(m) \in \{\pm 1/\sqrt{N}\}$, $m = 1, \ldots, N$. The channel vector is $\mathbf{h}(i) = [h_0(i) \ldots h_{L_p-1}(i)]^T$, $\boldsymbol{\eta}_k(i)$ is the inter-symbol interference (ISI), $\mathbf{n}(i) = [n_0(i) \ldots n_{M-1}(i)]^T$ is the complex Gaussian noise vector with zero mean and $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$, where σ^2 is the noise variance, $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose, respectively.

3. LINEARLY CONSTRAINED RECEIVERS AND BLIND ADAPTIVE CCM-RLS ALGORITHM

In this section, we describe the multipath blind adaptive CCM-RLS algorithm for estimating the parameters of the linear receiver first, and then we generalize the GVFF scheme [10] for the multipath adaptive CCM-RLS receiver.

3.1. Multipath Blind Adaptive CCM-RLS Algorithm

Consider the cost function $J_{CM} = E[(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2]$ subject to the constraint $\mathbf{w}_k^H(i)\mathbf{C}_k\mathbf{h}(i) = \nu$, where ν is a constant. Using

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the method of Lagrange multipliers we obtain the CCM receive filter

$$\mathbf{w}_0 = \bar{\mathbf{Q}}_k^{-1}(\bar{\mathbf{d}}_k - (\mathbf{h}^H \mathbf{C}_k^H \bar{\mathbf{Q}}_k^{-1} \mathbf{C}_k \mathbf{h})^{-1} \times (\mathbf{h}^H \mathbf{C}_k^H \bar{\mathbf{Q}}_k^{-1} \bar{\mathbf{d}}_k \mathbf{C}_k \mathbf{h} - \nu \mathbf{C}_k \mathbf{h})), \quad (2)$$

where $\bar{\mathbf{Q}}_k = E[|z_0(i)|^2 \mathbf{r}(i)\mathbf{r}^H(i)]$, $\bar{\mathbf{d}}_k = E[z_0^*(i)\mathbf{r}(i)]$ and $z_0(i) = \mathbf{w}_0^H \mathbf{r}(i)$. The adaptive blind CCM-RLS receiver for multipath CDMA systems is given as follows [2]

$$\mathbf{s}_k(i) = \frac{\mathbf{Q}_k^{-1}(i-1)\mathbf{u}_k(i)}{\gamma + \mathbf{u}_k^H(i)\mathbf{Q}_k^{-1}(i-1)\mathbf{u}_k(i)} \quad (3)$$

$$\mathbf{Q}_k^{-1}(i) = \gamma^{-1}\mathbf{Q}_k^{-1}(i-1) - \gamma^{-1}\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{Q}_k^{-1}(i-1) \quad (4)$$

$$\mathbf{u}_k(i) = z_k(i)\mathbf{r}(i) \quad (5)$$

$$\mathbf{w}_k(i) = \mathbf{Q}_k^{-1}(i)(\mathbf{d}_k(i) - (\mathbf{h}^H \mathbf{C}_k^H \mathbf{Q}_k^{-1}(i) \mathbf{C}_k \mathbf{h})^{-1} \times (\mathbf{h}^H \mathbf{C}_k^H \mathbf{Q}_k^{-1}(i) \mathbf{d}_k(i) \mathbf{C}_k \mathbf{h} - \nu \mathbf{C}_k \mathbf{h})) \quad (6)$$

$$\mathbf{d}_k(i) = \gamma \mathbf{d}_k(i-1) + z_k^*(i)\mathbf{r}(i) \quad (7)$$

where $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i)$, and γ denotes the forgetting factor. The estimated symbol of user k is given by $\hat{b}_k(i) = \text{sign}\{\Re[\mathbf{w}_k^H(i)\mathbf{r}(i)]\}$, where the operator $\Re[.]$ retains the real part of the argument and sign$\{.\}$ is the signum function.

3.2. GVFF Scheme in Multipath Channels

We extend the GVFF scheme in [10] to the adaptive CCM-RLS algorithm in multipath CDMA channels. By taking the gradient of the instantaneous CM cost function $(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2$ with respect to the variable forgetting factor $\gamma(i)$ we obtain the adaptive rule

$$\gamma(i+1) = \left[\gamma(i) - \frac{\partial\big((|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2\big)}{\partial \gamma}\right]_{\gamma^-}^{\gamma^+}, \quad (8)$$

where $\frac{\partial\left((|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2-1)^2\right)}{\partial\gamma} = \mu(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)\Re[\mathbf{Y}_k^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_k(i)]$, and $\mathbf{Y}_k(i) = \frac{\partial \mathbf{w}_k(i)}{\partial \gamma}$, $[.]_{\gamma^-}^{\gamma^+}$ denotes the truncation to the limits of the range $[\gamma^-, \gamma^+]$, μ denotes a step-size. The updated equation of $\mathbf{Y}_k(i)$ can be obtained by taking the gradient of (6) with respect to $\gamma(i)$. Thus, we generate two new quantities $\frac{\partial \mathbf{Q}_k^{-1}(i)}{\partial \gamma}$ and $\frac{\partial \mathbf{d}_k(i)}{\partial \gamma}$, updated equations of which can be obtained by following the same approach using (4) and (7). Note that we generate another new quantity $\frac{\partial \mathbf{s}_k(i)}{\partial \gamma}$ by computing $\frac{\partial \mathbf{Q}_k^{-1}(i)}{\partial \gamma}$. The updated equation of $\frac{\partial \mathbf{s}_k(i)}{\partial \gamma}$ can be similarly obtained by differentiating (3). For the sake of simplicity, we do not list all the expressions here. The CCM-RLS receiver with the GVFF mechanism is implemented by using (3)-(8) and the updated equations of $\mathbf{Y}_k(i)$, $\frac{\partial \mathbf{Q}_k^{-1}(i)}{\partial \gamma}$, $\frac{\partial \mathbf{d}_k(i)}{\partial \gamma}$ and $\frac{\partial \mathbf{s}_k(i)}{\partial \gamma}$ with initial values.

4. PROPOSED TAVFF SCHEME

In this section, we first introduce the proposed low-complexity VFF scheme that adjusts the forgetting factor of the adaptive CCM-RLS algorithm. A steady-state analysis of the proposed VFF scheme is carried out and then we present the computational complexity analysis for the proposed TAVFF scheme and the GVFF scheme.

4.1. TAVFF Mechanism

The proposed low-complexity VFF scheme is given by

$$\gamma(i) = \left[\frac{1}{1+\bar{\gamma}(i)}\right]_{\gamma^-}^{\gamma^+}, \quad (9)$$

where $\bar{\gamma}(i)$ denotes an updated component related to the time-averaged CM cost function. It uses the following adaptive rule,

$$\bar{\gamma}(i) = \delta_1\bar{\gamma}(i-1) + \delta_2(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2 \quad (10)$$

where $0 < \delta_1 < 1$, and $\delta_2 > 0$. Normally, δ_1 is close to 1, and δ_2 is set to be a small value. It is worth to point out that other rules have been experimented and the TAVFF scheme is a result of several attempts to devise a simple and yet effective mechanism.

Let us derive the steady-state first order and second order statistical properties for the variable forgetting factor. Based on (9), when $i \to \infty$, we have

$$E[\gamma(\infty)] \approx (1 + E[\bar{\gamma}(\infty)])^{-1} \quad (11)$$

and

$$E[\gamma^2(\infty)] \approx (1 + 2E[\bar{\gamma}(\infty)] + E[\bar{\gamma}^2(\infty)])^{-1}. \quad (12)$$

Next, we show the convergence and derive the expressions for $E[\bar{\gamma}(\infty)]$ and $E[\bar{\gamma}^2(\infty)]$. Since $0 < \delta_1 < 1$, from (10) we can see that $\bar{\gamma}(i)$ converges. Assume $\lim_{i\to\infty} E[(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2] = \xi_{min} + \xi_{ex}(\infty)$ [1], where ξ_{min} denotes the CCM minima which roughly corresponds to the minimum mean square error $\xi_{min} \approx 1 - \mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}$, where $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ and $\xi_{ex}(\infty)$ denotes the steady-state excess error of the CM cost function, $\xi_{min} \gg \xi_{ex}(\infty)$ [8]. Subsequently, we obtain

$$E[\bar{\gamma}(\infty)] = \frac{\delta_2(\xi_{min} + \xi_{ex}(\infty))}{1-\delta_1} \approx \frac{\delta_2\xi_{min}}{1-\delta_1}. \quad (13)$$

Using (10), by computing the square of $\bar{\gamma}(i)$ we obtain $\bar{\gamma}^2(i) = \delta_1^2\bar{\gamma}^2(i-1) + 2\delta_1\delta_2\bar{\gamma}(i-1)(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2-1)^2 + \delta_2^2(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2-1)^4$. Taking the expectation we have $E[\bar{\gamma}^2(i)] \approx \delta_1^2 E[\bar{\gamma}^2(i-1)] + 2\delta_1\delta_2 E[\bar{\gamma}(i-1)]E[(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2]$, where we assume that $\bar{\gamma}(i)$ and $(|\mathbf{w}_k^H(i)\mathbf{r}(i)|^2 - 1)^2$ are uncorrelated, when $i \to \infty$. Since δ_2^2 is very small, we neglect the last term. Due to $0 < \delta_1^2 < 1$, we know that $E[\bar{\gamma}^2(i)]$ converges. When $i \to \infty$, we obtain

$$E[\bar{\gamma}^2(\infty)] \approx \frac{2\delta_1\delta_2^2(\xi_{min} + \xi_{ex}(\infty))^2}{(1-\delta_1^2)(1-\delta_1)} \approx \frac{2\delta_1\delta_2^2\xi_{min}^2}{(1-\delta_1^2)(1-\delta_1)}, \quad (14)$$

where we assume $(\xi_{min} + \xi_{ex}(\infty))^2 \approx \xi_{min}^2$, since $\xi_{min} \gg \xi_{ex}(\infty)$. By substituting (13) and (14) into (11) and (12), respectively, we have the steady-state statistical properties for the variable forgetting factor

$$E[\gamma(\infty)] = (1-\delta_1)(1 + \delta_2\xi_{min} - \delta_1)^{-1} \quad (15)$$

$$E[\gamma^2(\infty)] = (1-\delta_1)^2(1+\delta_1)\big((1-\delta_1)^2(1+\delta_1) + 2\delta_2(1-\delta_1)(1+\delta_1)\xi_{min} + 2\delta_1\delta_2\xi_{min}^2\big)^{-1}. \quad (16)$$

4.2. Computational Complexity

We show the computational complexity of the proposed TAVFF and GVFF mechanisms. Table 1 shows the additional computational complexity of the algorithms for multipath channels. We estimate the number of arithmetic operations by considering the number of complex additions and multiplications required by the mechanisms. An important advantage of the proposed adaptation rule is that it requires only a few fixed number of operations while the GVFF technique has an additional complexity proportional to the processing gain N and to the number of propagation paths L_p.

Table 1. Additional Computational Complexity.

Algorithm	Number of operations per symbol	
	Multiplications	Additions
TAVFF	4	2
GVFF	$10M^2+16M+7$	$10M^2+6M-1$

5. CONVERGENCE ANALYSIS

This section first theoretically shows the convergence of the adaptive CCM-RLS weight vector. Then, the steady-state MSE expression of the CCM-RLS receiver with the proposed TAVFF scheme is derived.

5.1. Convergence of the Mean Weight Vector

Let $\beta(i)=\frac{1}{\mathbf{h}^H\mathbf{C}_k^H\mathbf{f}(i)}$, where $\mathbf{f}(i)=\mathbf{Q}_k^{-1}(i)\mathbf{C}_k\mathbf{h}$. By using (4), we have

$$\beta(i)=\gamma(i)[\beta(i-1)+\beta(i-1)\Gamma(i)\mathbf{u}_k^H(i)\mathbf{f}(i-1)], \tag{17}$$

where $\Gamma(i)=\frac{\beta(i-1)\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)}{1-\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{f}(i-1)\beta(i-1)}$. By defining $\boldsymbol{\omega}(i)=\mathbf{h}^H\mathbf{C}_k^H\mathbf{Q}_k^{-1}(i)\mathbf{d}_k(i)\mathbf{C}_k\mathbf{h}-\nu\mathbf{C}_k\mathbf{h}$ and using $\mathbf{s}_k(i)=\mathbf{Q}_k^{-1}(i)\mathbf{u}(i)$ [9], we rewrite (6) as

$$\begin{aligned}\mathbf{w}_k(i)&=\beta(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\\&=\beta(i-1)\mathbf{Q}_k^{-1}(i-1)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\\&\quad-\mathbf{Q}_k^{-1}(i)\mathbf{u}_k(i)\mathbf{u}_k^H(i)\beta(i-1)\mathbf{Q}_k^{-1}(i-1)\\&\quad\times(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\\&\quad+\gamma(i)\Gamma(i)e(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i)).\end{aligned} \tag{18}$$

where $e(i)=\beta(i-1)\mathbf{u}_k^H(i)\mathbf{f}(i-1)$. Based on the assumption in [7] and using the convexity of the CM cost function [2] and the adaptive CCM-RLS expressions we have the following assumptions

$$\begin{aligned}\lim_{i\to\infty}\mathbf{Q}_k^{-1}(i)&\approx\lim_{i\to\infty}E[\mathbf{Q}_k^{-1}(i)]=(1-E[\gamma(i)])\bar{\mathbf{Q}}_k^{-1}\\&\approx(1-E[\gamma(i)])E[|z_k(i)|^2]^{-1}\mathbf{R}^{-1},\end{aligned} \tag{19}$$

$$\lim_{i\to\infty}\mathbf{d}_k(i)\approx\lim_{i\to\infty}E[\mathbf{d}_k(i)]=\frac{1}{1-E[\gamma(i)]}\bar{\mathbf{d}}_k. \tag{20}$$

Thus, when $i\to\infty$ we assume $\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i)\approx\beta^{-1}(i-1)\mathbf{d}_k(i-1)-\boldsymbol{\omega}(i-1)$. Subsequently, we rewrite (18) as

$$\begin{aligned}\mathbf{w}_k(i)&\approx\mathbf{w}_k(i-1)-\mathbf{Q}_k^{-1}(i)\mathbf{u}_k(i)\mathbf{u}_k^H(i)\mathbf{w}_k(i-1)\\&\quad+\gamma(i)\Gamma(i)e(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i)).\end{aligned} \tag{21}$$

By multiplying $\mathbf{Q}_k(i)$ and substituting $\mathbf{Q}_k(i)=\gamma(i)\mathbf{Q}_k(i-1)+\mathbf{u}_k(i)\mathbf{u}_k^H(i)$ we obtain $\mathbf{Q}_k(i)\mathbf{w}_k(i)=\gamma(i)\mathbf{Q}_k(i-1)\mathbf{w}_k(i-1)+\gamma(i)\Gamma(i)e(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))$. Let $\boldsymbol{\epsilon}(i)=\mathbf{w}_k(i)-\mathbf{w}_0$, we have

$$\mathbf{Q}_k(i)\boldsymbol{\epsilon}(i)=\gamma(i)\mathbf{Q}_k(i-1)\boldsymbol{\epsilon}(i-1)+\mathbf{y}(i), \tag{22}$$

where $\mathbf{y}(i)=\beta(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\beta(i-1)\mathbf{f}(i-1)-\mathbf{u}_k(i)\mathbf{u}_k^H(i)\mathbf{w}_0$, where we use $\gamma(i)\Gamma(i)=\beta(i)\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)$ [9]. Note that $\beta(i-1)\mathbf{f}(i-1)=\frac{\mathbf{Q}^{-1}(i-1)\mathbf{C}_k\mathbf{h}}{\mathbf{h}^H\mathbf{C}_k^H\mathbf{Q}^{-1}(i-1)\mathbf{C}_k\mathbf{h}}\approx\frac{\mathbf{R}^{-1}(i-1)\mathbf{C}_k\mathbf{h}}{\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}(i-1)\mathbf{C}_k\mathbf{h}}$ is the linear CMV receiver in multipath channels. Defining $\mathbf{v}_0$ as the optimum minimum variance receiver [4], we obtain

$$\begin{aligned}\mathbf{Q}_k^{-1}(i)\mathbf{y}(i)&=\beta(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\\&\quad\times\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\boldsymbol{\epsilon}'(i-1)\\&\quad+\beta(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\\&\quad\times\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{v}_0-\mathbf{Q}_k^{-1}(i)\mathbf{u}_k(i)\mathbf{u}_k^H(i)\mathbf{w}_0,\end{aligned} \tag{23}$$

where $\boldsymbol{\epsilon}'(i-1)=\beta(i-1)\mathbf{f}(i-1)-\mathbf{v}_0$. When $i\to\infty$, we have $\boldsymbol{\epsilon}'(i-1)\approx 0$ [9] and $\beta(i)\mathbf{Q}_k^{-1}(i)(\beta^{-1}(i)\mathbf{d}_k(i)-\boldsymbol{\omega}(i))\approx\mathbf{w}_0$. Thus, we obtain

$$\begin{aligned}\lim_{i\to\infty}\mathbf{Q}_k^{-1}(i)\mathbf{y}(i)&\approx\mathbf{w}_0\mathbf{h}_k^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{v}_0\\&\quad-\mathbf{Q}_k^{-1}(i)\mathbf{u}_k(i)\mathbf{u}_k^H(i)\mathbf{w}_0.\end{aligned} \tag{24}$$

Multiplying $\mathbf{Q}_k^{-1}(i)$ at the both sides of (22) we have $\boldsymbol{\epsilon}(i)=\gamma(i)\mathbf{Q}_k^{-1}(i)\mathbf{Q}_k(i-1)\boldsymbol{\epsilon}(i-1)+\mathbf{Q}_k^{-1}(i)\mathbf{y}(i)$. When $i\to\infty$, it is given by

$$\boldsymbol{\epsilon}(i)\approx\gamma(i)\boldsymbol{\epsilon}(i-1)+\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{s}_k(i)\mathbf{u}_k^H(i)\mathbf{v}_0-\mathbf{Q}_k^{-1}(i)\mathbf{u}_k(i)\mathbf{u}_k^H(i)\mathbf{w}_0, \tag{25}$$

where $\mathbf{Q}_k^{-1}(i)\mathbf{Q}_k(i-1)\approx\mathbf{I}$. By taking the expectation and due to $E[\mathbf{s}_k(i)\mathbf{u}_k^H(i)]\approx(1-E[\gamma(i)])\bar{\mathbf{Q}}_k^{-1}E[\mathbf{u}_k(i)\mathbf{u}_k^H(i)]=(1-E[\gamma(i)])\mathbf{I}$, we have

$$\begin{aligned}E[\boldsymbol{\epsilon}(i)]&\approx E[\gamma(i)]E[\boldsymbol{\epsilon}(i-1)]+(1-E[\gamma(i)])\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{v}_0\\&\quad-(1-E[\gamma(i)])\mathbf{w}_0.\end{aligned} \tag{26}$$

Using $\mathbf{h}^H\mathbf{C}_k^H\mathbf{v}_0=1$ [4], finally we obtain

$$E[\boldsymbol{\epsilon}(i)]\approx E[\gamma(i)]E[\boldsymbol{\epsilon}(i-1)]. \tag{27}$$

Since $0<E[\gamma(i)]<1$, the expected weight error converges to zero.

5.2. Convergence of MSE

When $i\to\infty$, we assume $E[\mathbf{u}_k(i)\mathbf{u}_k^H(i)]\approx E[|z_k(i)|^2]\mathbf{R}$. Using (25) we have

$$\begin{aligned}\boldsymbol{\Theta}(i)&=E[\boldsymbol{\epsilon}(i)\boldsymbol{\epsilon}^H(i)]\approx E[\gamma^2(i)]\boldsymbol{\Theta}(i-1)+(1-E[\gamma(i)])^2\bar{\zeta}_1\\&\quad\times\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}\mathbf{w}_0^H-(1-E[\gamma(i)])^2\bar{\zeta}_2\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\\&\quad-(1-E[\gamma(i)])^2\bar{\zeta}_3\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}\mathbf{w}_0^H+(1-E[\gamma(i)])^2\bar{\zeta}_4\mathbf{R}^{-1},\end{aligned} \tag{28}$$

where $\bar{\zeta}_1=E[|\mathbf{v}_0^H\mathbf{r}(i)|^2]=\mathbf{v}_0^H\mathbf{R}\mathbf{v}_0$, $\bar{\zeta}_2=E[\mathbf{w}_0^H\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{v}_0]=\mathbf{w}_0^H\mathbf{R}\mathbf{v}_0$, $\bar{\zeta}_3=E[\mathbf{v}_0^H\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_0]=\mathbf{v}_0^H\mathbf{R}\mathbf{w}_0$, and $\bar{\zeta}_4=E[\mathbf{w}_0^H\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_0]=\mathbf{w}_0^H\mathbf{R}\mathbf{w}_0$. The steady-state MSE is

$$\begin{aligned}\lim_{i\to\infty}\xi(i)&=\lim_{i\to\infty}E[|A_kb(i)-\mathbf{w}_k^H(i-1)\mathbf{r}(i)|^2]\\&\approx\lim_{i\to\infty}\Xi(i)+(1-2\nu)A_k^2,\end{aligned} \tag{29}$$

where

$$\begin{aligned}\Xi(i)&=E[(\boldsymbol{\epsilon}^H(i-1)+\mathbf{w}_0^H)\mathbf{r}(i)\mathbf{r}^H(i)(\boldsymbol{\epsilon}(i-1)+\mathbf{w}_0)]\\&=\bar{\zeta}_4+tr[\mathbf{R}\boldsymbol{\Theta}(i-1)]+\boldsymbol{\epsilon}^H(i-1)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{w}_0\\&\quad+\mathbf{w}_0^H\mathbf{r}(i)\mathbf{r}^H(i)\boldsymbol{\epsilon}(i-1).\end{aligned} \tag{30}$$

Since $\lim_{i\to\infty} \boldsymbol{\epsilon}(i-1) = 0$, we have $\Xi(i) \approx \bar{\zeta}_4 + \Xi_{ex}(i)$, where $\Xi_{ex}(i) = tr[\mathbf{R}\boldsymbol{\Theta}(i-1)]$ denotes the steady-state excess MSE. Multiplying (28) by $\mathbf{R}$ we have

$$\begin{aligned} tr[\mathbf{R}\boldsymbol{\Theta}(i)] \approx\ & E[\gamma^2(i)]tr[\mathbf{R}\boldsymbol{\Theta}(i-1)] + (1-E[\gamma(i)])^2\bar{\zeta}_1 \\ & \times tr[\mathbf{R}\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}\mathbf{w}_0^H] \\ & - (1-E[\gamma(i)])^2\bar{\zeta}_2\nu - (1-E[\gamma(i)])^2\bar{\zeta}_3\nu \\ & + (1-E[\gamma(i)])^2\bar{\zeta}_4 M. \end{aligned} \quad (31)$$

Since $0 < E[\gamma^2(i)] < 1$, $tr[\mathbf{R}\boldsymbol{\Theta}(i)]$ converges. Finally, we have

$$\begin{aligned} \Xi_{ex}(\infty) = & \frac{(1-E[\gamma(\infty)])^2}{1-E[\gamma^2(\infty)]}\{\bar{\zeta}_1 tr[\mathbf{R}\mathbf{w}_0\mathbf{h}^H\mathbf{C}_k^H\mathbf{R}^{-1}\mathbf{C}_k\mathbf{h}\mathbf{w}_0^H] \\ & - \bar{\zeta}_2\nu - \bar{\zeta}_3\nu + \bar{\zeta}_4 M\}, \end{aligned} \quad (32)$$

where $E[\gamma(\infty)]$ and $E[\gamma^2(\infty)]$ are given in (15) and (16).

6. SIMULATIONS

In this section, we evaluate the performance of the proposed TAVFF scheme with the blind adaptive CCM-RLS receiver and compare it with the blind GVFF scheme and the adaptive CCM-RLS and CMV-RLS receivers with fixed forgetting factor. The DS-CDMA system employs random sequences as the spreading codes, and the spreading gain is $N = 16$. The sequence of channel coefficients for each path is $h_f(i) = p_f\alpha_f(i) (f = 0, 1, 2)$. All channels have a profile with three paths whose powers are $p_0 = 0$ dB, $p_1 = -7$ dB and $p_2 = -10$ dB, respectively, where $\alpha_f(i)$ is computed according to the Jakes model. We optimized the parameters of the adaptive TAVFF scheme with $\delta_1 = 0.99$, $\delta_2 = 1.5 \times 10^{-5}$, and initial value $\bar{\gamma}(0) = 0$. The parameters for the GVFF scheme are $\gamma(0) = 0.999$, $\mu = 10^{-4}$, and initial values $\frac{\partial \mathbf{Q}_k^{-1}(0)}{\partial \gamma} = \mathbf{I}$, $\mathbf{Y}_k(0) = 0.01 \times \mathbf{1}$, where $\mathbf{1}$ denotes an all-one vector, $\frac{\partial \mathbf{d}_k(0)}{\partial \gamma} = \mathbf{0}$, $\mathbf{s}_k(0) = \mathbf{0}$. For the VFF schemes we set $\gamma^- = 0.999$ and $\gamma^+ = 0.99998$. For the blind RLS algorithms we set $\mathbf{d}_k(0) = \mathbf{0}$. $\mathbf{Q}_k^{-1}(0) = \mathbf{I}$, $\mathbf{w}_k(0) = \mathbf{C}_k\hat{\mathbf{h}}(0)$, where we employ the blind channel estimation algorithm in [11], and the fixed forgetting factor $\gamma = 0.9996$.

Fig. 1 shows the signal to interference plus noise ratio (SINR) performance of the desired user versus the number of received symbols in a nonstationary scenario for the proposed TAVFF scheme, the GVFF scheme and the conventional fixed forgetting factor schemes. In the simulation, the system starts with five users including one high power level interferer with 3 dB and after 1000 symbols, four new users including a 10 dB, a 6 dB and two 3 dB high power level users enter the system, where $f_dT_s = 5 \times 10^{-5}$. We can see that the CCM-RLS algorithm with the TAVFF scheme converges much faster than the GVFF scheme and fixed forgetting factor algorithms in multipath fading channels. Fig. 2 illustrates the BER performance of the desired user versus SNR and number of users K, where we set $f_dT_s = 5 \times 10^{-5}$. We can see that the best performance is achieved by the CCM-RLS receiver with the TAVFF scheme. In particular, the CCM-RLS receiver with the TAVFF scheme can save up to over 5dB and support up to 2 more users in comparison with the CCM-RLS receiver with a fixed forgetting factor, at the BER level of 10^{-2}.

7. CONCLUSION

We proposed a low-complexity VFF scheme for adaptive CCM-RLS algorithms and extended the conventional GVFF scheme to the CCM criterion. We conducted a convergence analysis of the proposed TAVFF scheme and derived expressions to predict the MSE of the CCM-RLS algorithm. The results showed that the proposed scheme significantly outperforms existing algorithms.

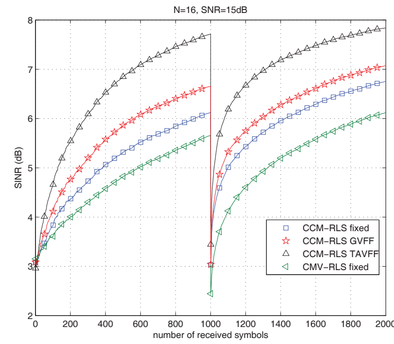

Fig. 1. SINR performance in nonstationary environment of multipath time varying channels.

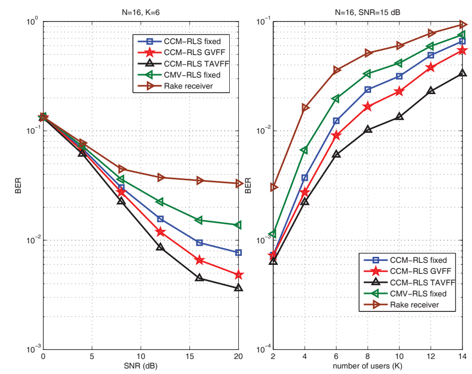

Fig. 2. BER performance in multipath time varying channels.

8. REFERENCES

[1] M. Honig. U. Madhow, and S. Verdu, "Blind adaptive multiuser detection," in *IEEE Trans. Inf. Theory,* vol. 41, pp. 944-960, Jul. 1995.

[2] R. C. de Lamare and R. Sampaio-Neto, "Blind Adaptive Code-Constrained Constant Modulus Algorithms for CDMA Interference Suppression in Multipath Channels," in *IEEE Commun. Letters,* vol. 9, no. 4, pp. 334-336, Apr. 2005.

[3] R. C. de Lamare, L. Wang, and R. Fa,"Adaptive reduced-rank LCMV beamforming algorithms based on joint iterative optimization of filters: Design and analysis," in *Signal Processing,* vol. 90, pp. 640C652, Aug. 2009.

[4] R. C. de Lamare and R. Sampaio-Neto, "Low-Complexity Variable Step-Size Mechanisms for Stochastic Gradient Algorithms in Minimum Variance CDMA Receivers," in *IEEE Trans. Signal Proc.,* vol. 54, no. 6, pp. 2302-2317, Jun. 2006.

[5] R. C. de Lamare, M. Haardt, and R. Sampaio-Neto, "Blind Adaptive Constrained Reduced-Rank Parameter Estimation Based on Constant Modulus Design for CDMA Interference Suppression," in *IEEE Trans. Signal Proc.* vol. 56, no. 6, pp. 2470-2482, Jun. 2008.

[6] Y. Cai and R. C. de Lamare, "Low-Complexity Variable Step-Size Mechanism for Code-Constrained Constant Modulus Stochastic Gradient Algorithms Applied to CDMA Interference Suppression," in *IEEE Trans. Signal Proc.* vol. 57, no. 1, pp. 313-323, Jan. 2009.

[7] T. Adali and S. H. Ardalan, "On the effect of input signal correlation on weight misadjustment in the RLS algorithm," in *IEEE Trans. Signal Proc.,* vol. 43, no. 4, pp. 988-991, Apr. 1995.

[8] H. Zeng, L. Tong and C. R. Johnson, "Relationships Between the Constant Modulus and Wiener Receivers," in *IEEE Trans. Inf. Theory,* vol. 44, no. 4, pp. 1523-1538, Jul. 1998.

[9] H. V. Poor and X. Wang, "Code-Aided Interference Suppression for DS/CDMA Communications-Part II: Parallel Blind Adaptive Implementations," in *IEEE Trans. Commun.,* vol. 45, no. 9, pp. 1112-1122, Sep. 1997.

[10] S. Haykin, "Adaptive Filter Theory," 4th ed. Englewood Cliffs, NJ: Prentice-Hall, 2002.

[11] X. G. Doukopoulos and G. V. Moustakides,"Adaptive Power Techniques for Blind Channel Estimation in CDMA Systems," in *IEEE Trans. Signal Processing,* vol. 53, No. 3, pp. 1110-1120, March, 2005.