PROBABILITY HYPOTHESIS DENSITY FILTERING WITH MULTIPATH-TO-MEASUREMENT ASSOCIATION FOR URBAN TRACKING

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ABSTRACT

We consider the particle probability hypothesis density filter (PPHDF) for tracking multiple targets in urban terrain. This is a filtering technique based on random finite sets, implemented using the particle filter. Unlike data association methods, the PPHDF can be modified to estimate both the number of targets and their corresponding tracking parameters. We propose a modified PPHDF algorithm that employs multipath-to-measurement association (PPHDF-MMA) to automatically and adaptively estimate the available types of measurements. By using the best matched measurement at each time step, the new algorithm results in improved radar coverage and scene visibility. Numerical simulations demonstrate the effectiveness of the PPHDF-MMA in improving the tracking performance of multiple targets and targets in clutter.

Index Terms— Urban terrain, multiple target tracking, probability hypothesis density filter, particle filter.

1. INTRODUCTION

In dense urban environments, most conventional radar tracking systems begin to fail due to the absence of line-of-sight returns, and the presence of multipath interference, obscuration from buildings, and high clutter [1]. Recently, waveform agile sensing has been integrated with multipath exploitation to further improve tracking performance in urban terrain [2]. Moreover, the problem of multiple target tracking is to instantaneously estimate both the number of targets present as well as each target's trajectory. Conventional multiple target tracking filtering techniques first couple the correct measurement to existing tracks through *measurement-to-track* associations and then estimate the target states using single target tracking techniques [3]. The most general data association method, multiple hypothesis tracking (MHT) [4], although exhaustive, is very computationally expensive [5]. As an alternative, the joint probabilistic data association (JPDA) method estimates the states by summing all the association hypothesis weighted by the probabilities from the likelihood [6]. Unfortunately, it requires that the number of targets is fixed and its performance is poor when the targets are close to each other [7]. The probability hypothesis density filter (PHDF) is a suboptimal but computationally tractable algorithm for multiple target tracking without data association [8]. It can also be implemented using sequential Monte Carlo methods such as particle filtering (PPHDF) [9]. Recently, the PPHDF was combined with data association to identify the trajectories of different targets [10]. In this paper, we propose a PPHDF with multipath-to-measurement association (PPHDFA) that can decide on the best matched measurement return path at each time step in realistic scenarios.

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2. TRACKING IN URBAN ENVIRONMENTS

For target tracking in urban environments, we consider two possible target state models: nearly constant velocity (NCV) model and coordinated turn (CT) model, that assumes that a target turns left or right with nearly constant velocity and nearly constant angular turning rate. [11]. The state vector $\mathbf{x}_k$ at time step k is given by $\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{n}_k$, where $\mathbf{n}_k$ is a modeling error zero-mean Gaussian random process and $\mathbf{F}$ depends on the state model [2].

An example urban scene of a target moving between two buildings is depicted in Fig. 1. We assume that the walls of the buildings are perfectly smooth so that all reflections can be assumed specular (that is, the angle of incidence equals the angle of reflection). The three-dimensional (3-D) Cartesian coordinates of the location of the radar receiver are given by (x_R, y_R, z_R), the transmitter and receiver are assumed to be stationary and collocated, the location and velocity of the target at time k are given by (x_k, y_k, z_k) and $(\dot{x}_k, \dot{y}_k, \dot{z}_k)$, respectively, and H is the street width. The measurement equation corresponding to the LOS return path depends on the range $r_{0,k} = ((x_k - x_R)^2 + (y_k - y_R)^2 + (z_k - z_R)^2)^{1/2}$ and range rate $\dot{r}_{0,k} = (\dot{x_k}(x_k - x_R) + \dot{y_k}(y_k - y_R) + \dot{z_k}(z_k - z_R))/r_{0,k}$. The measurement equation due to the multipath returns depends on the number of reflections off different objects. In this case, the range from the radar to the target after m bounces off the ith building, $i = 1, 2$, is given by

$$r_{m,k,i} = ((x_k - x_R)^2 + (z_k - z_R)^2)^{1/2} + \left[(-1)^{m+1}\left(2\,[m/2]_i\,H - (-1)^{i+1}\,y_k\right) - y_R\right]^2 . \quad (1)$$

It is assumed that the first bounce is off the ith building, with

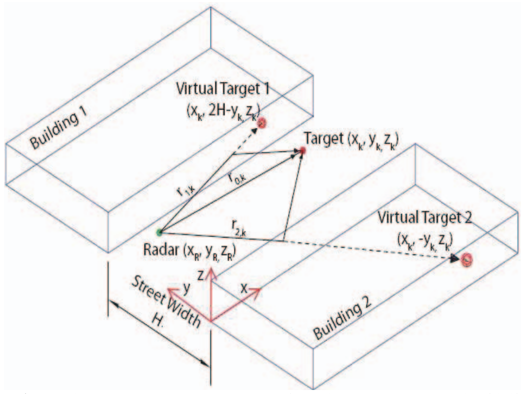


Fig. 1. Urban scene:target moving between two buildings.

$[m/2]_1 = \lceil m/2 \rceil$ and $[m/2]_2 = \lfloor m/2 \rfloor$ [2]. The range-rate is obtained as the derivative of the range in (1) with respect to time. Note that no return is observed at the receiver when shadowing or obscuring occurs.

The measurement model is given by $\mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{w}_k$, where $\mathbf{w}_k$ is a zero-mean Gaussian noise process with covariance matrix R_k, and $h_k(\mathbf{x}_k)$ is given by

$$h_k(\mathbf{x}_k) = \begin{bmatrix} r_{0,k} & r_{1,k} & \cdots & r_{P_k,k} \\ \dot{r}_{0,k} & \dot{r}_{1,k} & \cdots & \dot{r}_{P_k,k} \end{bmatrix},$$

where $r_{p,k}$ and $\dot{r}_{p,k}$ are the range and range-rate measurements of the pth path at time step k, $p = 1, \ldots, P_k$, respectively, and P_k is the total number of paths at time step k.

3. PROBABILITY HYPOTHESIS DENSITY FILTER

We assume that a target generates only one observation at each time step k, and that each target generates measurements independently of each other. Assuming N_k targets at time k, the multiple-target state random finite set (RFS) is given by $\mathbf{X}_k = \{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,N_k}\}$. The multiple target measurement RFS at time k is $\mathbf{Z}_k = \{\mathbf{z}_{k,1}, \ldots \mathbf{z}_{k,M_k}\}$ where M_k is the number of measurements at the receiver at time k. Here, $\mathbf{x}_{k,i}$ is the unknown state of the ith target and $\mathbf{z}_{k,i}$ is the corresponding measurement at time step k. Note that there may be more measurements than targets at any given time k since measurements may also be obtained from clutter. Given $\mathbf{X}_{k-1}$ at time $(k-1)$, $\mathbf{X}_k$ is formed by combining the surviving and spawned target RFS, $\mathbf{X}^{\text{(surv)}}_{k|k-1}$ and $\mathbf{X}^{\text{(spn)}}_{k|k-1}$, respectively, from the previous time step $(k-1)$, and the spontaneous target birth RFS $\mathbf{X}^{\text{(birth)}}_k$. Also, due to the presence of clutter, the received multiple-target measurement RFS $\mathbf{Z}_k$ is formed by the combination of two types of measurement RFS: $\mathbf{Z}^{\text{(trg)}}_k$ generated by the existing targets and $\mathbf{Z}^{\text{(clt)}}_k$ generated by false alarms or clutter at time k. It is assumed that the clutter RFS is independent of the target measurement RFS and that the target measurement RFS are mutually independent.

The PHDF assumes that the predicted multiple-target posterior density $p(\mathbf{X}_k|\mathbf{Z}_{k-1})$ can be completely characterized by the corresponding intensity function $\lambda(\mathbf{x}_k|\mathbf{Z}_{k-1})$. Thus, given the posterior intensity $\lambda(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})$ at time step $(k-1)$, the predicted intensity $\lambda(\mathbf{x}_k|\mathbf{Z}_{k-1})$ can be obtained as

$$\begin{aligned}\lambda(\mathbf{x}_k|\mathbf{Z}_{k-1}) = \int [\, \mathrm{P}^{\text{(surv)}}_{k|k-1}(\mathbf{x}_{k-1})\, p(\mathbf{x}_k|\mathbf{x}_{k-1}) + \\ \lambda^{\text{(spn)}}(\mathbf{x}_k|\mathbf{Z}_{k-1})\,]\lambda(\mathbf{x}_{k-1}|\mathbf{Z}_{k-1})\, d\mathbf{x}_{k-1} + \lambda^{\text{(birth)}}(\mathbf{x}_k|\mathbf{Z}_k)\,,\end{aligned}$$

where $\lambda^{\text{(spn)}}(\mathbf{x}_k|\mathbf{Z}_{k-1})$ is the intensity of the targets spawned at the previous time step $(k-1)$, $\lambda^{\text{(birth)}}(\mathbf{x}_k|\mathbf{Z}_k)$ is the intensity of new target births, and $\mathrm{P}^{\text{(surv)}}_{k|k-1}(\mathbf{x}_{k-1})$ is the probability that a target present at time step $(k-1)$ will survive to time step k. The posterior intensity is given by

$$\begin{aligned}\lambda(\mathbf{x}_k|\mathbf{Z}_k) = (1 - \mathrm{P}^{\text{(det)}}_k(\mathbf{x}_k))\, \lambda(\mathbf{x}_k|\mathbf{Z}_{k-1}) + \\ \sum_{\mathbf{z}_k \in \mathbf{Z}_k} \frac{\mathrm{P}^{\mathrm{D}}_k(\mathbf{x}_k)\, p(\mathbf{z}_k|\mathbf{x}_k)\, \lambda(\mathbf{x}_k|\mathbf{Z}_{k-1})}{\lambda^{\text{(clt)}}(\mathbf{z}_k) + \int \mathrm{P}^{\text{(det)}}_k(\tilde{\mathbf{x}}_k)\, p(\mathbf{z}_k|\tilde{\mathbf{x}}_k)\, \lambda(\tilde{\mathbf{x}}_k|\mathbf{Z}_{k-1})\, d\tilde{\mathbf{x}}_k}\end{aligned}$$

where $\lambda^{\text{(clt)}}(\mathbf{z}_k)$ is the clutter intensity and $\mathrm{P}^{\text{(det)}}_k(\mathbf{x}_k)$ is the probability of detecting a target at time k.

In maneuvering target tracking, the target may change its motion model at any time, according to a transitional probability matrix $\Pi = \{\pi_{mn}\}$ [12]. The model number $\varpi^{(i)}_k$ of the ith particle, $i = 1, \ldots, L_k$ at time k, follows the transitional matrix, where L_k is the number of particles that still exist at time k. Specifically, if at time $k-1$ a particle has model index number $m = \varpi^{(i)}_{k-1}$, then at time k, the model index transfers to model number n with probability π_{mn}. The multiple-model particle filter is used to generate $\varpi^{(i)}_k$ from $\varpi^{(i)}_{k-1}$ according to the transitional matrix Π.

4. PPHDF FOR URBAN ENVIRONMENTS

Although the PPHDF avoids conventional data association, it requires prior knowledge of path-to-measurement associations [9]. Specifically, at the receiver, when the radar observes all range-range rate pairs, it is assumed that it can successfully distinguish which range-range rate pair corresponds to the different paths. In practice, however, the matched filter may not receive path information in any particular order.

We propose a modified version of PPHDF with multipath-to-measurement association (PPHDF-MMA) that can be used to automatically and adaptively estimate the measurement types available at each time step. This is done by associating the ith particle at time k to a possible path parameter $\rho^{(i)}_k$. If the particle is due to a target spawned from the previous time step, then the new prediction process can be updated using

$$\mathbf{x}^{(i)}_k, \rho^{(i)}_k \sim q_k(\cdot|\mathbf{x}^{(i)}_{k-1}, \mathbf{Z}_k, \varpi^{(i)}_k),\ i = 1, \ldots, L_{k-1}$$

$$w^{(i)}_{k|k-1} = \frac{\phi_{k|k-1}(\mathbf{x}^{(i)}_k, \mathbf{x}^{(i)}_{k-1}) w^{(i)}_{k-1}}{q_k(\mathbf{x}^{(i)}_k|\mathbf{x}^{(i)}_{k-1}, \mathbf{Z}_k)},\ i = 1, \ldots, L_{k-1}$$

where L_{k-1} is the number of particles from the previous step. If the particle is due to a newly appeared target, then the prediction process can be updated using

$$\mathbf{x}_k^{(i)}, \rho_k^{(i)} \sim p_k(\cdot|\mathbf{Z}_k),\ i = L_{k-1}+1, \ldots, L_{k-1}+J_k$$
$$w_{k|k-1}^{(i)} = \frac{\lambda^{(\text{birth})}(\mathbf{x}_k^{(i)}|\mathbf{Z}_k)}{J_k\, p_k(\mathbf{x}_k^{(i)}|\mathbf{Z}_k)},\ i = L_{k-1}+1, \ldots, L_{k-1}+J_k,$$

where J_k is the number of particles needed to represent the new birth target RFS and $\phi_{k|k-1}(\mathbf{x}_k^{(i)}, \mathbf{x}_{k-1}^{(i)}) = \mathrm{P}_{k|k-1}^{(\text{surv})}(\mathbf{x}_{k-1}^{(i)})\, p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{(i)}) + \lambda^{(\text{sp})}(\mathbf{x}_k^{(i)}|\mathbf{Z}_{k-1})$.

When the radar observes all the range-range rate pairs, the proposed filter updates the particle weights as

$$w_{k|k-1}^{(i)} = \left[1 - \mathrm{P}_k^{(\text{det})}(\mathbf{x}_k^{(i)}) + \sum_{\mathbf{z}_k \in \mathbf{Z}_k} \frac{\psi_{k,\mathbf{z}_k}(\mathbf{x}_k^{(i)}, \rho_k^{(i)})}{\lambda^{(\text{clt})}(\mathbf{z}_k) + C_k(\mathbf{z}_k)}\right] w_{k|k-1}^{(i)}$$

where $C_k(\mathbf{z}_k) = \sum_{j=1}^{L_{k-1}+J_k} \psi_{k,\mathbf{z}_k}(\mathbf{x}_k^{(j)}, \rho_k^{(j)})\, w_{k|k-1}^{(j)}$ and $\psi_{k,\mathbf{z}_k}(\mathbf{x}_k^{(j)}, \rho_k^{(j)}) = \mathrm{P}_k^{(\text{det})}(\mathbf{x}_k^{(j)}) g_k(\mathbf{z}_k|\mathbf{x}_k^{(j)}, \rho_k^{(j)})$. The likelihood $g_k(\mathbf{z}_k|\mathbf{x}_k^{(j)}, \rho_k^{(j)})$ is calculated by first generating another prediction RFS according to the jth particle $\mathbf{x}_k^{(j)}$ and its path index parameter $\rho_k^{(j)}$, whose elements are all the possible range-range rate pairs corresponding to particle $\mathbf{x}_k^{(j)}$. Then, the likelihood of each element in the generated prediction RFS is calculated individually and the largest one is selected. From the particle representation of the posterior intensity after resampling, the states of the individual targets are estimated using a clustering algorithm such as k-means. The average error distance is then calculated and compared to a threshold. Finally, the number of targets is equal to the number of clusters, and the estimated target states correspond to the centroid of each cluster.

5. SIMULATIONS

In order to demonstrate the performance of the proposed algorithm, we provide simulations for tracking multiple targets in urban terrain. Our numerical simulations are based on a 3-D environment, consisting of three buildings, an airborne radar, 1.4 km in height, located about 8 km southeast of the scene. The targets considered in the simulations are ground vehicles moving in 2-D. Each target can switch between a constant velocity or coordinated turn model with angular turning rate $\omega = -2$. The process noise intensity coefficient was chosen to be 0.04 and the probability of target survival is 0.95. The PPHDF and PPHDF-MMA algorithms used 2,000 particles for each target. The clutter RFS is considered to be Poisson, and each clutter is assumed to be uniformly distributed in the region $[-50, 150] \times [-100, 50]$. The clutter density is assumed to be 3.33×10^{-4}, resulting in an average rate of 10 points per scan. We consider the results of two different simulations, both based in the urban scene depicted in Fig. 2 that

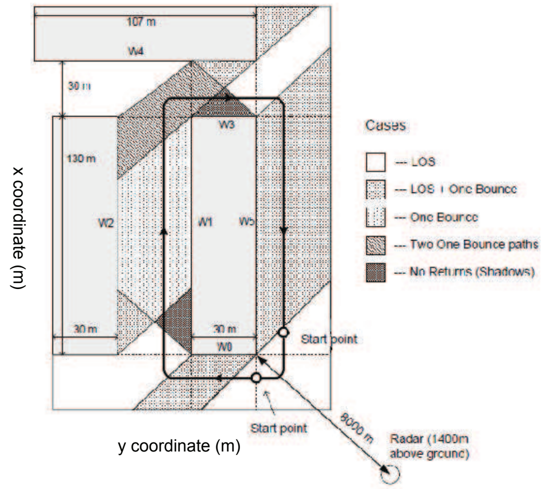


Fig. 2. Measurement map of simulated urban terrain [2].

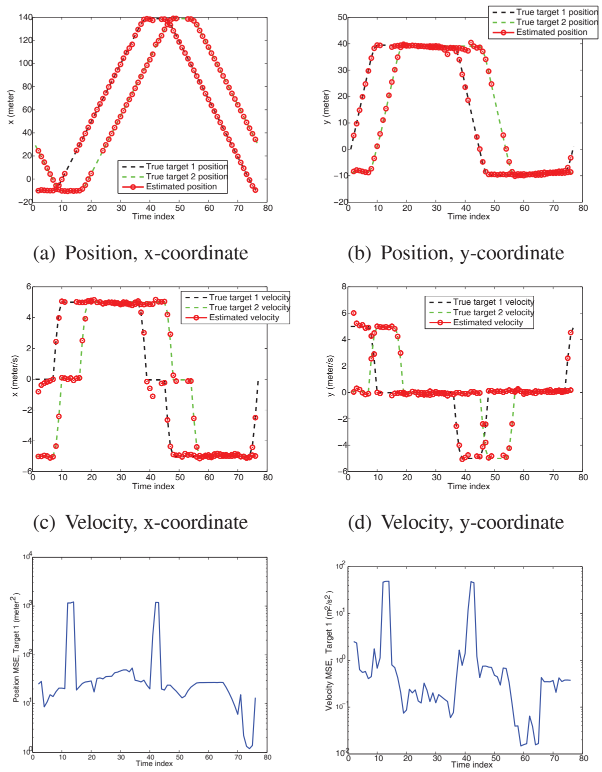
includes LOS regions, one-bounce regions, two one-bounce regions, and shadow regions [2].

Case 1: Two targets moving in the same direction. We assume that that there are two ground vehicles, whose loop trajectory and starting points are marked in the measurement map in Figure 2. In this example, we assumed that there is no spawning and that no new targets appear. The simulations results, shown in Figure 3, demonstrate the estimation performance and mean-squared error (MSE) of the proposed PPHDF-MMA algorithm based on 300 Monte Carlo simulations. Note that, as expected, due to no signal returns, the target track is lost when a target enters the shadowing region. The MSE error is small when the target is in the LOS region or in the LOS plus one-bounce region; the error increases when the target is in the one-bounce or two one-bounce regions.

Case 2: Time varying number of targets moving in different directions. Figure 4 shows the measurement map and true trajectory that initially assumed that there are 2 targets, moving in the same direction. Then, at time step k=5, a new target appears and moves in the opposite direction of the two targets; at k=12, a target leaves the scene and at k=15 another target leaves the scene. No spawning is assumed; new born targets are assumed to have Gaussian distributions. The simulations results, based on 100 Monte Carlo simulations, are shown in Figure 5. From Figure 5(d), other than in the shadow region, the proposed algorithm can accurately estimate the time-varying number of targets.

6. CONCLUSIONS

We demonstrated the applicability of the PPHDF for tracking multiple targets in urban environments. We proposed a modification to the algorithm to address the realistic scenario of associating measurements with different paths at each time step in the complex urban scene. Using simulations, we demonstrated that the resulting algorithm can adaptively estimate the type of measurement at each time step so that we can track



(a) Position, x-coordinate (b) Position, y-coordinate

(c) Velocity, x-coordinate (d) Velocity, y-coordinate

(e) Position MSE, Target 1 (f) Velocity MSE, Target 1

Fig. 3. (a)-(d) PPHDF-MMA estimated positions and velocities; Target 1 (e) position and (f) velocity MSE for Case 1.

both the number of targets as well as their parameters.

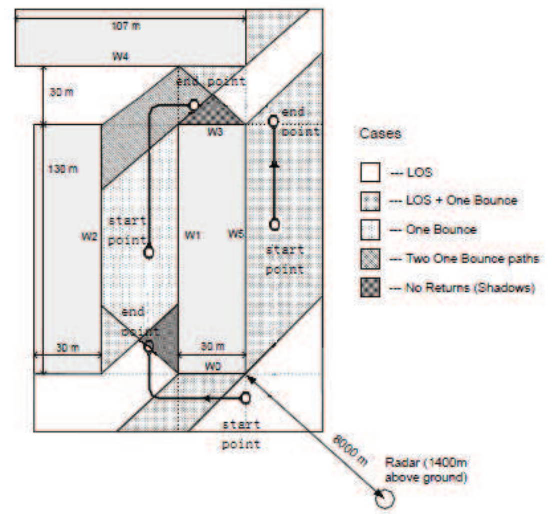
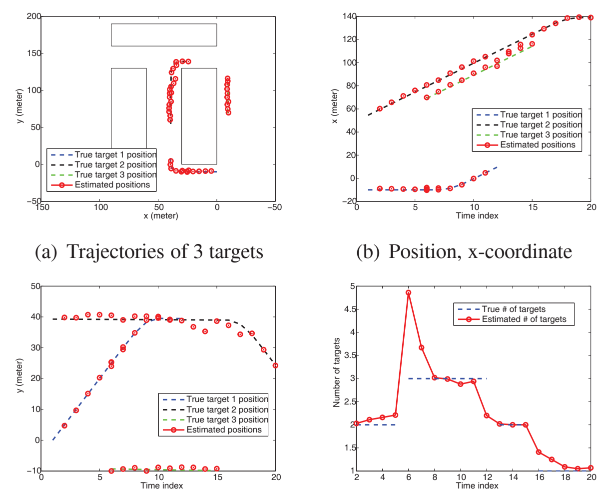


Fig. 4. Measurement map of urban terrain in Case 2.



(a) Trajectories of 3 targets (b) Position, x-coordinate

(c) Position, y-coordinate (d) Estimated number of targets

Fig. 5. PPHDF-MMA estimated (a) trajectory; (b), (c) position and (d) varying number of targets for Case 2.

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